3 A curve, defined for $x > 0$, has equation $y = \frac{8}{x^3} + 3x$.

(a) Find $\frac{dy}{dx}$.

[3 marks]

(b) Find an equation of the normal to the curve at the point $(2, 7)$.

[3 marks]

2 A curve is defined for $x > 0$ by the equation

$$y = 3x + x^{\frac{3}{2}} - 7$$

(a) Find $\frac{dy}{dx}$.

[2 marks]

(b) The point $P(4, k)$ lies on the curve.

(i) Find the value of the integer k.

[1 mark]

(ii) Find an equation of the normal to the curve at the point P.

[3 marks]

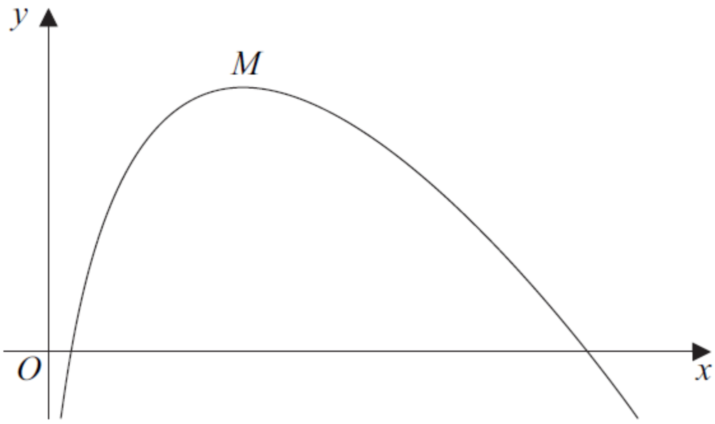
(iii) The normal to the curve at P intersects the x-axis at the point Q. Find the x-coordinate of Q.

[2 marks]

Q	Solution	Mark	Total	Comment
3(a)	$\frac{8}{x^3} = 8x^{-3}$	B1		PI by its derivative as $24x^{-4}$ or $-24x^{-4}$
	$\frac{dy}{dx} = -24x^{-4} + 3$	B2, 1	3	ACF. If not **B2**, award **B1** for either $-24x^{-4}$ OE or for $kx^{-4} + 3$, $k \neq 0, 32$
(b)	At $P(2, 7)$ $\frac{dy}{dx} = -24(2)^{-4} + 3 \quad (= 1.5)$	M1		Attempt to find c's $\frac{dy}{dx}$ when $x = 2$.
	Gradient of normal = $-\frac{2}{3}$	dM1		$m \times m' = -1$ used
	Eqn of normal $y - 7 = -\frac{2}{3}(x - 2)$	A1	3	ACF eg $y = -\frac{2}{3}x + \frac{25}{3}$
	Total		6	

Q	Solution	Mark	Total	Comment
2(a)	$\frac{dy}{dx} = 3 + \frac{3}{2}x^{\frac{1}{2}}$	B2,1	2	ACF. If not **B2**, award **B1** for correct differentiation of either $x^{3/2}$ or $3x - 7$
(b)(i)	$(k =)$ 13	B1	1	13
(ii)	At $P(4,k)$ $\frac{dy}{dx} = 3 + \frac{3}{2}(4)^{0.5} \quad (= 6)$	M1		Attempt to find c's $\frac{dy}{dx}$ when $x = 4$. M0 if c's answer (a) is a constant
	Gradient of normal = $-\frac{1}{6}$	dM1		$m \times m' = -1$ used
	Eqn of normal $y - 13 = -\frac{1}{6}(x - 4)$	A1F	3	ACF **only** ft on c's non-zero value of k ie check c's equation is equivalent to $6y+x=4+6k$, for c's non-zero value of k
(iii)	When $y = 0$, $0 - 13 = -\frac{1}{6}(x - 4)$	M1		Attempts to find x when y=0 in c's **linear** equation answer to (b)(ii)
	$x = 82$	A1	2	82 82 with or without working scores 2/2
	Total		8	

3 The diagram shows a curve with a maximum point M.



The curve is defined for $x > 0$ by the equation

$$y = 6x^{\frac{1}{2}} - x - 3$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

(b) **Hence** find the y-coordinate of the maximum point M.

[3 marks]

(c) Find an equation of the normal to the curve at the point $P(4,\ 5)$.

[3 marks]

(d) It is given that the normal to the curve at P, when translated by the vector $\begin{bmatrix} k \\ 0 \end{bmatrix}$, passes through the point M. Find the value of the constant k.

[3 marks]

8 The point A lies on the curve with equation $y = x^{\frac{1}{2}}$. The tangent to this curve at A is parallel to the line $3y - 2x = 1$. Find an equation of this tangent at A.

[5 marks]

Q3	Solution	Mark	Total	Comment
(a)	$\left(\frac{dy}{dx}=\right)\ \frac{6}{2}x^{-0.5}-1\ =3x^{-0.5}-1$	B2,1	2	ACF. If not B2, award B1 for correct differentiation of either $6x^{1/2}$ or $-x-3$
(b)	$3x^{-0.5}-1=0$	M1		Evidence of c's $\frac{dy}{dx}$ equated to 0 to form an equation in x.
	$3x^{-0.5}=1,\quad x=9$	A1F		Only ft if c's $\frac{dy}{dx}=ax^{-0.5}-1$ ie $x_M=a^2$
	(y-coordinate of M is) 6	A1	3	NMS scores 0/3
(c)	At $P(4,5)$ $\frac{dy}{dx}=3(4)^{-0.5}-1$ (=0.5)	M1		Attempt to find c's $\frac{dy}{dx}$ when $x=4$.
	Gradient of normal = -2	m1		$m\times m'=-1$ used
	Eqn of normal $y-5=-2(x-4)$	A1	3	ACF eg $y+2x=13$
(d)	Translated normal: $y-5=-2(x-k-4)$	M1		Either $x\to x-k$ or $x\to x+k$ with no change to y in cand's eqn of normal seen or used
	Passes through M (9, 6) so $6-5=-2(9-k-4)$	m1		Subst of c's M coordinates (both +'ve) into cand's eqn of normal with $x\to x-k$ and no change to y.
	$k=5.5$	A1	3	A correct value of k with no errors seen.

Q8	Solution	Mark	Total	Comment
	Gradient of the line $3y-2x=1$ is $\frac{2}{3}$	B1		(Gradient) $\frac{2}{3}$ seen or used. Condone 0.66, 0.67 or better for $\frac{2}{3}$.
	$\frac{dy}{dx}=\frac{1}{2}x^{-0.5}$	B1		Correct differentiation of $x^{\frac{1}{2}}$
	At A, $\frac{1}{2}x^{-0.5}=\frac{2}{3}$	M1		c's $\frac{dy}{dx}$ expression = c's numerical gradient of given line.
	$A\left(\frac{9}{16},\frac{3}{4}\right)$	A1		Correct exact coordinates of A
	Eqn of tang at A: $y-\frac{3}{4}=\frac{2}{3}\left(x-\frac{9}{16}\right)$	A1	5	ACF eg $y=\frac{2}{3}x+\frac{3}{8}$ or eg $3y-2x=\frac{9}{8}$ must be exact
	Total		5	

3 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 18t^2 + 54t, \qquad 0 \leqslant t \leqslant 5$$

(a) Find:

(i) $\frac{\mathrm{d}x}{\mathrm{d}t}$;

[2 marks]

(ii) $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$.

[1 mark]

(b) Verify that x has a stationary value when $t = 3$, and determine, with a reason, whether this stationary value is a maximum value or a minimum value.

[4 marks]

(c) Find the rate of change of x with respect to t in cm per second when $t = 1$.

[2 marks]

(d) Determine, with a reason, whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$.

[2 marks]

Q3	Solution	Mark	Total	Comment
(a)(i)	$\left(\frac{dx}{dt}=\right)\ \frac{4t^3}{2}-36t+54$	M1 A1	2	two terms correct all correct
(ii)	$\left(\frac{d^2x}{dt^2}=\right)\ 6t^2-36$	B1 ft	1	**FT** their $\frac{dx}{dt}$
(b)	$\frac{4\times 27}{2}-36\times 3+54$	M1		sub $t=3$ into "their" $\frac{dx}{dt}$
	$=54-108+54=0$ so stationary	A1		shown $=0$ plus statement
	$\left(\text{when } t=3, \frac{d^2x}{dt^2}\right)=(54-36)=18$	M1		sub $t=3$ into their $\frac{d^2x}{dt^2}$ and evaluate correctly
	$18>0$ therefore **minimum** (point)	A1	4	with **correct** reason such as $\frac{d^2x}{dt^2}>0$
(c)	$\left(\frac{dx}{dt}=\right)\ 2\times 1^3-36\times 1+54$	M1		Sub $t=1$ into "their" $\frac{dx}{dt}$
	$=20$	A1	2	
(d)	$\left(\frac{dx}{dt}=\right)\ 2\times 2^3-36\times 2+54=-2$	M1		Sub $t=2$ into "their" $\frac{dx}{dt}$
	$-2<0$ therefore decreasing	E1	2	must have reason (with no incorrect notation) and all working correct
	Total		**11**	

2 A curve has equation $y = 20x - x^2 - 2x^3$. The curve has a stationary point at the point M where $x = -2$.

(a) Find the x-coordinate of the other stationary point of the curve.

[4 marks]

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point M, and hence determine, with a reason, whether M is a minimum point or a maximum point.

[3 marks]

(c) Sketch the curve.

[2 marks]

8 The water level in a reservoir rises and falls during a four-hour period of heavy rainfall. The height, h cm, of water above its normal level, t hours after it starts to rain, can be modelled by the equation

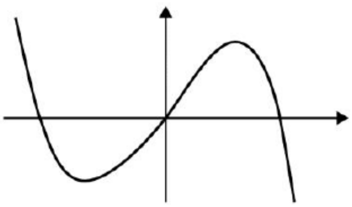
$$h = 4t^3 - \frac{59}{2}t^2 + 72t, \quad 0 \leqslant t \leqslant 4$$

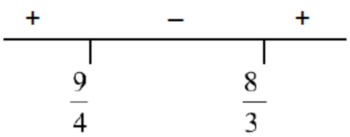
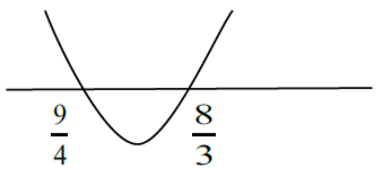
(a) Find the rate of change of the height of water, in cm per hour, 3 hours after it starts to rain.

[4 marks]

(b) Find the values of t for which the height of the water is decreasing.

[5 marks]

Q2	Solution	Mark	Total	Comment
(a)	$\left(\frac{dy}{dx}=\right)\quad 20-2x-6x^2$	M1 A1		two terms correct all correct
	$(10-6x)(2+x)\quad (=0)$ **OE**	A1		correct factors or correct use of formula for correct quadratic possibly multiplied by -1 or divided by ± 2
	(other stationary point when) $x=\frac{5}{3}$	A1cso	4	**OE eg** $\frac{20}{12}$, $1\frac{2}{3}$, $1.\dot{6}$ but not $\frac{-20}{-12}$
(b)	$\left(\frac{d^2y}{dx^2}=\right)\quad -2-12x$	B1		
	$\left(\text{when } x=-2, \frac{d^2y}{dx^2}\right)=(-2+24)=22$	B1ft		Sub $x=-2$ into their $\frac{d^2y}{dx^2}$ and evaluate correctly
	$22>0$ therefore **minimum** (point)	E1ft	3	**FT** their value of $\frac{d^2y}{dx^2}$ but must have reason
(c)	Cubic graph **through origin** with one max & one min on either side of y-axis	M1		may be reflection of given graph in x-axis for **M1**
		A1	2	Graph roughly as shown in all 4 quadrants

Q8	Solution	Mark	Total	Comment
(a)	$\left(\frac{dh}{dt}=\right)\quad 12t^2-59t+72$ **OE**	M1 A1		two terms correct (may have x for t) all correct – must have t
	$t=3\Rightarrow\left(\frac{dh}{dt}=\right)12\times 3^2-59\times 3+72$	dM1		substituting $t=3$ into their $\frac{dh}{dt}$
	(108 – 177 + 72 =) 3	A1	4	
(b)	(Decreasing) $\Rightarrow 12t^2-59t+72<0$	B1ft		**FT** their $\frac{dh}{dt}$ but must have "<0"
	$(4t-9)(3t-8)$	M1		attempt at factors or correct use of formula
	CVs are $(t=)\frac{9}{4}$, $(t=)\frac{8}{3}$	A1		
		M1		use of sign diagram or sketch 
	$\frac{9}{4}<t<\frac{8}{3}$	A1	5	fractions must be simplified & **B1** earned for final **A1**
	may have $t<\frac{8}{3}$ **AND** $t>\frac{9}{4}$			no **ISW** here

8 The gradient, $\frac{dy}{dx}$, at the point (x, y) on a curve is given by

$$\frac{dy}{dx} = 54 + 27x - 6x^2$$

(a) (i) Find $\frac{d^2y}{dx^2}$.

[2 marks]

(ii) The curve passes through the point $P\left(-1\frac{1}{2}, 4\right)$.

Verify that the curve has a minimum point at P.

[4 marks]

(b) (i) Show that at the points on the curve where y is decreasing

$$2x^2 - 9x - 18 > 0$$

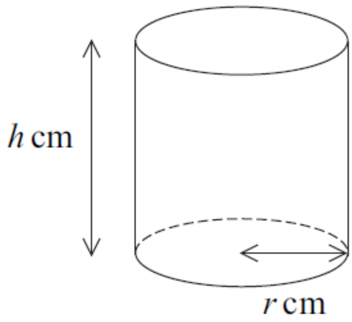
[2 marks]

(ii) Solve the inequality $2x^2 - 9x - 18 > 0$.

[4 marks]

Q7	Solution	Mark	Total	Comment
(a)(i)	$\left(\frac{dy}{dx}=\right)\quad -2x-9x^2$	M1 A1		one term correct all correct (no +c etc)
	when $x=-2$, $\frac{dy}{dx}=(4-36=)-32$	A1		
	$y="their -32"x+c$ & attempt to find c using $x=-2$ and $y=24$	m1		or $y-24="their -32"(x--2)$
	$y=-32x-40$	A1	5	must write in this form; no **ISW** here
(ii)	$y=0\Rightarrow x=-\frac{5}{4}$ **OE**	B1F	1	**strict FT** from their answer to **(a)(i)**
(b)(i)	$4x-\frac{x^3}{3}-\frac{3x^4}{4}(+c)$	M1 A1		two terms correct all correct
	$\left[4\times1-\frac{1^3}{3}-\frac{3\times1^4}{4}\right]-\left[4\times(-2)-\frac{(-2)^3}{3}-\frac{3(-2)^4}{4}\right]$	m1		"their" F(1) – F(–2)
	$\left[4-\frac{1}{3}-\frac{3}{4}\right]-\left[-8+\frac{8}{3}-\frac{48}{4}\right]$	A1		**correct** with powers of 1 and (–2) and minus signs handled correctly
	$=20\frac{1}{4}$	A1	5	20.25 , $\frac{81}{4}$, $\frac{243}{12}$ **OE**
(ii)	Area of missing triangle $=\left(\frac{1}{2}\times24\times\frac{3}{4}=\right)9$	B1		or correct **single** equivalent fraction
	Area of region = "their"**(b)(i)** –"their" Δ	M1		"their" $(20\frac{1}{4}-9)$
	$=11\frac{1}{4}$	A1	3	11.25 , $\frac{45}{4}$, $\frac{135}{12}$ **OE**
	Total		**14**	

6 The diagram shows a cylindrical container of radius r cm and height h cm. The container has an **open** top and a circular base.



The **external** surface area of the container's curved surface and base is $48\pi\ \text{cm}^2$.

When the radius of the base is r cm, the volume of the container is $V\ \text{cm}^3$.

(a) (i) Find an expression for h in terms of r.

[3 marks]

(ii) Show that $V = 24\pi r - \dfrac{\pi}{2}r^3$.

[2 marks]

(b) (i) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$.

[2 marks]

(ii) Find the positive value of r for which V is stationary, and determine whether this stationary value is a maximum value or a minimum value.

[4 marks]

Q6	Solution	Mark	Total	Comment
(a)(i)	$(SA =)\ \pi r^2 + 2\pi rh$	B1		correct surface area
	$\pi r^2 + 2\pi rh = 48\pi$ $\Rightarrow 2rh = 48 - r^2 \quad \Rightarrow h = \ldots$	M1		equating *"their" SA* to 48π **and** attempt at $h =$
	$h = \dfrac{48 - r^2}{2r}$	A1	3	**or** $h = \dfrac{24}{r} - \dfrac{r}{2}$ **OE**
(ii)	$V = \pi r^2 h = \ldots$ $= \pi f(r)$	M1		correct volume expression & elimination of h using "their" **(a)(i)**
	$V = \pi r^2 \left(\dfrac{48 - r^2}{2r} \right) = 24\pi r - \dfrac{\pi}{2} r^3$	A1	2	**AG** (*be convinced)*
(b)(i)	$\left(\dfrac{dV}{dr} = \right)\ 24\pi - \dfrac{3}{2}\pi r^2$	M1 A1	2	one term correct all correct, must simplify r^0
(ii)	$24\pi - \dfrac{3}{2}\pi r^2 = 0 \Rightarrow r^2 = \dfrac{48\pi}{3\pi}$	M1		"their" $\dfrac{dV}{dr} = 0$ and attempt at $r^n = \ldots$
	$r = 4$	A1		from correct $\dfrac{dV}{dr}$
	$\dfrac{d^2V}{dr^2} = -\dfrac{6\pi r}{2}$	B1√		FT "their" $\dfrac{dV}{dr}$
	$\dfrac{d^2V}{dr^2} < 0$ when $r = 4 \Rightarrow$ Maximum	A1cso	4	explained convincingly, all working and notation correct
	Total		**11**	

3 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = \ln(3x + 4)$.

[4 marks]

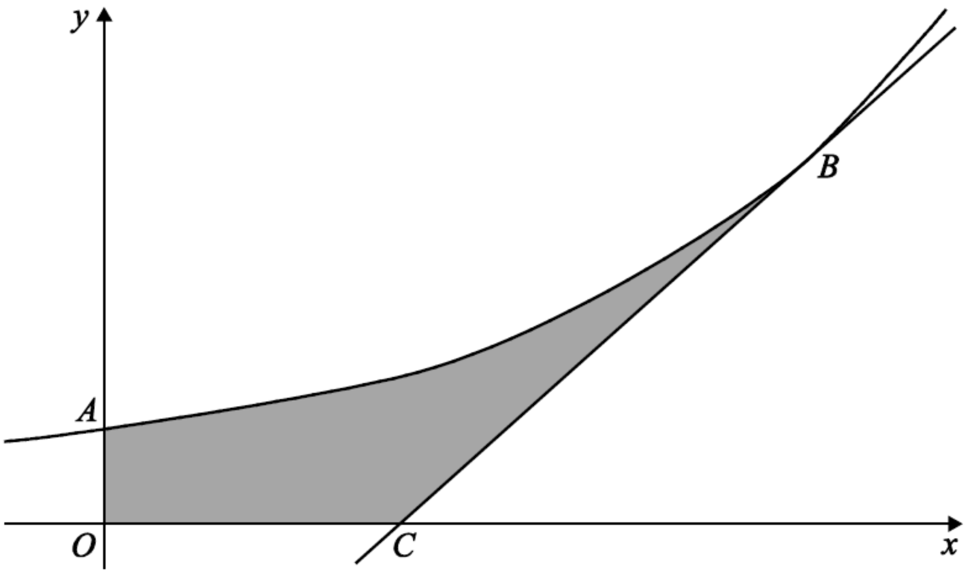
(b) The **normal** to the curve $y = \ln(3x + 4)$ at the point $P(2,\ \ln 10)$ intersects the x-axis at the point A.

Find the exact value of the x-coordinate of A.

[4 marks]

Q3	Solution	Mark	Total	Comment
(a)	Stretch I Parallel to x-axis II SF 1/3 III then	M1 A1		I and II or III (or line $y = 0$) I + II + III
	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	B1		
	$k = -4/3$	B1		
	OR			
	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	(B1)		
	$k = -4$	(B1)		
	then Stretch I Parallel to x-axis II SF 1/3 III	(M1) (A1)		As above
	OR (either order) Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	(B1)		
	$k = -4/3$	(B1)		
	Translation $\begin{bmatrix} 0 \\ k \end{bmatrix}$	(M1)		
	$k = \ln 3$	(A1)		
			4	
(b)	$\frac{dy}{dx} = \frac{3}{3x+4}$ oe	M1		
	At $x = 2$, Gradient $= 0.3$	A1		
	Equation of normal: $y - \ln 10 = -\frac{10}{3}(x - 2)$	M1		Allow -1/(their grad)
	Intersects x – axis: $x = 0.3\ln 10 + 2$ **ACF**	A1	4	

10 The diagram shows the curve $y = e^{2x}$, intersecting the y-axis at the point A, and the tangent to this curve at the point B, where $x = \ln 4$, intersecting the x-axis at the point C.



(a) (i) Find an equation of the tangent to the curve at B.

[3 marks]

(ii) Hence show that the coordinates of C are $\left(\ln 4 - \frac{1}{2},\ 0\right)$.

[1 mark]

Q10	Solution	Mark	Total	Comment
(a)(i)	$x=\ln 4,\quad y=e^{2\ln 4}$ $y=\left(e^{\ln 16}=\right)16$	B1		
	$\frac{dy}{dx}=2e^{2x}$	M1		
	$y-16=32(x-\ln 4)$ OE	A1		With no exponentials
			3	
(ii)	$[y=0]\quad -\frac{16}{32}=x-\ln 4$ $x=\ln 4-\frac{1}{2}$ **or** $[x=\ln 4-0.5]$ $y-16=32(\ln 4-0.5-\ln 4)$ $y=32\times -0.5+16=0$	B1		Must see this line oe **AG** All correct and no errors seen. Must be using a correct equation from **(i)**, (condone $e^{2\ln 4}$ unsimplified)
			1	

4 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{2x-5}$.

[4 marks]

(b) The **normal** to the curve $y = e^{2x-5}$ at the point $P(2,\ e^{-1})$ intersects the x-axis at the point A and the y-axis at the point B.

Show that the area of the triangle OAB is $\dfrac{(e^2+1)^m}{e^n}$, where m and n are integers.

[6 marks]

Q4	Solution	Mark	Total	Comment
a	Stretch I [Parallel to] x[-axis] II (or line $y = 0$) [SF] 0.5 III	M1 A1		I and II or III I + II + III
	then Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	M1		**or** (2nd) Stretch [parallel to] y[-axis]
	$\begin{bmatrix} 2.5 \\ 0 \end{bmatrix}$	A1		SF e^{-5} (for the '2 stretch' method, if the 'y' direction stretch is first, marks **can only** be earned if there is a **second** stretch in 'x' direction. The stretches can be in either order)
	OR Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	(M1)		
	$\begin{bmatrix} 5 \\ 0 \end{bmatrix}$	(A1)		
	then Stretch I [Parallel to] x[-axis] II [SF] 0.5 III	(M1) (A1)	4	I and II or III I + II + III
b	$\frac{dy}{dx} = 2e^{2x-5}$	B1		
	Grad normal $= -\frac{1}{\text{their gradient}}$	B1F		Condone expression in terms of x
	(equation normal) $y - e^{-1} = -\frac{e}{2}(x-2)$ oe	B1		Must be exact values
	(At A $y = 0$) $x = 2 + \frac{2}{e^2}$ oe	M1		Attempt to find at least one intercept from 'their' normal, subst $x = 0$ **or** $y = 0$ in any straight line equation
	(At B $x = 0$) $y = e + \frac{1}{e} = \frac{e^2+1}{e}$ oe	A1		Both x **and** y values correct
	$\left(\text{(Area =)}\ 0.5 \times \frac{(e^2+1)}{e} \times \frac{2(1+e^2)}{e^2}\right)$ $= \frac{(e^2+1)^2}{e^3}$	A1	6	
	Total		10	

(b) A curve has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 6\ln x + x^2 - 8x + 3$.

(i) Find the exact values of the coordinates of the stationary points of the curve.

[5 marks]

(ii) Hence, or otherwise, find the exact values of the coordinates of the stationary points of the curve with equation

$$y = 2\mathrm{f}(x-4)$$

[2 marks]

2. It is given that

$$y = 15x + 108x^{\frac{1}{2}} + 4x^{\frac{5}{2}} \qquad x > 0$$

Find, in simplest form,

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

(2)

(c) Find the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = 9$

(1)

bi	$\frac{dy}{dx} = \frac{6}{x} + 2x - 8$	**B1**		Condone $\frac{6x^5}{x^6}$
	$(\frac{dy}{dx} = 0)\ 6 + 2x^2 - 8x = 0$	**M1**		Equate to zero (PI) and eliminate their fraction correctly.
	$x = 1,\quad x = 3$	**A1**		
	$(x = 1),\quad y = -4$	**A1**		
	$(x = 3),\quad y = 6\ln 3 - 12$ **or** $\ln 729 - 12$	**A1**	5	Oe for other exact correct values If **M0** then SC1 for (1, -4) and/or $(3, 6\ln 3 - 12)$
ii	$x = 5,\quad y = -8$	**M1**		their x + 4 and $2 \times$ their y on either of their 'pairs'
	$x = 7,\quad y = 12\ln 3 - 24$	**A1**	2	All correct : oe exact

Question Number	Scheme	Notes	Marks
2.(a)		For $15x \rightarrow 15$ or $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ or $x^{\frac{5}{2}} \rightarrow x^{\frac{3}{2}}$	M1
		Any 2 correct terms, may be simplified or unsimplified.	A1
	$\left(\frac{dy}{dx} =\right) 15 + 54x^{-\frac{1}{2}} + 10x^{\frac{3}{2}}$	All correct and simplified on one line. Allow equivalent forms for the powers of x e.g. $\frac{1}{x^{\frac{1}{2}}}, \frac{1}{\sqrt{x}}$ for $x^{-\frac{1}{2}}$ $\sqrt{x^3}, x\sqrt{x}$ for $x^{\frac{3}{2}}$	A1
			[3]
(b)		For $x^n \rightarrow x^{n-1}$ on one of their terms from (a) but **not** for $k \rightarrow 0$	M1
	$\left(\frac{d^2y}{dx^2} =\right) -27x^{-\frac{3}{2}} + 15x^{\frac{1}{2}}$	For a correct simplified answer on one line or for $Ax^{-\frac{3}{2}} + Bx^{\frac{1}{2}}$ where A and B are simplified and follow their "54" and "10" from (a) all on one line.	A1ft
			[2]
	Penalise the occurrence of "+ c" in (a) or (b) only once and penalise it the first time it occurs.		
(c)	When $x = 9$, $\left(\frac{d^2y}{dx^2} =\right) -1 + 45 = 44$	44 only	B1
			[1]
			6 marks

9.

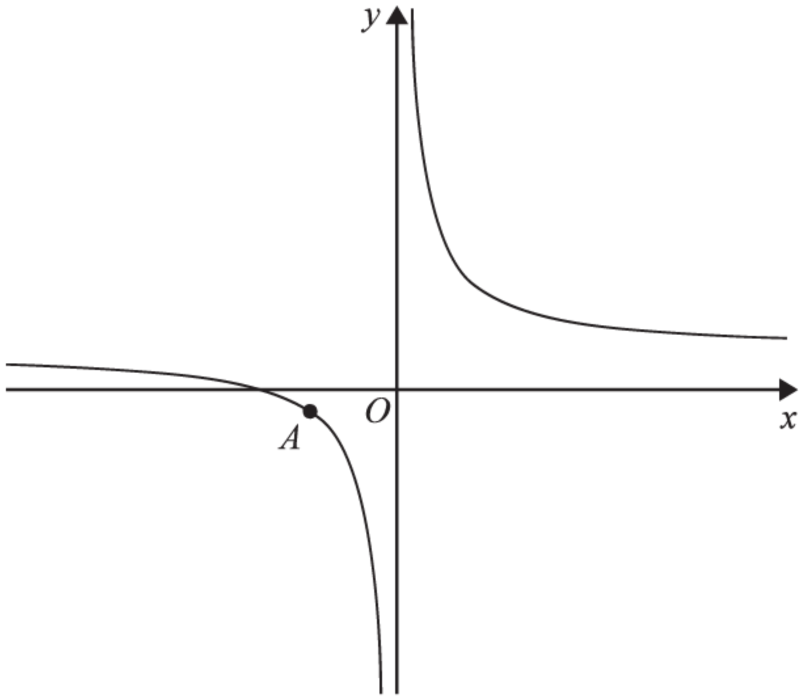


Figure 1

Figure 1 shows a sketch of part of the curve H with equation

$$y = \frac{12}{x} + 5 \quad x \neq 0$$

(a) Find an equation for the normal to H at the point A (–2, –1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

The points B and C also lie on the curve H.
The normal to H at the point B and the normal to H at the point C are each parallel to the straight line with equation $4y = 3x + 5$

(b) Find the coordinates of the points B and C, given that the x coordinate of B is positive.

(5)

Question Number	Scheme	Notes	Marks
9.(a)	$y=\frac{12}{x}+5 \Rightarrow \frac{dy}{dx}=-\frac{12}{x^2}$	$\frac{12}{x}\to\frac{k}{x^2}\left(\text{or } kx^{-2}\right)$	M1
	At $x=-2$ $\frac{dy}{dx}=-\frac{12}{4}$ or -3	Correct value (may be implied by later work)	A1
	Gradient of normal is $-1\div-\frac{12}{4}\left(=\frac{1}{3}\right)$	Correct application of the perpendicular gradient rule. May be implied by use of $-\frac{1}{12}$ as the normal gradient for those candidates who think the gradient is 12.	M1
	$y+1=\text{"}\frac{1}{3}\text{"}(x+2)$ **or** $y=\text{"}\frac{1}{3}\text{"}x+c$ **and** $-1=\text{"}\frac{1}{3}\text{"}(-2)+c\Rightarrow c=...$	A correct straight line method using their changed gradient and the point (–2, –1). This must follow use of calculus to find the gradient.	M1
	$3y-x+1=0$	Correct equation in the required form. (Allow any integer multiple)	A1
			[5]
(b)	Gradient of given line is $\frac{3}{4}$	May be implied by use of $-\frac{4}{3}$	B1
	$\frac{x^2}{12}=\frac{3}{4}\Rightarrow x=...$	Sets up a correct equation using what they think is the gradient of the given line and attempts to solve.	M1
	$x=\pm 3$	Both correct values required	A1
	$x=...\Rightarrow\frac{12}{x}+5=...$	Uses at least one x to find a value for y using $y=\frac{12}{x}+5$. **Dependent on the first method mark.**	**d**M1
	(3 , 9) and (–3, 1) or e.g. $x=3, y=9$ $x=-3, y=1$	Correct coordinates correctly paired	A1
			[5]
			10 marks

10.

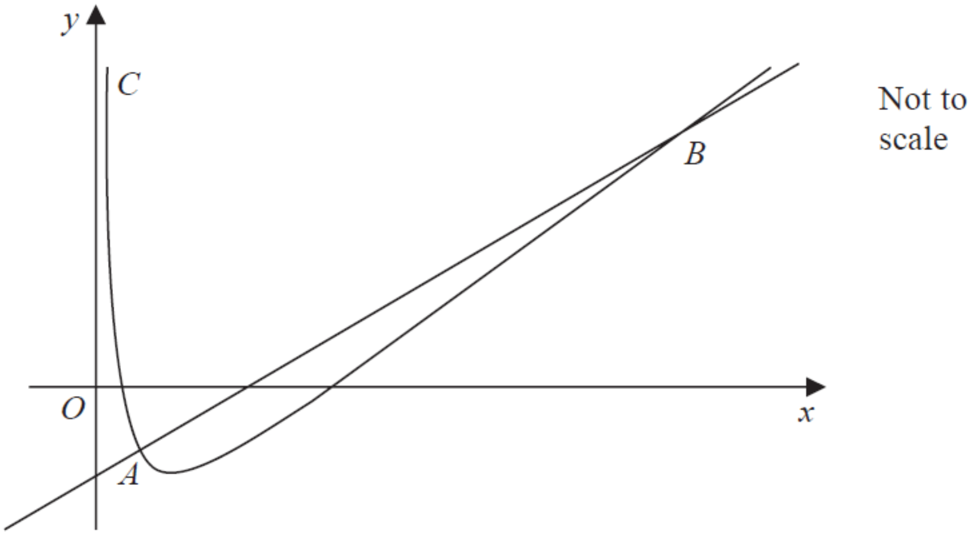


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2}x + \frac{27}{x} - 12, \qquad x > 0$$

The point A lies on C and has coordinates $\left(3, -\frac{3}{2}\right)$.

(a) Show that the equation of the normal to C at A can be written as $10y = 4x - 27$

(5)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

Question Number	Scheme		Marks
10(a)	$\frac{dy}{dx}=\frac{1}{2}-\frac{27}{x^2}$	M1: $\frac{1}{2}$ or $-\frac{27}{x^2}$ A1: $\frac{dy}{dx}=\frac{1}{2}-\frac{27}{x^2}$ oe e.g. $\frac{1}{2}x^0-27x^{-2}$	M1A1
	$x=3\Rightarrow\frac{dy}{dx}=\frac{1}{2}-\frac{27}{9}=\left(-\frac{5}{2}\right)$	Substitutes $x=3$ into their $\frac{dy}{dx}$ to obtain a numerical gradient	M1
	$m_T=-\frac{5}{2}\Rightarrow m_N=-1\div-\frac{5}{2}$ $\Rightarrow y-\left(-\frac{3}{2}\right)=\frac{2}{5}(x-3)$	The correct method to find the equation of a normal. Uses $-\frac{1}{m_T}$ with $\left(3,-\frac{3}{2}\right)$ where m_T has come from calculus. If using $y=mx+c$ must reach as far as $c=$ …	M1
	$10y=4x-27$*	Cso (correct equation must be seen in (a))	A1*
			(5)
(b)	$\frac{1}{2}x+\frac{27}{x}-12=\frac{4x-27}{10}$ or $y=\frac{10y+27}{8}+\frac{108}{10y+27}-12$	Equate equations to produce an equation just in x or just in y. Do not allow e.g. $\frac{1}{2}x^2+27-12x=\frac{4x-27}{10}$ Unless $\frac{1}{2}x+\frac{27}{x}-12=\frac{4x-27}{10}$ was seen previously. **Allow sign slips only.**	M1
	$x^2-93x+270=0$ or $20y^2-636y-999=0$	Correct 3 term quadratic equation (or any multiple of). Allow terms on both sides e.g. $x^2-93x=-270$ (The "= 0" may be implied by their attempt to solve)	A1
	$(x-90)(x-3)=0\Rightarrow x=\ldots$ or $x=\frac{93\pm\sqrt{93^2-4\times270}}{2}$ or $(10y-333)(2y+3)=0\Rightarrow y=\ldots$ or $y=\frac{636\pm\sqrt{636^2-4\times20\times(-999)}}{2\times20}$	Attempt to solve a 3TQ (see general guidance) leading to at least one for x or y. **Dependent on the first method mark.**	dM1
	$x=90$ **or** $y=33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
	$x=90$ **and** $y=33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
			(5)

2. Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(5)

Question Number	Scheme		Marks
2.	$y=\sqrt{x}+\frac{4}{\sqrt{x}}+4=x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+4$		
	$x^n \to x^{n-1}$	Decreases any power by 1. Either $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \to x^{-\frac{3}{2}}$ or $4 \to 0$ or $x^{\text{their } n} \to x^{\text{their } n-1}$ for fractional n.	M1
	$\left(\frac{dy}{dx}=\right)\frac{1}{2}x^{-\frac{1}{2}}+4\times-\frac{1}{2}x^{-\frac{3}{2}}$ $\left(=\frac{1}{2}x^{-\frac{1}{2}}-2x^{-\frac{3}{2}}\right)$	Correct derivative, simplified or un-simplified including indices. E.g. allow $\frac{1}{2}-1$ for $-\frac{1}{2}$ and allow $-\frac{1}{2}-1$ for $-\frac{3}{2}$	A1
	$x=8 \Rightarrow \frac{dy}{dx}=\frac{1}{2}8^{-\frac{1}{2}}+4\times-\frac{1}{2}8^{-\frac{3}{2}}$	Attempts to substitute $x = 8$ into their 'changed' (even integrated) expression that is clearly not y. If they attempt algebraic manipulation of their dy/dx before substitution, this mark is still available.	M1
	$=\frac{1}{2\sqrt{8}}-\frac{2}{\left(\sqrt{8}\right)^3}=\frac{1}{2\sqrt{8}}-\frac{2}{8\sqrt{8}}=\frac{1}{8\sqrt{2}}=\frac{1}{16}\sqrt{2}$	B1: $\sqrt{8}=2\sqrt{2}$ seen or implied anywhere, including from substituting $x = 8$ into y. May be seen explicitly or implied from e.g. $8^{\frac{3}{2}}=16\sqrt{2}$ or $8^{\frac{5}{2}}=128\sqrt{2}$ or $4\sqrt{8}=8\sqrt{2}$ A1: cso $\frac{1}{16}\sqrt{2}$ or $\frac{\sqrt{2}}{16}$ and allow rational equivalents for $\frac{1}{16}$ e.g. $\frac{32}{512}$ Apply isw so award this mark as soon as a correct answer is seen.	B1A1
			(5 marks)

10.

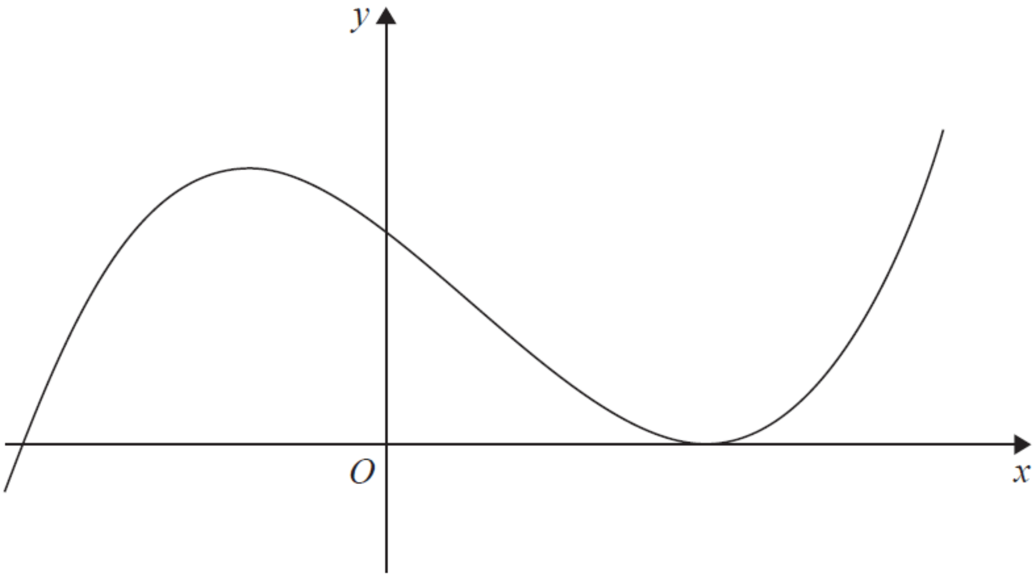


Figure 2

Figure 2 shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where

$$f(x) = (2x - 5)^2 (x + 3)$$

(a) Given that

(i) the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(b) Show that $f'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = f(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(c) find the x coordinate of point B.

(5)

10.(a)(i)	$k=(-5)^2\times3=75$	M1: Attempts to find the 'y' intercept. Accept as evidence $(-5)^2\times3$ with or without the bracket. If they expand f(x) to polynomial form here **then they must then select their constant to score this mark**. May be implied by sight of 75 on the diagram.	M1A1
		A1: $k=75$. Must clearly be identified as k. Allow this mark even from an incorrect or incomplete expansion as long as the constant $k=75$ is obtained. Do not isw e.g. if 75 is seen followed by $k=-75$ score M1A0.	
(ii)	$c=\frac{5}{2}$ only	$c=\frac{5}{2}$ oe (and no other values). Do not award just from the diagram – must be stated as the value of c.	B1
			(3)
(b)	$f(x)=(2x-5)^2(x+3)=(4x^2-20x+25)(x+3)=4x^3-8x^2-35x+75$ Attempts f(x) as a cubic polynomial by attempting to square the first bracket and multiply by the linear bracket or expands $(2x-5)(x+3)$ and then multiplies by $2x-5$ **Must be seen or used in (b) but may be done in part (a).** Allow poor squaring e.g. $(2x-5)^2=4x^2\pm25$		M1
	$(f'(x)=)12x^2-16x-35$ *	M1: Reduces powers by 1 in **all** terms including any constant $\to 0$	M1A1*
		A1: Correct proof. Withhold this mark if there have been any errors including missing brackets earlier e.g. $(2x-5)^2(x+3)=4x^2-20x+25(x+3)=...$	
			(3)
(c)	$f'(3)=12\times3^2-16\times3-35$	Substitutes $x=3$ into **their** $f'(x)$ **or the given** $f'(x)$. Must be a changed function i.e. not into f(x).	M1
	$12x^2-16x-35='25'$	Sets **their** $f'(x)$ **or the given** $f'(x)=$ their $f'(3)$ with a consistent f'. **Dependent on the previous method mark.**	**d**M1
	$12x^2-16x-60=0$	$12x^2-16x-60=0$ or equivalent 3 term quadratic e.g. $12x^2-16x=60$. (A correct quadratic equation may be implied by later work). **This is cso so must come from correct work – i.e. they must be using the <u>given</u>** $f'(x)$.	A1 cso
	$(x-3)(12x+20)=0\Rightarrow x=...$	Solves 3 term quadratic by suitable method – see General Principles. **Dependent on both previous method marks.**	**dd**M1
	$x=-\frac{5}{3}$	$x=-\frac{5}{3}$ oe clearly identified. If $x=3$ is also given and not rejected, this mark is withheld. (allow -1.6 recurring as long as it is clear i.e. a dot above the 6). **This is cso and must come from correct work – i.e. they must be using the <u>given</u>** $f'(x)$.	A1 cso
			(5)

7. Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0$$

find $\frac{dy}{dx}$. Give each term in your answer in its simplified form.

(6)

	$\frac{2x^3-7}{3\sqrt{x}} = \frac{2x^3}{3\sqrt{x}} - \frac{7}{3\sqrt{x}} = \frac{2}{3}x^{\frac{5}{2}} - \frac{7}{3}x^{-\frac{1}{2}}$	Attempts to split the fraction into 2 terms and obtains either $\alpha x^{\frac{5}{2}}$ or $\beta x^{-\frac{1}{2}}$. This may be implied by a correct power of x in their differentiation of one of these terms. But beware of $\beta x^{-\frac{1}{2}}$ coming from $\frac{2x^3-7}{3\sqrt{x}} = 2x^3 - 7 + 3x^{-\frac{1}{2}}$	M1
	$x^n \to x^{n-1}$	Differentiates by reducing power by one for any of their powers of x	M1
	$\left(\frac{dy}{dx} =\right) 6x + 2x^{-\frac{2}{3}} + \frac{5}{3}x^{\frac{3}{2}} + \frac{7}{6}x^{-\frac{3}{2}}$	A1: $6x$. Do not accept $6x^1$. Depends on second M mark only. Award when first seen and isw. A1: $2x^{-\frac{2}{3}}$. Must be simplified so do not accept e.g. $\frac{2}{1}x^{-\frac{2}{3}}$ but allow $\frac{2}{\sqrt[3]{x^2}}$. Depends on second M mark only. Award when first seen and isw. A1: $\frac{5}{3}x^{\frac{3}{2}}$. Must be simplified but allow e.g. $1\frac{2}{3}x^{1.5}$ or e.g. $\frac{5}{3}\sqrt{x^3}$. Award when first seen and isw. A1: $\frac{7}{6}x^{-\frac{3}{2}}$. Must be simplified but allow e.g. $1\frac{1}{6}x^{-1\frac{1}{2}}$ or e.g. $\frac{7}{6\sqrt{x^3}}$. Award when first seen and isw.	A1A1A1A1
	In an otherwise fully correct solution, penalise the presence of + c by deducting the final A1		
			[6]

11. The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of k,

(4)

(c) the value of the y coordinate of P,

(2)

(d) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

11. (a)	$y = 2x^3 + kx^2 + 5x + 6$		
	$\left(\frac{dy}{dx} = \right) 6x^2 + 2kx + 5$	M1: $x^n \to x^{n-1}$ for one of the terms including $6 \to 0$ A1: Correct derivative	M1 A1
			[2]
(b)	Gradient of given line is $\frac{17}{2}$	Uses or states $\frac{17}{2}$ or equivalent e.g. 8.5. Must be stated or used in (b) and not just seen as part of $y = \frac{17}{2}x + \frac{1}{2}$.	B1
	$\left(\frac{dy}{dx}\right)_{x=-2} = 6(-2)^2 + 2k(-2) + 5$	Substitutes $x = -2$ into their derivative **(not the curve)**	M1
	$"24 - 4k + 5" = "\frac{17}{2}" \Rightarrow k = \frac{41}{8}$	dM1: Puts their expression = their $\frac{17}{2}$ (Allow BOD for 17 or -17 but **not** the normal gradient) and solves to obtain a value for k. **Dependent on the previous method mark**. A1: $\frac{41}{8}$ or $5\frac{1}{8}$ or 5.125	dM1 A1
	Note: $6x^2 + 2kx + 5 = \frac{17}{2}x + \frac{1}{2}$ scores no marks on its own but may score the first M mark if they substitute x = -2 into the lhs. If they rearrange this equation and then substitute x = -2, this scores no marks.		
			[4]
(c)	$y = -16 + 4k - 10 + 6 = 4"k" - 20 = \frac{1}{2}$	M1: Substitutes $x = -2$ and their numerical k into $y = \ldots$ A1: $y = \frac{1}{2}$	M1 A1
	Allow the marks for part (c) to be scored in part (b).		
			[2]
(d)	$y - "\frac{1}{2}" = "\frac{17}{2}"(x - {}^{-}2) \Rightarrow -17x + 2y - 35 = 0$ or $y = "\frac{17}{2}"x + c \Rightarrow c = \ldots \Rightarrow -17x + 2y - 35 = 0$ or $2y - 17x = 1 + 34 \Rightarrow -17x + 2y - 35 = 0$	M1: Correct attempt at linear equation with their 8.5 gradient (not the normal gradient) using x = -2 and their $\frac{1}{2}$ A1: cao (allow any integer multiple)	M1 A1
			[2]

6. The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \quad x \neq 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

(5)

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

6(a)	$(x^2+4)(x-3)=x^3-3x^2+4x-12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
	$\frac{x^3-3x^2+4x-12}{2x}=\frac{x^2}{2}-\frac{3}{2}x+2-6x^{-1}$	M1: Attempt to divide each term by $2x$. The powers of x of at least two terms must follow from their expansion. Allow an attempt to multiply by $2x^{-1}$ A1: Correct expression. May be un-simplified but powers of x must be combined e.g. $\frac{x^2}{2}$ not $\frac{x^3}{2x}$	M1A1
	$\frac{dy}{dx}=x-\frac{3}{2}+\frac{6}{x^2}$ oe e.g. $\frac{2x^3-3x^2+12}{2x^2}$	ddM1: $x^n \to x^{n-1}$ or $2 \to 0$ **Dependent on both previous method marks.** A1: $x-\frac{3}{2}+\frac{6}{x^2}$ oe and **isw** Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0. If they lose the previous A1 because of an **incorrect constant only** then allow recovery here and in part (b) for a correct derivative.	ddM1A1
			(5)
(b)	At $x=-1,\ y=10$	Correct value for y	B1
	$\left(\frac{dy}{dx}=\right)-1-\frac{3}{2}+\frac{6}{1}=3.5$	M1: Substitutes $x=-1$ into their expression for dy/dx A1: 3.5 oe cso	M1A1
	$y-'10'='3.5'(x--1)$	Uses their **tangent** gradient **which must come from calculus** with $x=-1$ and their numerical y with a correct straight line method. If using $y=mx+c$, this mark is awarded for correctly establishing a value for c.	M1
	$2y-7x-27=0$	$\pm k(2y-7x-27)=0$ cso	A1
			(5)

10.

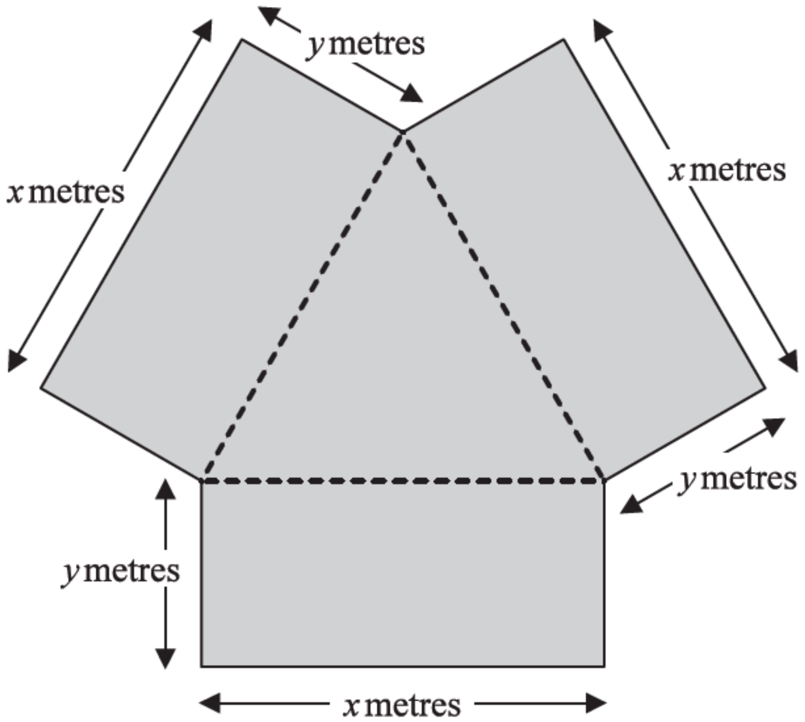


Figure 4

Figure 4 shows a plan view for a flower bed. Its shape is an equilateral triangle of side x metres with three congruent rectangles attached to the triangle along its sides. Each rectangle has length x metres and width y metres, as shown in Figure 4.

Given that the total area of the flower bed is $3\,\text{m}^2$ and that $0 < x < 2.632$ (3d.p.),

(a) show that the perimeter P metres, around the outside of the flower bed, is given by the equation

$$P = 3x + \frac{6}{x} - \frac{\sqrt{3}}{2}x$$

(6)

(b) Use calculus to find the minimum value of P, giving your answer to 3 significant figures.

(5)

(c) Justify, using calculus, that the value you have found in part (b) is a minimum value.

(2)

Question	Scheme	Marks
10(a)	$\{A =\}\ \frac{1}{2}x^2 \sin 60^\circ + 3xy$ or $\frac{1}{2}x\sqrt{x^2 - \frac{x^2}{4}} + 3xy$	M1 A1
	$3 = \frac{\sqrt{3}x^2}{4} + 3xy \Rightarrow y = \frac{1}{x} - \frac{\sqrt{3}x}{12}$ o.e.	A1
	$\{P =\}\ 3x + 6y$	B1
	$P = 3x + 6\left(\frac{1}{x} - \frac{\sqrt{3}x}{12}\right)$	M1
	So $P = 3x + \frac{6}{x} - \frac{\sqrt{3}}{2}x$	A1 *
		(6)
(b)	$\left(\frac{dP}{dx} =\right) 3 - \frac{6}{x^2} - \frac{\sqrt{3}}{2}$	M1 A1
	Puts their $P' = 0$ and attempts to solve for x^2 or x	M1
	$\Rightarrow x = \sqrt{\frac{12}{6 - \sqrt{3}}} = \sqrt{\frac{24 + 4\sqrt{3}}{11}}$ or 1.68	A1
	$\Rightarrow P = 7.16...$ (m)	A1
		(5)
(c)	$\frac{d^2P}{dx^2} = \frac{12}{x^3} > 0 \Rightarrow$ Minimum	M1
		A1 **(2)**
		(13 marks)

9.

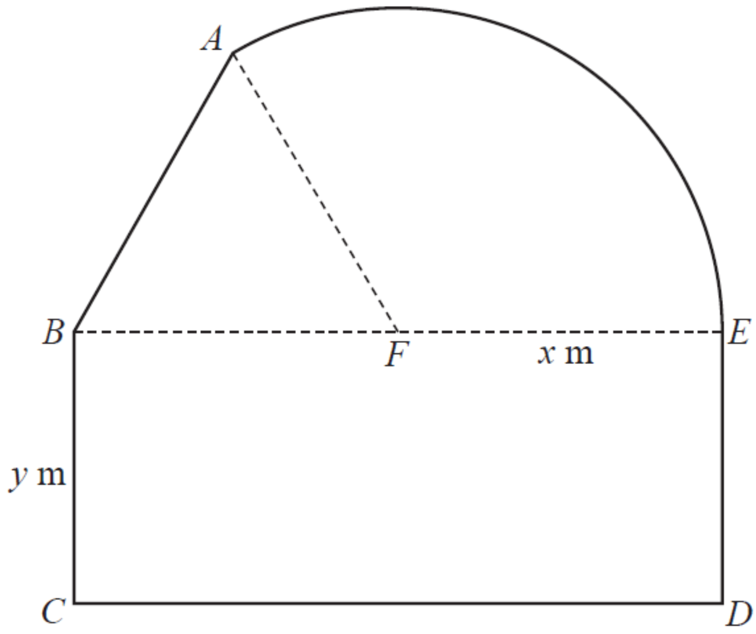


Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.

(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\,m^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre.

(5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum.

(2)

Question	Scheme	Notes	Marks
9. (a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right)$; $= \frac{\pi x^2}{3}$	$\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360}\times \pi x^2$ simplified or un-simplified	M1
		$\frac{\pi x^2}{3}$	A1
			[2]
	Parts (b) and (c) may be marked together		
(b)	$\{A =\}\ \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	Attempt to sum 3 areas (at least one correct)	M1
		Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ *	Correct proof.	A1 *
			[3]
(c)	$\{P =\}\ x + x\theta + y + 2x + y\ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	Correct expression in x and y for their θ measured in rads	B1ft
	$...2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$	Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$		
	$\Rightarrow P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$ *	Correct proof.	A1 *
			[3]
	Parts (d) and (e) should be marked together		
(d)	$\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}$; $= 0$	$\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$	M1
		Correct differentiation (need not be simplified).	A1;
		Their $P' = 0$	M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ $(= 16.63392808...)$	$\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..\ (\text{m})$	awrt 120	A1
			[5]
(e)	$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow \text{Minimum}$	Finds P'' **and** considers sign.	M1
		$\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
			[2]

9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

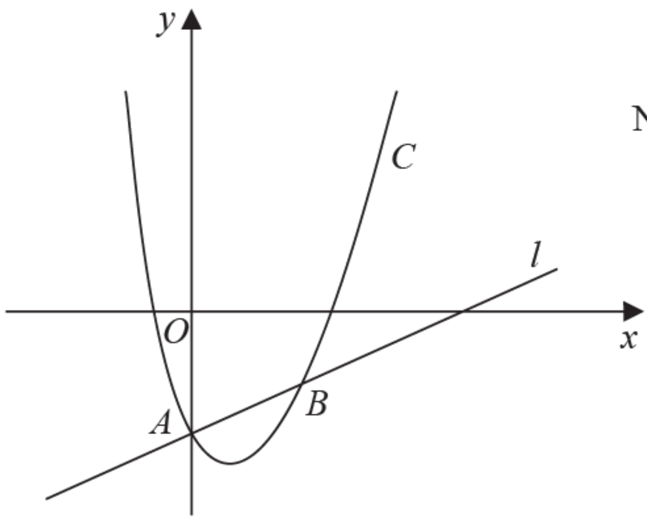
(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

4.



Not to scale

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

(a) Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

9. (a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ **or** (Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ — Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{dC}{dr}=\right\}12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ **so** $r^k=$ value where $k=\pm 2,\pm 3,\pm 4$	dM1
	Use **cube** root to obtain $r=\left(\text{their }\frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow $r=3$, and thus $C=$	ddM1
	Then $C=$ awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1)
		[10]

4.(a)	$\dfrac{dy}{dx}=-2e^{-2x}+2x$	M1A1
	At $x=0$ $\dfrac{dy}{dx}=-2\Rightarrow\dfrac{dx}{dy}=\dfrac{1}{2}$	M1
	Equation of normal is $y-(-2)=\dfrac{1}{2}(x-0)\Rightarrow y=\dfrac{1}{2}x-2$	M1 A1
		(5)

5.

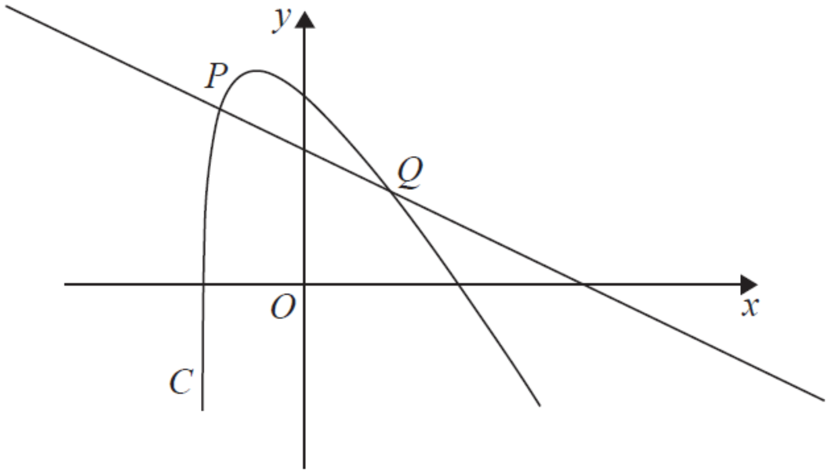


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate -2 lies on C.

(a) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

5. (a)	At P $x=-2 \Rightarrow y=3$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4}{2x+5}-\frac{3}{2}$	M1, A1
	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=-2}=\frac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\frac{2}{5}\left(x-(-2)\right)$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\frac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\frac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)