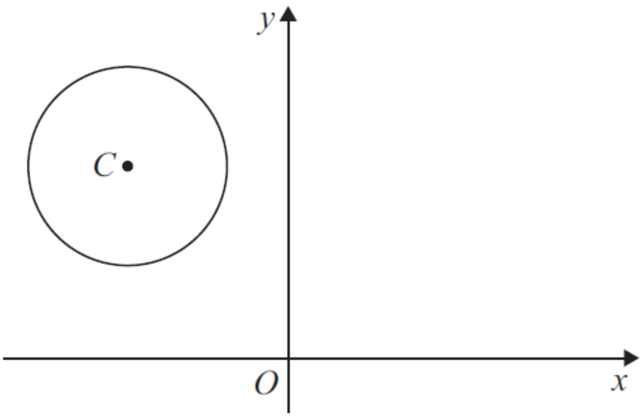
6 The diagram shows a circle and the origin O.



The circle has centre $C(-8, 12)$ and radius $\sqrt{13}$.

(a) Write down the equation of the circle in the form

$$(x-a)^2+(y-b)^2=k$$

[2 marks]

(b) The point P has coordinates $(-5, 10)$.

(i) Verify that P lies on the circle.

[1 mark]

(ii) Find an equation of the tangent to the circle at the point P, giving your answer in the form $px+qy+r=0$, where p, q and r are integers.

[5 marks]

(c) Find the coordinates of the point on the circle that is closest to O.

[4 marks]

Q6	Solution	Mark	Total	Comment
(a)	$(x+8)^2+(y-12)^2$ **=…** **or** RHS=13	M1		
	$(x+8)^2+(y-12)^2=13$	A1	2	**or** $(x--8)^2+(y-12)^2=13$
(b)(i)	$(-5+8)^2+(10-12)^2=9+4=13$ Therefore P lies on the circle	B1	1	correct convincing arithmetic plus **statement** (not just ticks etc)
(ii)	Gradient $PC=\dfrac{10-12}{-5+8}$	M1		correct unsimplified
	$=-\dfrac{2}{3}$	A1		
	Gradient of tangent $=\dfrac{3}{2}$	A1ft		
	$y-10=\text{"}their\dfrac{3}{2}\text{"}(x+5)$	dM1		must be attempting tangent and not normal
	Equation of tangent $3x-2y+35=0$ **or** $0=2y-3x-35$ **etc**	A1	5	terms all on one side with integer coefficients
(c)	Eqn of OC **:** $y=-\dfrac{3}{2}x$	B1		
	$(x+8)^2+(-1.5x-12)^2=13$	M1		Sub into circle eqn or equate to radius
	$\dfrac{13}{4}x^2+52x+208=13\Rightarrow x^2+16x+60=0$			
	$x=-6,-10$ **OE**	A1		both values of x
	Closest point $(-6,9)$	A1	4	must simplify to integers
	Total		**12**	

5 A circle with centre $C(7, -8)$ passes through the point $P(2, -2)$.

(a) Find the gradient of the normal to the circle at the point P.

[2 marks]

(b) Find the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$

[3 marks]

(c) The point Q is the point on the circle that is closest to the x-axis.
Find the exact value of the y-coordinate of Q.

[2 marks]

(d) The point R also lies on the circle. The length of the chord PR is 8. Show that the shortest distance from C to PR is $n\sqrt{5}$, where n is an integer.

[3 marks]

6 A circle with centre C has equation $x^2 + y^2 + 20x - 14y + 49 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$

[3 marks]

(b) Show that the circle touches the y-axis and crosses the x-axis in two distinct points.

[4 marks]

(c) A line has equation $y = kx + 2$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the circle and the line satisfy the equation

$$(1+k^2)x^2 + 10(2-k)x + 25 = 0$$

[2 marks]

(ii) Hence, find the value of k for which the line is a tangent to the circle.

[3 marks]

Q5	Solution	Mark	Total	Comment
(a)	Grad $PC = \frac{-2--8}{2-7}$	M1		condone one sign error in one term
	$= -\frac{6}{5}$ **OE**	A1	2	withhold **A1** if gradient of perpendicular attempted. No **ISW** here.
(b)	$(x-7)^2+(y+8)^2$ **=…**	M1		**or** $(x-7)^2+(y--8)^2$ **=…**
	5^2+6^2 **or** $25+36$ **or** 61	B1		**or** seen under square root
	$(x-7)^2+(y+8)^2=61$	A1	3	**or** $(x-7)^2+(y--8)^2=61$
(c)	$-8+\text{"their"}\sqrt{k}$ **or** $-8\pm\text{"their"}\sqrt{k}$	M1		also allow $-8-\text{"their"}\sqrt{k}$ for **M1**
	$-8+\sqrt{61}$	A1	2	
(d)	M is midpoint of PR			
	$(CM^2=)$ "their 61"-4^2	M1		Pythagoras used correctly with "4" and with hyp^2 = " *their* " k or correct
	$(CM^2=)$ 45	A1		or $(CM=)$ $\sqrt{45}$
	(shortest distance $=$) $3\sqrt{5}$	A1cso	3	all notation correct
	Total		10	

(a)	$(x+10)^2+(y-7)^2$ **=…**	M1		one of these terms correct
		A1		LHS correct ignoring any extra constants **or** $(x--10)^2+(y-7)^2$ **=…**
	$(x+10)^2+(y-7)^2=10^2$ (or …=100)	A1	3	**or** $(x--10)^2+(y-7)^2=10^2$ (or …=100)
(b)	$10^2+(y-7)^2=10^2$ $\Rightarrow (y-7)^2=0$ $\Rightarrow y=7$	M1		putting x=0 in "their" equation and attempt to solve for y
	Repeated root means circle touches y-axis	E1		completely correct working and **both parts** of the conclusion
	$(x+10)^2+7^2=100$ $(x+10)^2=51$ $\Rightarrow x=-10\pm\sqrt{51}$	M1		putting $y=0$ in "their" equation and attempt to solve for x
	Two roots so circle crosses x–axis twice	E1	4	completely correct working and **both parts** of the conclusion
(c)(i)	$(x+10)^2+(kx-5)^2=100$ $x^2+20x+100+k^2x^2-10kx+25=100$	M1		sub $y=kx+2$ into "their" circle equation and attempt to multiply out brackets
	$(1+k^2)x^2+10(2-k)x+25=0$	A1cso	2	**AG** be convinced -
	must have terms exactly as printed answer			condone $0=(1+k^2)x^2+10(2-k)x+25$
(ii)	$10^2(2-k)^2-4\times25(1+k^2)$	M1		correct discriminant unsimplified
	$400-400k+100k^2-100-100k^2$ $(=0)$	A1		multiplying out correctly
	$k=\frac{3}{4}$	A1cso	3	must see "=0" before final answer

5 A circle with centre $C(5, -3)$ passes through the point $A(-2, 1)$.

(a) Find the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$

[3 marks]

(b) Given that AB is a diameter of the circle, find the coordinates of the point B.

[2 marks]

(c) Find an equation of the tangent to the circle at the point A, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers.

[5 marks]

(d) The point T lies on the tangent to the circle at A such that $AT = 4$.

Find the length of CT.

[3 marks]

4 A circle with centre C has equation $x^2 + y^2 + 2x - 6y - 40 = 0$.

(a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = d$$

[3 marks]

(b) (i) State the coordinates of C.

[1 mark]

(ii) Find the radius of the circle, giving your answer in the form $n\sqrt{2}$.

[2 marks]

(c) The point P with coordinates $(4, k)$ lies on the circle. Find the possible values of k.

[3 marks]

(d) The points Q and R also lie on the circle, and the length of the chord QR is 2. Calculate the shortest distance from C to the chord QR.

[2 marks]

Q5	Solution	Mark	Total	Comment
(a)	$(x-5)^2+(y+3)^2$ =...	M1		or $(x-5)^2+(y--3)^2$ =...
	7^2+4^2 **or** $49+16$ **or** 65	B1		**or** seen under square root
	$(x-5)^2+(y+3)^2=65$	A1	3	**or** $(x-5)^2+(y--3)^2=65$
(b)	$x_B=12$	B1		
	$y_B=-7$	B1	2	$B(12,-7)$
(c)	Grad $AC=\dfrac{1--3}{-2-5}$	M1		condone one sign error in one term FT their B if grad AB or grad BC is used.
	$=-\dfrac{4}{7}$	A1		
	Grad tgt $=\dfrac{7}{4}$	B1F		
	Equation of tgt: $y-1=\text{"their"}\dfrac{7}{4}(x--2)$	m1		or $y=\text{"their"}\dfrac{7}{4}x+c$ & attempt to find c using $x=-2$ and $y=1$
	$7x-4y+18=0$	A1	5	any multiple – must have integer coefficients and all terms on one side
(d)	$CT^2=AT^2+AC^2$ $(CT^2=)\ 4^2+\text{"their"}65$	M1		Pythagoras with hyp=CT & AC^2="*their*" k or correct
	$(CT^2=)\ 81$	A1		or $(CT=)\sqrt{81}$
	$(CT=)9$	A1	3	all notation correct; must simplify $\sqrt{81}$
	Total		**13**	

(a)	$(x+1)^2+(y-3)^2...$	M1		one of these terms correct
		A1		LHS correct with perhaps extra constant terms
	$(x+1)^2+(y-3)^2=50$	A1	3	
(b)(i)	$C(-1,3)$	B1√	1	correct or **FT** from their equation in **(a)**
(ii)	$(r=)\sqrt{50}$	M1		correct or **FT** their $\sqrt{RHS}$ provided $RHS>0$
	$=5\sqrt{2}$	A1	2	
(c)	$4^2+k^2+2\times 4-6k-40=0$ **or** "*their*" $(4+1)^2+(k-3)^2=50$	M1		sub $x=4$, correctly into given circle equation (or their circle equation)
	$k^2-6k-16(=0)$ **or** $(k-3)^2=25$	A1		
	$k=-2, k=8$	A1	3	
(d)	$D^2+1^2=$ " their r^2 "	M1		Pythagoras used correctly with 1 and r
	$D^2=50-1=49$ (distance =) 7	A1	2	Do not accept $\sqrt{49}$ or ± 7
	Total		**11**	

5. The circle C has equation

$$x^2 + y^2 - 3x + 6y = 1$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the value of the radius of C.

(2)

The point P $(5, -3)$ lies on the circle C.

(c) Find an equation of the tangent to C at the point P.

(2)

5. The circle C has equation

$$x^2 + y^2 - 2x + 14y = 0$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the exact value of the radius of C,

(2)

(c) the y coordinates of the points where the circle C crosses the y-axis.

(2)

(d) Find an equation of the tangent to C at the point $(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(a)	Obtain LHS as $\underline{(x \pm \frac{3}{2})^2} + \underline{\underline{(y \pm 3)^2}} = \ldots$	M1	
	Centre is $(\frac{3}{2}, -3)$.	A1	(2)
(b)	Uses $\underline{(x \pm \frac{3}{2})^2} - \frac{9}{4} + \underline{\underline{(y \pm 3)^2}} - 9 = 1$ to give $r = \sqrt{1 + "\frac{9}{4}" + "9"}$ or just $r^2 = 1 + "\frac{9}{4}" + "9"$	M1	
	$r = \frac{7}{2}$	A1	(2)
Special case	Uses (5, -3) from (c) to find radius $\underline{(5 - \frac{3}{2})^2} + \underline{\underline{(-3 + 3)^2}} = \ldots$	M1	
	$r = \frac{7}{2}$	A1	(2)
(c)	Way 1: Deduces gradient is infinite (from diagram or from perpendicular to zero gradient	M1	
	So equation is $x = 5$	A1	(2)
	Way 2: Implicit differentiation $\frac{dy}{dx} = \frac{3 - 2x}{2y + 6} = \frac{3 - 10}{0}$ **so infinite gradient o.e.**	M1	
	So equation is $x = 5$	A1	(2) **(6 marks)**

5	**You may mark (a) and (b) together** $x^2 + y^2 - 2x + 14y = 0$		
(a)	Obtain LHS as $\underline{(x \pm 1)^2} + \underline{\underline{(y \pm 7)^2}} = \ldots$	M1	
	Centre is $(1, -7)$.	A1	(2)
(b)	Uses $r^2 = a^2 + b^2$ or $r = \sqrt{a^2 + b^2}$ where their centre was at $(\pm a, \pm b)$	M1	
	$r = \sqrt{50}$ or $5\sqrt{2}$	A1	(2)
(c)	Substitute $x = 0$ in either form of equation of circle and solve resulting quadratic to give $y =$	M1	
	$y^2 + 14y = 0$ *so* $y = 0$ *and* -14 **or** $\underline{\underline{(y \pm 7)^2 - 49}} = 0$ *so* $y = 0$ *and* -14	A1	(2)
(d)	Gradient of radius joining centre to (2,0) is $\frac{"-7" - 0}{"1" - 2}$ $(= 7)$	M1	
	Gradient of tangent is $\frac{-1}{m}$ $(= -\frac{1}{7})$	M1	
	So equation is $y - 0 = -\frac{1}{7}(x - 2)$ and so $x + 7y - 2 = 0$	M1, A1	(4) **(10 marks)**

5. The circle C has equation

$$x^2 + y^2 - 10x + 6y + 30 = 0$$

Find

(a) the coordinates of the centre of C,

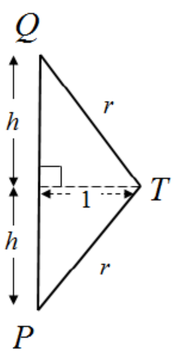
(2)

(b) the radius of C,

(2)

(c) the y coordinates of the points where the circle C crosses the line with equation $x = 4$, giving your answers as simplified surds.

(3)

Question	Scheme	Marks	
5	$x^2 + y^2 - 10x + 6y + 30 = 0$		
(a)	Uses any appropriate method to find the coordinates of the centre, e.g achieves $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = \ldots$. Accept (±5,±3) as indication of this.	M1	
	Centre is $(5, -3)$.	A1	**(2)**
(b) Way 1	Uses $\underline{(x \pm "5")^2 - "5^2"} + \underline{\underline{(y \pm "3")^2 - "3^2"}} + 30 = 0$ to give $r = \sqrt{"25"+"9"-30}$ or $r^2 = "25"+"9"-30$ (not $30 - 25 - 9$)	M1	
	$r = 2$	A1cao	**(2)**
Or Way 2	Using $\sqrt{g^2 + f^2 - c}$ from $x^2 + y^2 + 2gx + 2fy + c = 0$ (Needs formula stated or correct working)	M1	
	$r = 2$	A1	**(2)**
(c) Way 1	Use $x = 4$ in *an* equation of circle and obtain equation in y only	M1	
	e.g $(4-5)^2 + (y+3)^2 = 4$ or $4^2 + y^2 - 10 \times 4 + 6y + 30 = 0$		
	Solve their quadratic in y and obtain **two** solutions for y	dM1	
	e.g. $(y+3)^2 = 3$ or $y^2 + 6y + 6 = 0$ so $y = -3 \pm \sqrt{3}$	A1	**(3)**
Or Way 2	 Divide triangle PTQ and use Pythagoras with $"r"^2 - ("5" - 4)^2 = h^2$,	M1	
	Find h and evaluate $"-3" \pm h$. May recognise $(1, \sqrt{3}, 2)$ triangle.	dM1	
	So $y = -3 \pm \sqrt{3}$	A1	**(3)** **[7]**

3.

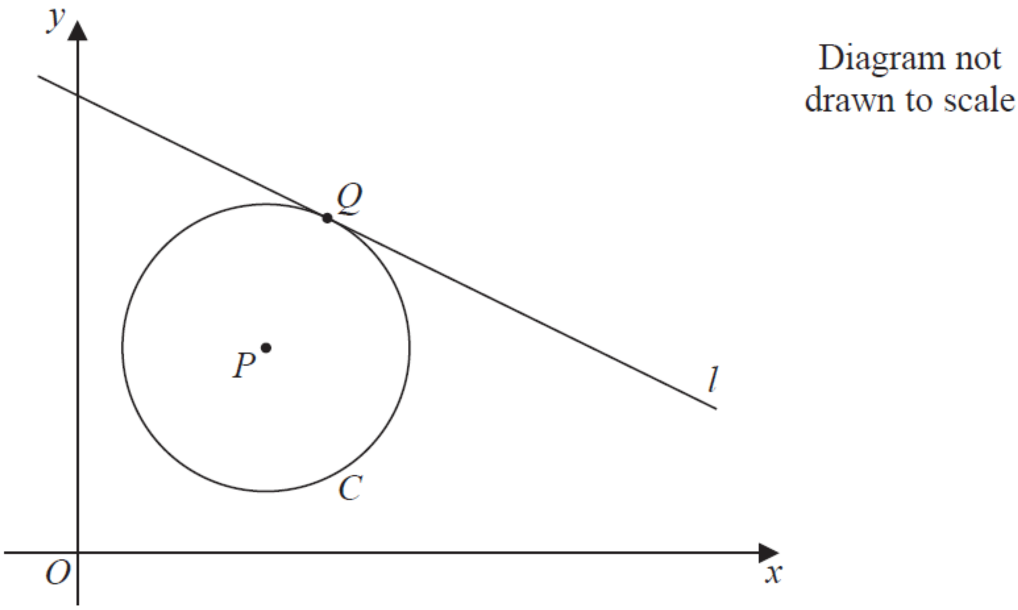


Figure 2

The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$, as shown in Figure 2.

(a) Find the length PQ, giving your answer as an exact value.

(2)

(b) Hence write down an equation for C.

(2)

The line l is a tangent to C at the point Q, as shown in Figure 2.

(c) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

2. A circle C with centre at the point $(2, -1)$ passes through the point A at $(4, -5)$.

(a) Find an equation for the circle C.

(3)

(b) Find an equation of the tangent to the circle C at the point A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

3.	$P(7, 8)$ and $Q(10, 13)$		
(a)	$\{PQ =\}\ \sqrt{(7-10)^2+(8-13)^2}$ or $\sqrt{(10-7)^2+(13-8)^2}$	Applies distance formula. Can be implied.	M1
	$\{PQ\} = \sqrt{34}$	$\sqrt{34}$ or $\sqrt{17}.\sqrt{2}$	A1
			[2]
(b) **Way 1**	$(x-7)^2+(y-8)^2 = 34\ \left(\text{or } \left(\sqrt{34}\right)^2\right)$	$(x \pm 7)^2 + (y \pm 8)^2 = k$, where k is a positive value.	M1
		$(x-7)^2+(y-8)^2 = 34$	A1 oe
			[2]
(c) **Way 1**	$\{\text{Gradient of radius}\} = \dfrac{13-8}{10-7}$ or $\dfrac{5}{3}$	This must be seen or implied in part (c).	B1
	Gradient of tangent $= -\dfrac{1}{m}\left(= -\dfrac{3}{5}\right)$	Using a perpendicular gradient method on their gradient. So Gradient of tangent $= -\dfrac{1}{\text{gradient of radius}}$	M1
	$y - 13 = -\dfrac{3}{5}(x-10)$	$y - 13 = (\text{their changed gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
(c) **Way 2**	$2(x-7) + 2(y-8)\dfrac{dy}{dx} = 0$	Correct differentiation (or equivalent). Seen or implied	B1
	$2(10-7) + 2(13-8)\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{3}{5}$	Substituting **both** $x = 10$ and $y = 13$ into a valid differentiation to find a value for $\dfrac{dy}{dx}$	M1
	$y - 13 = -\dfrac{3}{5}(x-10)$	$y - 13 = (\text{their gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]

	Way 1	Way 2		
2 (a)	$(x \mp 2)^2 + (y \pm 1)^2 = k,\ k > 0$	$x^2 + y^2 \mp 4x \pm 2y + c = 0$	M1	
	Attempts to use $r^2 = (4-2)^2 + (-5+1)^2$	$4^2 + (-5)^2 - 4\times 4 + 2\times -5 + c = 0$	M1	
	Obtains $(x-2)^2 + (y+1)^2 = 20$	$x^2 + y^2 - 4x + 2y - 15 = 0$	A1	**(3)**
	N.B. Special case: $(x-2)^2 - (y+1)^2 = 20$ is not a circle equation but earns M0M1A0			
(b) Way 1	Gradient of radius from centre to (4, -5) = -2 (must be correct)		B1	
	Tangent gradient $= -\dfrac{1}{\text{their numerical gradient of radius}}$		M1	
	Equation of tangent is $(y+5) = '\tfrac{1}{2}'(x-4)$		M1	
	So equation is $x - 2y - 14 = 0$ (or $2y - x + 14 = 0$ or other integer multiples of this answer)		A1	**(4)**