8.

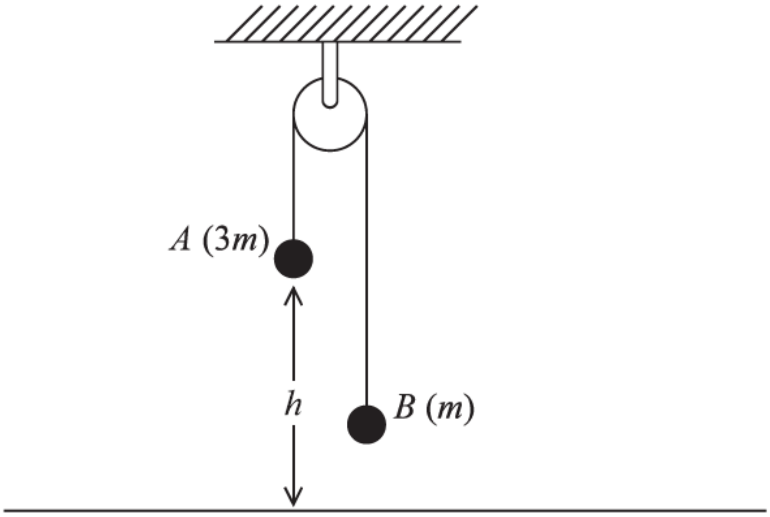


Figure 3

Two particles, A and B, have masses $3m$ and m respectively. The particles are attached to the ends of a light inextensible string which passes over a light smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and A is at a height h above a horizontal floor, as shown in Figure 3. The system is now released from rest and in the subsequent motion B does not reach the pulley.

For the motion of A and B before A hits the floor,

(a) (i) write down an equation of motion for A,

(ii) write down an equation of motion for B.

(4)

(b) Hence show that, until A hits the floor, the acceleration of A is $0.5g$

(2)

(c) State how, in your solution, you have used the fact that the string is modelled as being inextensible.

(1)

The speed of A at the instant immediately before it hits the floor is V.

(d) Find V in terms of g and h.

(2)

As a result of hitting the floor, A rebounds with speed $\frac{1}{2}V$.

(e) Find, in terms of m, g and h, the magnitude of the impulse exerted by the floor on A.

(3)

(f) Find, in terms of h, the height of A above the floor when A next comes to rest.

(2)

8(a)	$3mg - T = 3ma$ $T - mg = ma$	M1 A1 M1 A1 (4)
(b)	$3mg - mg = 4ma$ $a = 0.5g$ Given answer	**DM**1 A1 (2)
(c)	The magnitude of the acceleration for both particles is the same	B1 (1)
(d)	$V = \sqrt{2 \times 0.5gh} = \sqrt{gh}$	M1 A1 (2)
(e)	$I = 3m(\frac{1}{2}V - -V)$ $= \frac{9m\sqrt{gh}}{2}$	M1 A1 A1 (3)
(f)	$0 = (\frac{1}{2}V)^2 - 2gd$ $d = \frac{h}{8}$	M1 A1 (2)
		(14)

7.

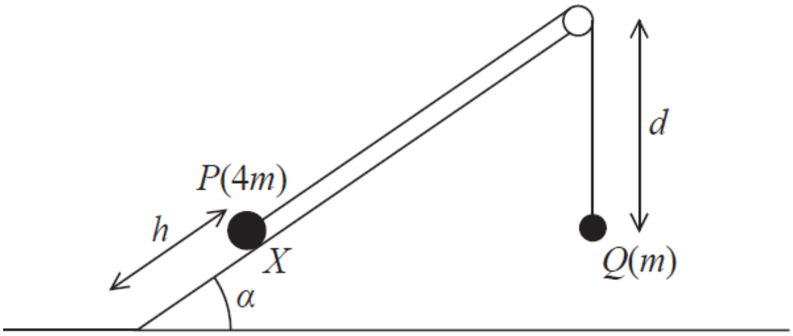


Figure 4

A particle P of mass $4m$ is held at rest at the point X on the surface of a rough inclined plane which is fixed to horizontal ground. The point X is a distance h from the bottom of the inclined plane. The plane is inclined to the horizontal at an angle α where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{4}$. The particle P is attached to one end of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which hangs freely at a distance d, where $d > h$, below the pulley, as shown in Figure 4.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The system is released from rest with the string taut and P moves down the plane.

For the motion of the particles before P hits the ground,

(a) state which of the information given above implies that the magnitudes of the accelerations of the two particles are the same,

(1)

(b) write down an equation of motion for each particle,

(5)

(c) find the acceleration of each particle.

(5)

When P hits the ground, it immediately comes to rest. Given that Q comes to instantaneous rest before reaching the pulley,

(d) show that $d > \frac{28h}{25}$.

(5)

7(a)	Inextensible string	B1 (1)
	MARK PARTS (b) and (c) together	
(b)	$4mg\sin\alpha - T - F = 4ma$	M1 A2
	$T - mg = ma$	M1 A1 (5)
(c)	$F = \frac{1}{4}R$	B1
	$R = 4mg\cos\alpha$	B1
	$\cos\alpha = \frac{4}{5}$ or $\sin\alpha = \frac{3}{5}$	B1
	Eliminating R, F and T	M1
	$a = \frac{3}{25}g = 1.2$ or 1.18 (m s^{-2})	A1 (5)
(d)	$v^2 = 2\times\frac{3}{25}gh = \frac{6}{25}gh$	M1
	$0^2 = \frac{6}{25}gh - 2gs$	
	$s = \frac{3}{25}h$	M1 A1
	$d > \frac{3}{25}h + h = \frac{28}{25}h$ GIVEN ANSWER	**DM**1 A1 (5)
		(16)

8.

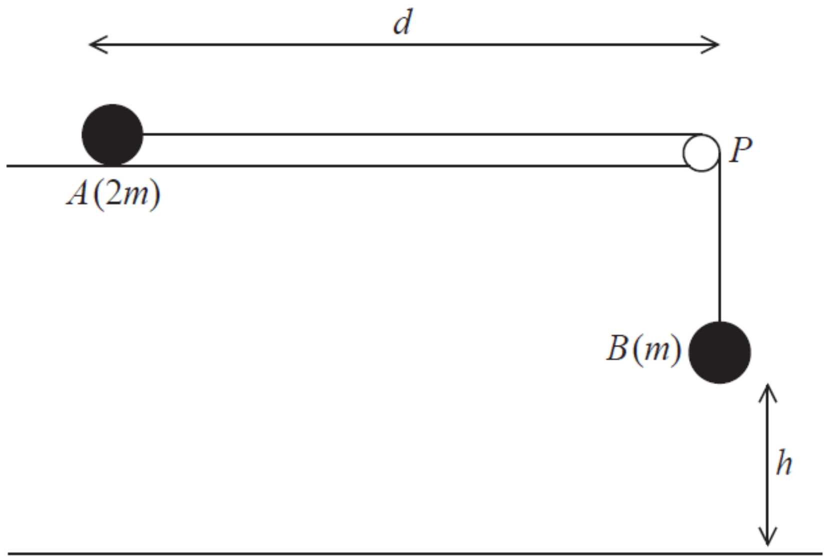


Figure 3

Two particles, A and B, have masses $2m$ and m respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a fixed rough horizontal table at a distance d from a small smooth light pulley which is fixed at the edge of the table at the point P. The coefficient of friction between A and the table is μ, where $\mu < \frac{1}{2}$. The string is parallel to the table from A to P and passes over the pulley. Particle B hangs freely at rest vertically below P with the string taut and at a height h, ($h < d$), above a horizontal floor, as shown in Figure 3. Particle A is released from rest with the string taut and slides along the table.

(a) (i) Write down an equation of motion for A.

(ii) Write down an equation of motion for B.

(4)

(b) Hence show that, until B hits the floor, the acceleration of A is $\frac{g}{3}(1 - 2\mu)$.

(3)

(c) Find, in terms of g, h and μ, the speed of A at the instant when B hits the floor.

(2)

After B hits the floor, A continues to slide along the table. Given that $\mu = \frac{1}{3}$ and that A comes to rest at P,

(d) find d in terms of h.

(5)

(e) Describe what would happen if $\mu = \frac{1}{2}$

(1)

8(a) **(i)** **(ii)**	For A : $T - F = 2ma$ For B : $mg - T = ma$	M1 A1 M1 A1 (4)
(b)	$R = 2mg$ $mg(1-2\mu) = 3ma$ $\frac{g}{3}(1-2\mu) = a$	B1 M1 A1 (3)
(c)	$v^2 = \frac{2gh}{3}(1-2\mu)$ $v = \sqrt{\frac{2gh}{3}(1-2\mu)}$	M1 A1 (2)
(d)	$-\mu R = 2ma'$ $0^2 = \text{their } u^2 - 2a's$ $0 = \frac{2gh}{3}(1-\frac{2}{3}) - 2(\frac{1}{3}g)s$ (or $s = (d-h)$) $s = \frac{1}{3}h$ $d = \frac{1}{3}h + h = \frac{4}{3}h$	M1 M1 A1 (A1) A1 A1 (5)
(e)	A (or B) would not move; **OR** A (or B) would remain in (limiting) equilibrium; **OR** the system would remain in (limiting) equilibrium	B1 (1) **15**

8.

Figure 3

Two particles P and Q have masses 1.5 kg and 3 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{5}$. The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle Q hangs freely at rest vertically below the pulley, as shown in Figure 3. Particle P is released from rest with the string taut and slides along the table.

Assuming that P has not reached the pulley, find

(a) the tension in the string during the motion,

(8)

(b) the magnitude and direction of the resultant force exerted on the pulley by the string.

(4)

8(a)	$F = \frac{1}{5}R$ $R = 1.5g$ $T - F = 1.5a$ $3g - T = 3a$ $T = 1.2g$ or 11.8 N or 12 N	M1 B1 M1 A1 M1 A1 **DM1** A1 (8)
(b)	$R = \sqrt{T^2 + T^2}$ or $2T\cos 45^0$ or $\frac{T}{\cos 45^0}$ $= 16.6$ (N) or 17(N) or $\frac{6g\sqrt{2}}{5}$ Direction is 45° below the horizontal oe	M1 A1 A1 B1 (4) **12**

8.

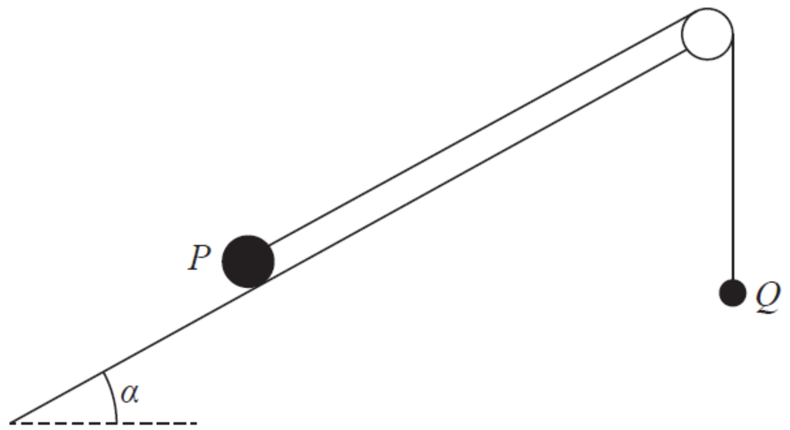


Figure 4

Two particles P and Q have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle α where $\tan \alpha = \frac{4}{3}$. The coefficient of friction between P and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle Q hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle P is released from rest with the string taut and slides down the plane.

Given that Q has not hit the pulley, find

(a) the tension in the string during the motion,

(11)

(b) the magnitude of the resultant force exerted by the string on the pulley.

(4)

8(a)	$R = 4g\cos\alpha$ $T - 0.5g = 0.5a$ $4g\sin\alpha - T - F = 4a$ (OR: $4g\sin\alpha - F - 0.5g = 4.5a$) $F = \frac{1}{2}R$; $\sin\alpha = \frac{4}{5}$ or $\cos\alpha = \frac{3}{5}$ Eliminating a or finding a Solving for T (must have had an a) $T = \frac{2g}{3}$N or 6.5N or 6.53N	M1 A1 M1 A1 M1 A1 B1; B1 M1 M1 A1 (11)
(b)	Magnitude $= 2T\cos\left(\frac{90-\alpha}{2}\right)$ $= 2 \text{ x } \frac{2g}{3} \text{ x } \frac{3}{\sqrt{10}}$ (0.94868..) $= 12N$ or $12.4N$ $\left(\frac{4g}{\sqrt{10}}\right)$	M1 A1 **A1 ft on *T*** A1 (4) **15**

6 A block of mass $20\,\text{kg}$ lies on a horizontal surface. The block is attached to a particle of mass $10\,\text{kg}$ by a light inextensible string which passes over a smooth fixed peg. Initially the block is held at rest so that the string supports the particle, as shown in the diagram.

The block is then released.

(a) If the surface is smooth, use two equations of motion to find the magnitude of the acceleration of the block and particle.

[4 marks]

(b) In reality the surface is rough and the acceleration of the block is $2.5\,\text{m}\,\text{s}^{-2}$

Find the coefficient of friction between the block and the surface.

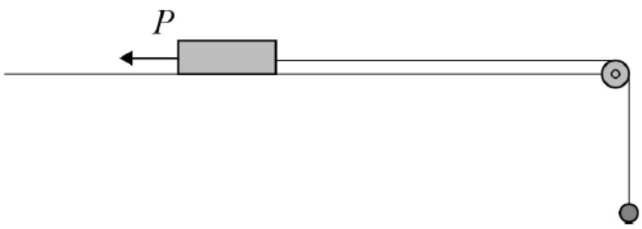
[6 marks]

(c) State one modelling assumption that you have made in this question.

[1 mark]

6 (a)	$10g - T = 10a$ $T = 20a$ $10g - 20a = 10a$ $a = \frac{10g}{30} = 3.27 \text{ ms}^{-2}$	**M1A1** **B1** **A1**	**4**	M1: Three term equation of motion for the particle. A1: Correct equation of motion for particle. B1: Equation of motion for the block. A1: Correct acceleration.
(b)	$R = 20 \times 9.8 = 196 \text{ N}$ $10g - T = 10 \times 2.5$ $T - F = 20 \times 2.5$ $10g - F = 75$ $F = 23$ $23 = \mu \times 196$ $\mu = \frac{23}{196} = 0.117$	**B1** **M1A1** **A1F** **dM1** **A1**	**6**	B1: Correct normal reaction. M1: Three term equation of motion for the particle and the block. A1: Both equations correct. A1: Correct F. dM1: Use of $F = \mu R$. A1: Correct μ.
(c)	No air resistance Horizontal String or **Block** is a particle	**B1**	**1**	B1: For acceptable assumption.

5 A block, of mass $4\,\mathrm{kg}$, is on a smooth horizontal surface. It is attached to a light inextensible string that passes over a smooth peg. A particle of mass $6\,\mathrm{kg}$ is attached to the other end of the string. The section of the string between the peg and the particle is vertical. A horizontal force of magnitude P newtons acts on the block, as shown in the diagram.



(a) P is such that the system remains at rest. By forming two equations, find P.

[3 marks]

(b) P is changed so that the block accelerates away from the peg at $0.6\,\mathrm{m\,s^{-2}}$. By forming two equations of motion, find P.

[4 marks]

(c) When the block is moving away from the peg at $2\,\mathrm{m\,s^{-1}}$, the force of magnitude P newtons is removed. Find the distance that the block travels as its speed reduces from $2\,\mathrm{m\,s^{-1}}$ to $0\,\mathrm{m\,s^{-1}}$.

[6 marks]

(d) Explain fully how your answer to part **(c)** would change if the effects of air resistance were included.

[2 marks]

5 (a)	$P-T=0$ $T-6g=0$ $P=6g=58.8$	**M1** **A1** **A1**	**3**	M1: Equations of equilibrium for both objects. Need correct terms but with any signs. A1: Both equations correct. A1: Correct value for *P*.
(b)	$P-T=4\times 0.6$ $T-6g=6\times 0.6$ $P-6g=6$ $P=6g+6=64.8$	**M1** **M1A1** **A1**	**4**	M1: Three term equation of motion for the block. . Need correct terms but with any signs. M1: Three term equation of motion for the particle. Need correct terms but with any signs. A1: Both equations correct. A1: Correct value of *P*. CAO
(c)	$-T=4a$ $T-6g=6a$ $-6g=10a$ $a=-\frac{3g}{5}=-5.88\text{ m s}^{-2}$ $0^2=2^2+2\times(-5.88)s$ $s=\frac{4}{11.76}=0.340\text{ m}$	**M1** **A1** **A1** **M1A1** **A1**	**6**	M1: Equations of motion for block and particle. Need correct terms but with any signs. A1: Both equations correct. A1: Correct acceleration. Allow +5.88 if consistent with signs. M1: Equation to find distance using their acceleration provided their acceleration is negative. A1: Correct equation. A1: Correct distance. Condone 0.34.
(d)	Distance is less. Air resistance produces a deceleration of greater magnitude.	**B1** **B1**	**2**	B1: Less distance stated. B1: Reason attributed to air resistance. Only award the second mark if the distance has been stated as less.

5 A car of mass $1600\ \mathrm{kg}$ tows a trailer of mass $400\ \mathrm{kg}$ on a straight horizontal road. The car starts from rest and accelerates uniformly. The car travels 45 metres in 12 seconds.

(a) Find the acceleration of the car.

[3 marks]

(b) A resistance force of magnitude 500 newtons acts on the car, and a resistance force of magnitude 80 newtons acts on the trailer. The trailer is connected to the car by a horizontal tow bar. A driving force of magnitude P newtons acts on the car.

(i) Find the tension in the tow bar.

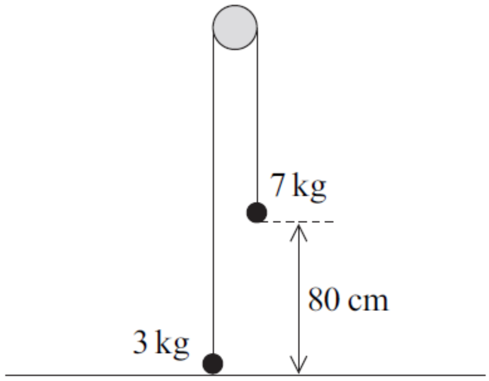
[3 marks]

(ii) Find P.

[3 marks]

5 (a)	$45 = 0 + \frac{1}{2} \times a \times 12^2$	M1 A1		M1: Use of a constant acceleration equation, with $u = 0$, to find a. A1: Correct equation.
	$a = \frac{45}{72} = 0.625 \text{ m s}^{-2}$	A1	3	A1: Correct acceleration. Accept $\frac{5}{8} \text{ m s}^{-2}$.
(b) (i)	$T - 80 = 400 \times 0.625$	M1 A1F		M1: Equation of motion for trailer with T and 80. Must have mass 400 × their acceleration. Allow sign errors. A1F: Correct equation with their acceleration from (a).
	$T = 330$	A1	3	A1: Correct T.
(b) (ii)	$P - 500 - 80 = 2000 \times 0.625$	M1 A1F		M1: Equation of motion for car and trailer combined with P, 500 and 80. Must have mass 2000 × their acceleration. Allow sign errors. A1F: Correct equation with their acceleration from (a).
	$P = 1830$	A1	3	A1: Correct P.
	OR			
	$P - T - 500 = 1600 \times 0.625$	(M1) (A1F)		M1: Equation of motion for car with their T, 500 and P. Must have mass 1600 × their acceleration. A1F: Correct equation with their acceleration from (a).
	$P = 1830$	(A1)		A1: Correct P.

5 Two particles, of masses 3 kg and 7 kg, are connected by a light inextensible string that passes over a smooth peg. The 3 kg particle is held at ground level with the string above it taut and vertical. The 7 kg particle is at a height of 80 cm above ground level, as shown in the diagram.



The 3 kg particle is then released from rest.

(a) By forming two equations of motion, show that the magnitude of the acceleration of the particles is $3.92\,\text{m}\,\text{s}^{-2}$.

[5 marks]

(b) Find the speed of the 7 kg particle just before it hits the ground.

[3 marks]

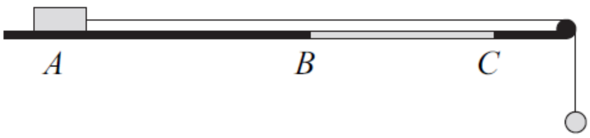
(c) When the 7 kg particle hits the ground, the string becomes slack and in the subsequent motion the 3 kg particle does not hit the peg.

Find the maximum height of the 3 kg particle above the ground.

[4 marks]

5. (a)	$7g - T = 7a$ $T - 3g = 3a$ $4g = 10a$ AG $a = \frac{4g}{10} = 3.92\ \mathrm{m\,s^{-2}}$	**M1** **M1** **A1** **dM1** **A1**	**5**	M1: Three term equation of motion for one particle. Accept: $7g - T = 7a$ $3g - T = 3a$ $T - 7g = 7a$ $T - 3g = 3a$ M1: Three term equation of motion from the list above for the other particle. A1: Two consistent equations, that is either $\begin{matrix} 7g - T = 7a \\ T - 3g = 3a \end{matrix}$ **or** $\begin{matrix} T - 7g = 7a \\ 3g - T = 3a \end{matrix}$. dM1: Solving equations to find a. A1: Obtaining 3.92 from consistent working.
5. (b)	$v^2 = 0^2 + 2 \times 3.92 \times 0.8$ $= 6.272$ $v = 2.50\ \mathrm{m\,s^{-1}}$	**M1A1** **A1**	**3**	M1: Using $v^2 = u^2 + 2as$ with $u = 0$, $s = 0.8$ or 80 and $a = 3.92$. A1: Correct equation. A1: Correct speed. Accept 2.5 or AWRT 2.50.
5.(c)	$0^2 = 6.272 + 2 \times (-9.8)s$ $s = \frac{6.272}{19.6} = 0.32\ \mathrm{m}$ Total height $= 32 + 80 = 112$ cm	**M1A1** **A1** **B1**	**4**	M1: Using $v^2 = u^2 + 2as$ with $v = 0$, their value for v from (b) for u and $a = \pm 9.8$. A1: Correct equation. Allow 6.25 from 2.5^2. A1: Obtaining AWRT ± 0.32 from correct working. B1: Adding 80 or 0.8 to the height from their intermediate working. Must use same units and obtain an answer greater than 80 or 0.8 depending on units used.
	Total		**12**	

5 A block, of mass $3m$, is placed on a horizontal surface at a point A. A light inextensible string is attached to the block and passes over a smooth peg. The string is horizontal between the block and the peg. A particle, of mass $2m$, is attached to the other end of the string. The block is released from rest with the string taut and the string between the peg and the particle vertical, as shown in the diagram.



Assume that there is no air resistance acting on either the block or the particle, and that the size of the block is negligible.

The horizontal surface is smooth between the points A and B, but rough between the points B and C. Between B and C, the coefficient of friction between the block and the surface is 0.8.

(a) By forming equations of motion for both the block and the particle, find the acceleration of the block between A and B.

[4 marks]

(b) Given that the distance between the points A and B is 1.2 metres, find the speed of the block when it reaches B.

[3 marks]

(c) By forming equations of motion for both the block and the particle, find the acceleration of the block between B and C.

[5 marks]

(d) Given that the distance between the points B and C is 0.9 metres, find the speed of the block when it reaches C.

[3 marks]

(e) Explain why it is important to assume that the size of the block is negligible.

[1 mark]

Q	Solution	Mark	Total	Comment
5. (a)	$2mg - T = 2ma$ $T = 3ma$ $2mg - 3ma = 2ma$ $a = \frac{2g}{5} = (3.92)\text{ m s}^{-2}$	M1A1 B1 A1	**4**	M1: Three term equation of motion for the particle. Must be either $2mg - T = 2ma$ or $T - 2mg = 2ma$ .OE . A1: Correct equation for the particle. B1: Correct equation of motion for the block. Must be consistent with first equation. A1: Correct acceleration. Allow 0.4g oe. Note that use of $g = 9.81$ gives 3.92. SC2: For "whole string method" leading to correct acceleration. Award either 2 or 0 marks.
5. (b)	$v^2 = 0^2 + 2 \times 3.92 \times 1.2$ $v = \sqrt{9.408} = 3.07\text{ m s}^{-1}$	M1A1 A1	**3**	M1: Use of constant acceleration equation with $u = 0$, $s = 1.2$ and their numerical value for a from part (a). A1: Correct equation. A1: Correct speed. AWRT 3.07.
5. (c)	$2mg - T = 2ma$ $T - F = 3ma$ $T - 0.8 \times 3mg = 3ma$ $-0.4mg = 5ma$ $a = -0.08g = -0.784\text{ m s}^{-2}$	B1 B1 M1A1 A1	**5**	B1: Three term equation of motion for the particle. Must be either $2mg - T = 2ma$ or $T - 2mg = 2ma$ OE. B1: Seeing $F = 0.8 \times 3mg$ OE. M1: Three term equation of motion for the block. Must be either $T - F = 3ma$ or $F - T = 3ma$ OE. A1: Correct equation for the block. Equation must be consistent with other equation. A1: Correct acceleration, sign consistent with working. Accept -0.08g oe.
5. (d)	$v^2 = 9.408 + 2 \times (-0.784) \times 0.9$ $v = \sqrt{7.9968} = 2.83\text{ m s}^{-1}$	M1A1 A1	**3**	M1: Use of constant acceleration equation with their answers to parts (b) and (c) with $s = 0.9$. A1: Correct equation. A1: Correct speed. AWRT 2.83.
5. (e)	If the size of the block is not negligible there will be mixed friction on the block as it passes from the smooth to rough sections of the surface.	B1	**1**	B1: Statement about issue of moving from smooth to rough.
	Total		**16**	