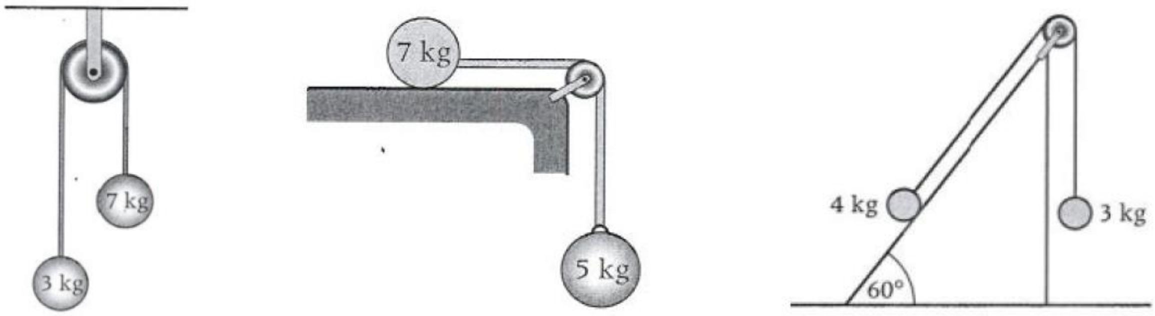
Mechanics Sector 2: Connected Particles

Pulleys

A system can involve two connected particles that are connected by a light, inextensible string that passes over a smooth pulley.



We assume that the pulley is **smooth** – there will be **no frictional forces** acting between the pulley and the string.

By Newton's 3rd Law, the **tension** forces acting on the particles will have the **same magnitude.**

We can assume that **the magnitude of the acceleration of each particle to be the same**. If this was not the case, then the string/cable/rod would stretch. By **assuming that the string/cable/rod is inextensible**, we can assume that the magnitude of the acceleration is the same.

However, the difference with pulley systems compared to other connected particle situations is that the **particles will not be moving in the same direction**. It is therefore important that you consider **each particle separately,** defining a positive direction for each particle.

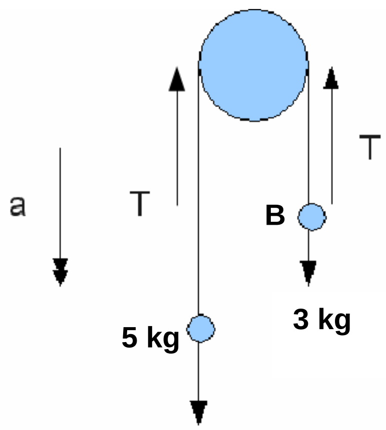
I would advise that the **positive direction** for each particle is the **initial direction of motion** for that particle.

By considering Newton's 2nd Law ($\boldsymbol{F_R} = \boldsymbol{ma}$) you will obtain equations that contain T and a, which are usually unknown. You can then solve these by considering them as **simultaneous equations.**

<u>Example 1</u>

Two particles A (of mass 5 kg) and B (of mass 3 kg) are connected by a light, inextensible string that passes over a smooth pulley. The objects are released at rest.

Find the acceleration of the particles and the tension in the string.



Example 2 (Harder)

Particle A of mass m and particle B of mass $\frac{3m}{2}$ are connected by a light inextensible string that passes over a smooth pulley. They are both positioned at a height of 1.5m above the ground

a) Find the acceleration of the particles in terms of g
b) Find the tension in the string in terms of m and g
c) After the particles move a distance of 1m, the string snaps. Assuming that in the resulting motion particle A does not hit the pulley, find the speed at which B hits the ground and the greatest height that A reaches above the ground in the subsequent motion.

NB
If the string snaps, or one particle stops moving (e.g. it might hit the floor) then each particle will no longer be subjected to tension. This means that there will be a new acceleration – you will have to consider Newton's 2nd law and the suvat equations again to calculate how the particles move from this point.

Example 3

Two particles, A and B, each of mass m, are connected by a light, inextensible string. Particle A is placed on a horizontal table, the string passes over a small, smooth pulley P fixed at the end of the table and B hangs vertically.

The particles are released from rest with both sections of the string taut and the section PB vertical.

If the table is rough and the coefficient of friction between A and the table is $\frac{1}{4}$, find:

a) The acceleration of the particles
b) The tension in the string
c) The speed at which B hits the floor, given that it starts at a height of 0.6m above the ground (assume that A does not reach the pulley P).

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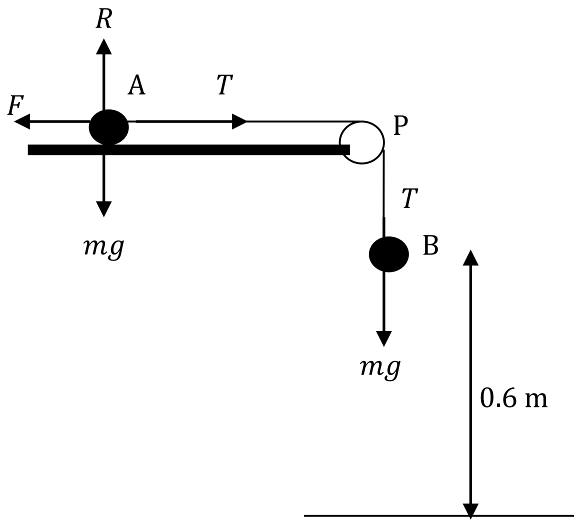
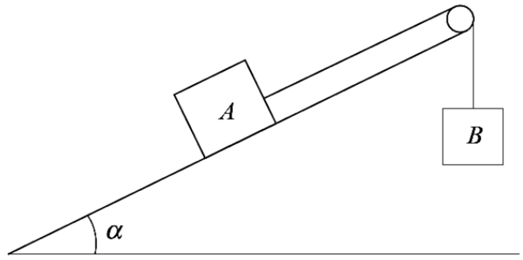


Example 4 (Harder)
The diagram shows a particle A, on a smooth inclined plane, joined by a light inextensible string passing over a smooth pulley to a particle B, which hangs freely. The plane is inclined at an angle α, where $\sin\alpha = \frac{5}{13}$.
The masses of A and B are 13kg and 15kg respectively and the string is in the same vertical plane as the line of greatest slope of the plane.

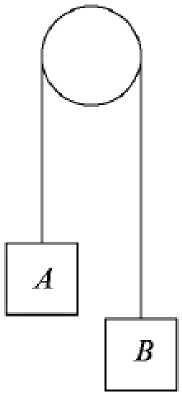
Initially, the particles are held at rest with the string taut. The system is then released. Calculate the magnitude of the acceleration of the particle and the tension in the string.



Exam Questions

WJEC M1 January 2012

5. The diagram shows two objects A and B, of mass 5 kg and 9 kg respectively, connected by a light inextensible string passing over a smooth peg. Initially, the objects are held at rest. The system is then released.



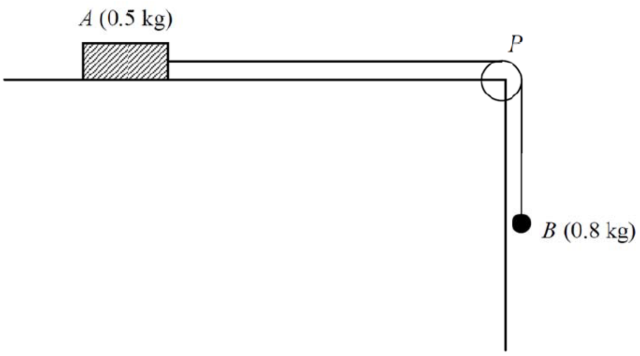
(a) Find the magnitude of the acceleration of A and the tension in the string. [7]

(b) What assumption did the word "light", underlined in the first sentence, enable you to make in your solution? [1]

Edexcel M1 January 2005

5.

Figure 4



A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in Figure 4. The coefficient of friction between A and the table is μ. The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Modelling A and B as particles, calculate

(*a*) the acceleration of B,

(3)

(*b*) the tension in the string,

(4)

(*c*) the value of μ.

(5)

(*d*) State how in your calculations you have used the information that the string is inextensible.

(1)

OCR M1 June 2010

6 A block B of mass 0.85 kg lies on a smooth slope inclined at 30° to the horizontal. B is attached to one end of a light inextensible string which is parallel to the slope. At the top of the slope, the string passes over a smooth pulley. The other end of the string hangs vertically and is attached to a particle P of mass 0.55 kg. The string is taut at the instant when P is projected vertically downwards.

(i) Calculate

(a) the acceleration of B and the tension in the string, **[5]**

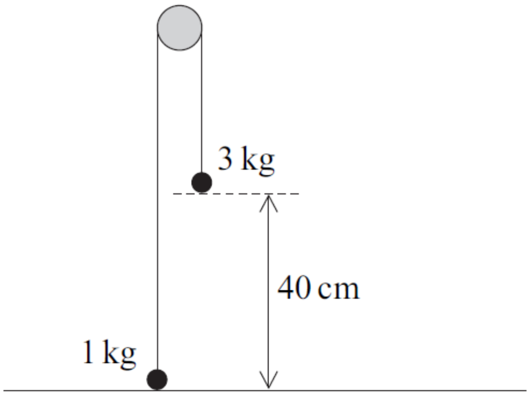
(b) the magnitude of the force exerted by the string on the pulley. **[2]**

The initial speed of P is $1.3\ \text{m s}^{-1}$ and after moving 1.5 m P reaches the ground, where it remains at rest. B continues to move up the slope and does not reach the pulley.

(ii) Calculate the total distance B moves up the slope before coming instantaneously to rest. **[6]**

AQA M1 June 2013

5 Two particles are connected by a light inextensible string that passes over a smooth peg. The particles have masses of 3 kg and 1 kg. The 1 kg particle is pulled down to ground level, where it is 40 cm below the level of the 3 kg particle, as shown in the diagram.



The particles are released from rest with the string vertical above each particle. Assume that no resistance forces act on the particles as they move.

(a) By forming two equations of motion, one for each particle, find the magnitude of the acceleration of the particles after they have been released but before the 3 kg particle hits the ground. *(5 marks)*

(b) Find the speed of the 1 kg particle when the 3 kg particle hits the ground. *(2 marks)*

(c) After the 3 kg particle has hit the ground, the 1 kg particle continues to move and the string is now slack. Find the maximum height above ground level reached by the 1 kg particle. *(3 marks)*

(d) If a constant air resistance force also acts on the particles as they move, explain how this would change your answer for the acceleration in part **(a)**. Give a reason for your answer. *(2 marks)*

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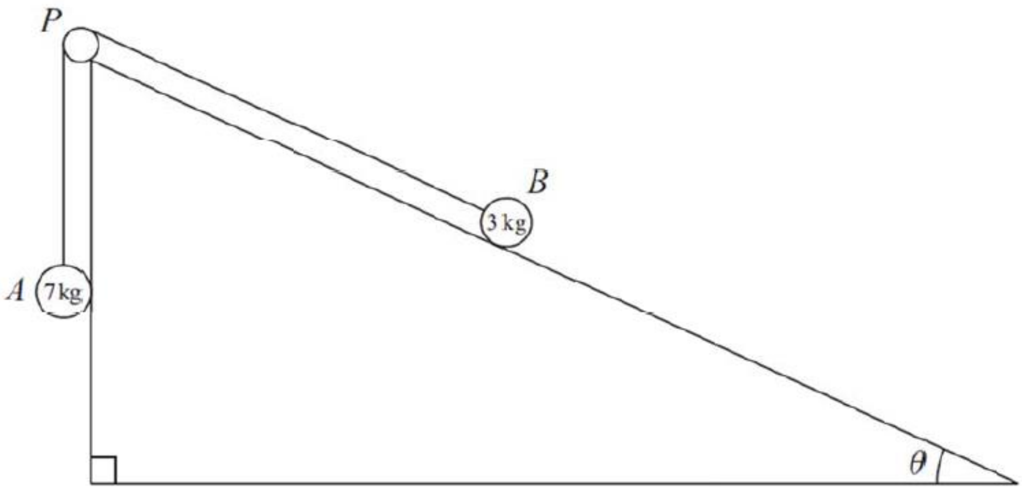


Figure 4

Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan \theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release.

(10)

(b) Find the speed of B when it has moved 1 m up the plane.

(2)

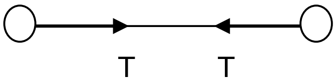
When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(c) find the time between the instants when the string breaks and when B comes to instantaneous rest.

(4)

The motion of two connected bodies

When two moving particles are connected by a string, cable or rod which is light and inextensible there will be tension acting through the string.
By Newton's 3rd Law, the tension forces acting on the particles will have the same magnitude but will be acting in opposite directions.



It is important on diagrams to show these separate tensions clearly.

By assuming that the string/cable/rod is **inextensible**, we can therefore say that **the acceleration of each particle will be the same**. If this was not the case, then the string/cable/rod would stretch, which contradicts our assumption.

When considering connected particle problems, you can consider the forces $(F_R = ma)$ on:

- The system
    - In this case the two tensions will cancel out, removing this unknown and allowing you to calculate other unknown forces

  NB
  Do not do this if you are considering pulley problems

- Each individual particle
    - Doing this afterwards will allow you to calculate the unknown tension
    - You can consider either particle
    - You can check your answer by considering the other particle

Example 5
A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find

(a) the acceleration of the car,
(b) the tension in the tow-rope.

Example 6 (Harder)

A car of mass 1500kg is connected to a caravan of mass 500kg by means of a horizontal, inextensible tow-bar. The driving force of the car, due to its engine, is 1700N and the resistive forces acting on the car and the caravan are 800N and 100N respectively.

a) Show that the acceleration of the car and caravan is 0.4 ms^{-2} and find the magnitude of the tension acting through the tow-bar.

The car then decelerates at a constant rate of 0.3 ms^{-2} whilst still applying a forward driving force. Assuming that the resistive forces acting on the car and the caravan, find

b) The magnitude and direction of the force acting on the car through the tow-bar
c) The new driving force

Exam Questions

AQA Jan 2013 M1

4 A tractor, of mass 3500 kg, is used to tow a trailer, of mass 2400 kg, across a horizontal field. The trailer is connected to the tractor by a horizontal tow bar. As they move, a constant resistance force of 800 newtons acts on the trailer and a constant resistance force of R newtons acts on the tractor. A forward driving force of 2500 newtons acts on the tractor. The trailer and tractor accelerate at $0.2\,\text{m s}^{-2}$.

(a) Find R. *(3 marks)*

(b) Find the magnitude of the force that the tow bar exerts on the trailer. *(3 marks)*

(c) State the magnitude of the force that the tow bar exerts on the tractor. *(1 mark)*

Edexcel Jan 2012 M1

2. A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

Given that the acceleration of the car and the caravan is $0.88\,\text{m s}^{-2}$,

(a) show that $R = 860$,

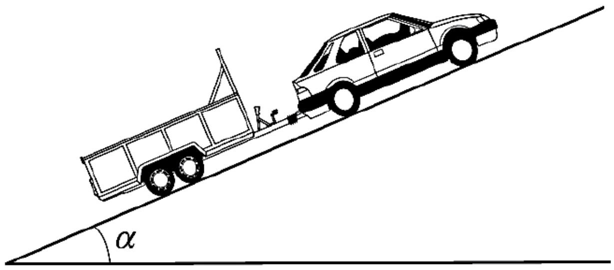
(3)

(b) find the tension in the tow-bar.

(3)

WJEC Jan 2014 M1

6. The diagram below shows a car of mass 1500 kg connected to a trailer of mass 600 kg by means of a rigid tow bar. The car is moving upwards along a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{7}{25}$. A constant resistance of magnitude 400 N acts on the car and a constant resistance of 300 N acts on the trailer. The car's engine produces a constant forward force of 8400 N.



(a) Calculate the acceleration of the car, giving your answer correct to three decimal places. [5]

(b) Determine the tension in the tow bar. [4]

OCR Jan 2009 M1

2 A trailer of mass 500 kg is attached to a car of mass 1250 kg by a light rigid horizontal tow-bar. The car and trailer are travelling along a horizontal straight road. The resistance to motion of the trailer is 400 N and the resistance to motion of the car is 900 N. Find both the tension in the tow-bar and the driving force of the car in each of the following cases.

(i) The car and trailer are travelling at constant speed. **[3]**

(ii) The car and trailer have acceleration $0.6\ \mathrm{m\,s^{-2}}$. **[6]**

Edexcel June 2008 M1

8.

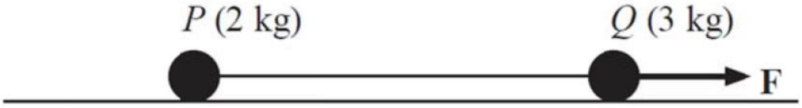


Figure 4

Two particles P and Q, of mass 2 kg and 3 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. A constant force **F** of magnitude 30 N is applied to Q in the direction PQ, as shown in Figure 4. The force is applied for 3 s and during this time Q travels a distance of 6 m. The coefficient of friction between each particle and the plane is μ. Find

(a) the acceleration of Q,

(2)

(b) the value of μ,

(4)

(c) the tension in the string.

(4)

(d) State how in your calculation you have used the information that the string is inextensible.

(1)

When the particles have moved for 3 s, the force **F** is removed.

(e) Find the time between the instant that the force is removed and the instant that Q comes to rest.

(4)