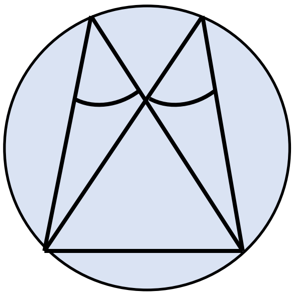
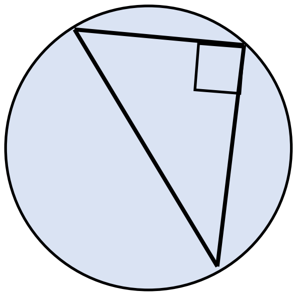
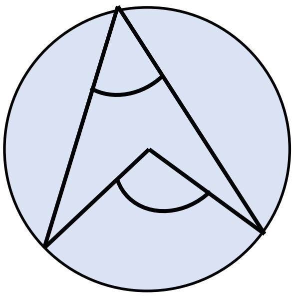
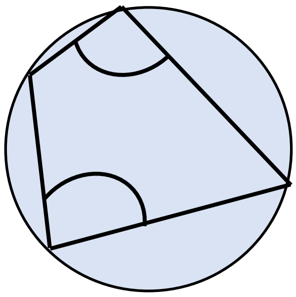
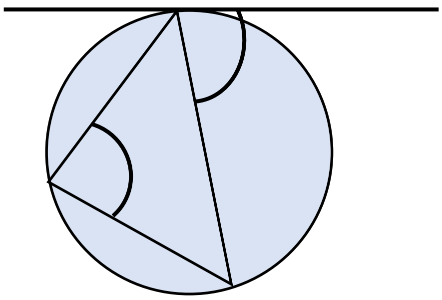
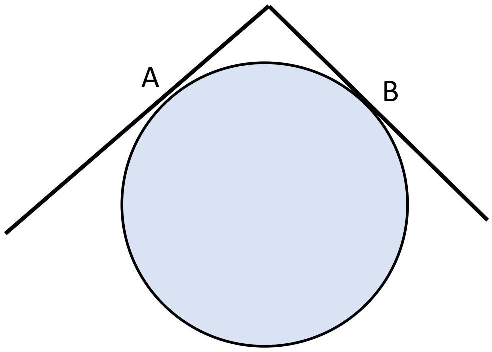
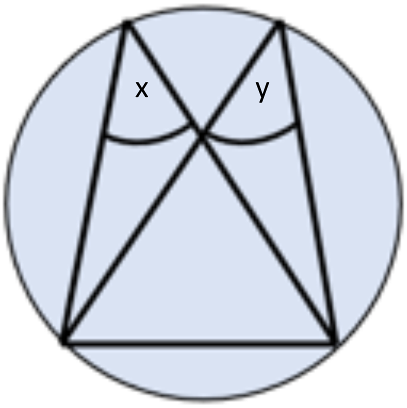
Proving Circle Theorems PLS Tutors Ltd

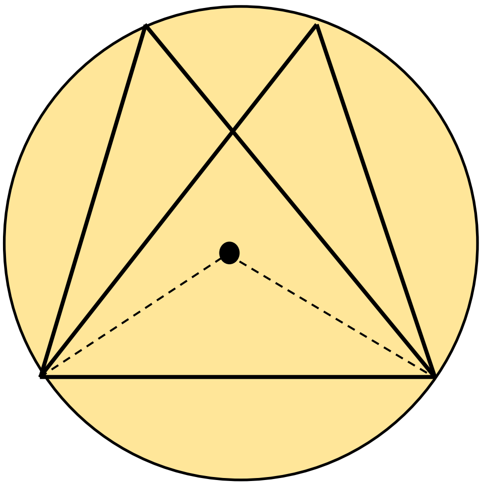
Angles in the same segment are always equal 	**The angle in a semicircle is always 90°** 
The angle at the centre of a circle is always double the angle at the circumference 	**Opposite angles in a cyclic quadrilateral sum to 180°** 
Angles in alternate segments are always equal 	**Tangents to the circle that meet at a point are the same length** 

Angles in the same segment are equal



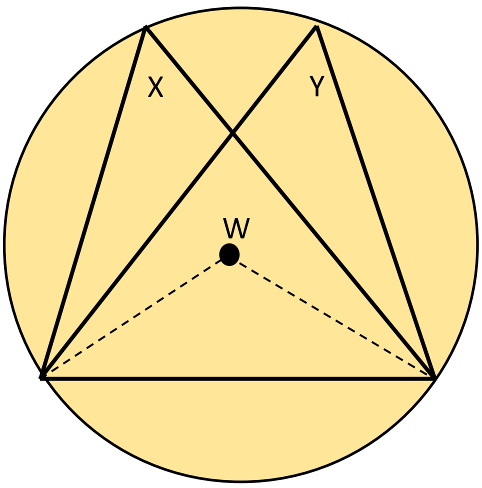
__Aim:__

Show that x = y



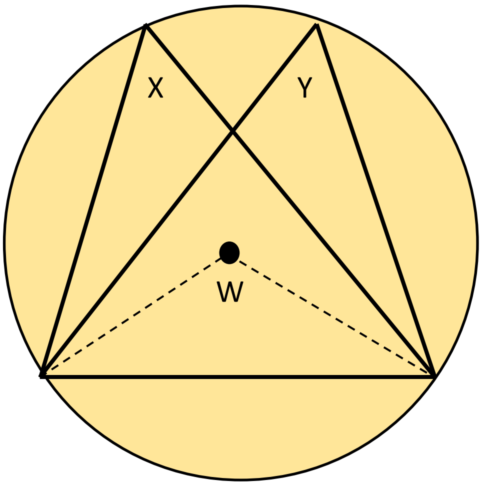
__Step One:__

Draw another segment from either end of the chord to the centre of the circle.



__Step Two:__

Notice that another circle theorem has been formed: ‘angles at the centre are double the size of angles at the circumference’.



Step Three:

Using the circle theorem 'angles at the centre are double the size of angles at the circumference', we know that angle W is twice the size of angle X and twice the size of angle Y.

Step Four:

We can now form two equations:

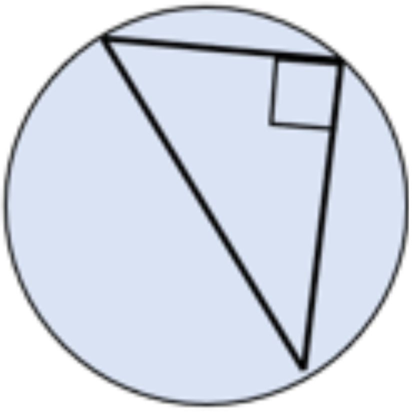
W = 2X and W = 2Y

This means that 2X = 2Y since both equal W

2X = 2Y can be simplified to X = Y

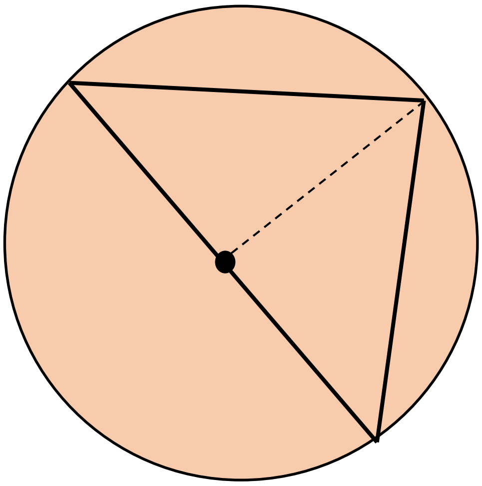
Therefore, we have proved the 'angles in the same segment are equal' theorem by showing that angle X = angle Y.

The angle in a semicircle is always 90˚



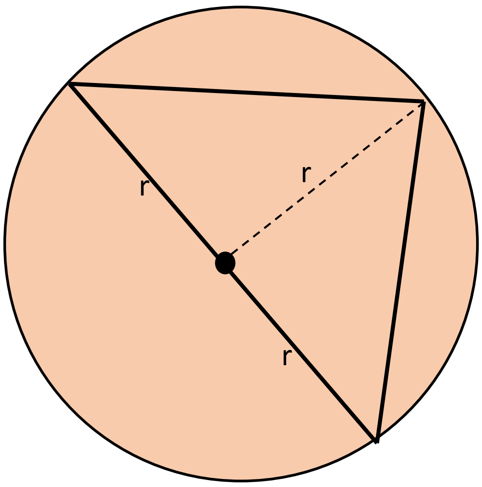
<u>Aim:</u>

Prove that the right angle in the diagram is in fact a right angle.



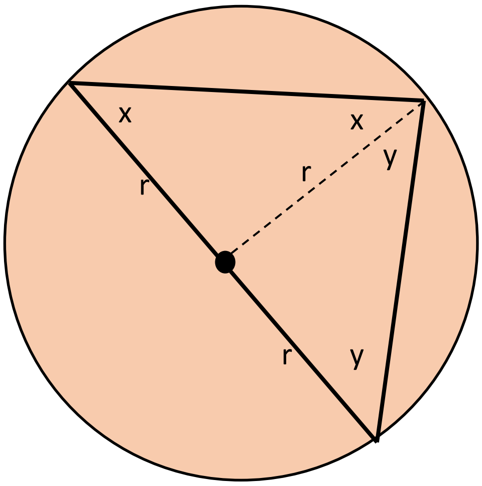
<u>Step One:</u>

Draw a line from the centre point to the point you are trying to show is 90˚.



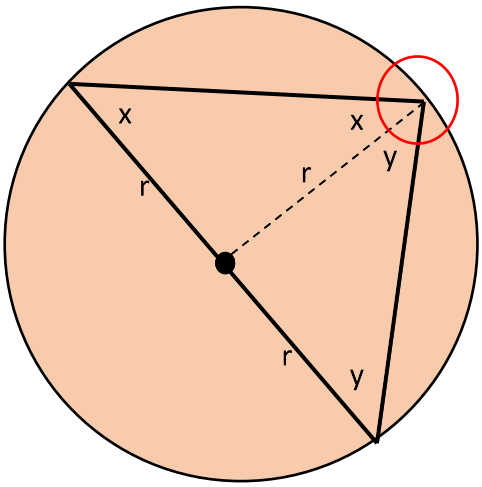
<u>Step Two:</u>

We now have two isosceles triangles since lines labelled ‘r’ are all the same length as they are all radius lines.



Step Three:

Knowing we have two isosceles triangles, we can start to label some angles that are equal.



Step Four:

We know that angles in a triangle sum to 180°.

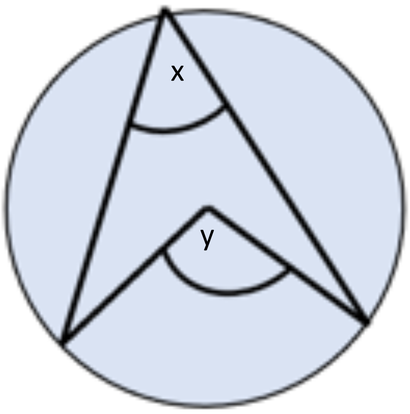
Using our angles that we just labelled, we can say that $x + x + y + y = 180^\circ$

This can be simplified to $2x + 2y = 180^\circ$,

Simplifying again makes the equation $x + y = 90^\circ$,

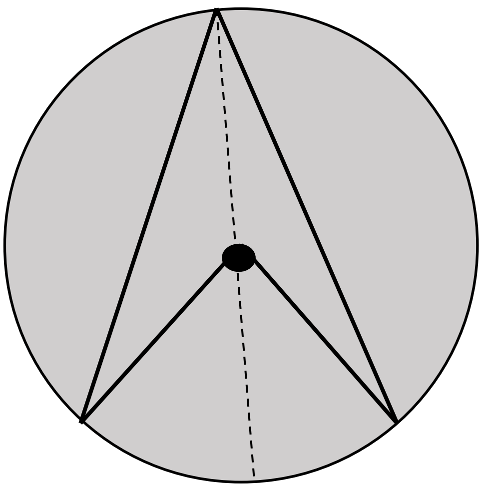
Looking back at the diagram, we can see the larger angle highlighted in red is the angle '$x + y$'. We have just proved that this angle equals 90°. We have proved that 'the angle in a semicircle is 90°'.

Angles at the centre are twice the size of angles at the circumference



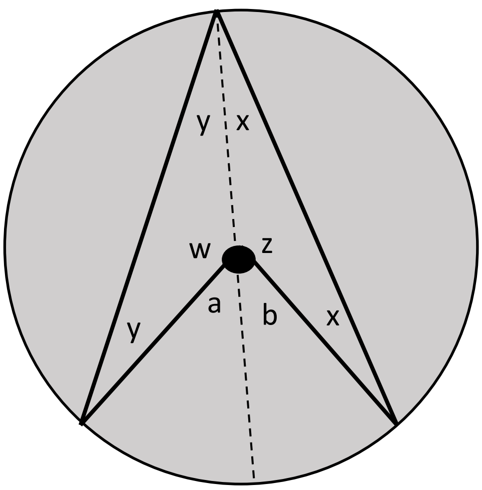
Aim:

Show that $y = 2x$



Step One:

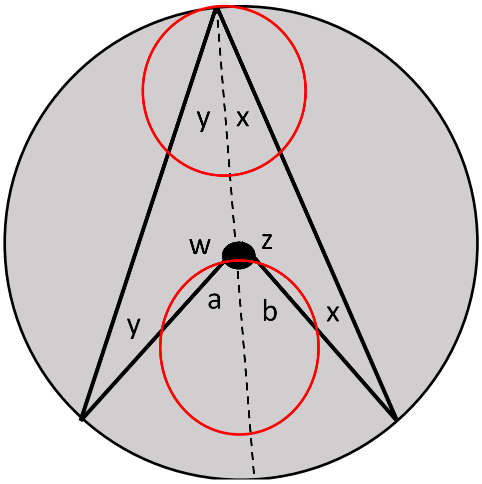
Draw a line from the point on the circumference through the point at the centre to make two isosceles triangles.



Step Two:

Label all angles that are equivalent, as well as angles around the centre point.

Step Three:



We can form the following equations based on knowledge of angles in a triangle:

$2y = 180 - w$
$2x = 180 - z$

Using our knowledge of straight lines, we can also infer that $w = 180 – a$ and $z = 180 – b$.

We can rearrange the four equations above:

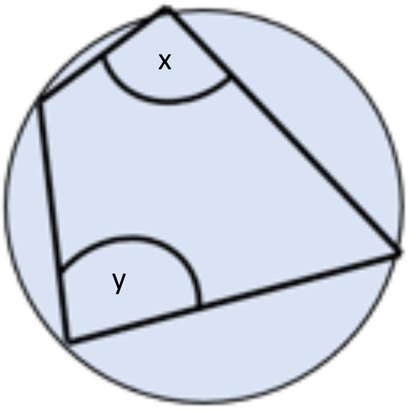
Sub '$w = 180 – a$' into '$2y = 180 – w$' to get
$2y = 180 – (180 - a)$ which simplifies to $2y = a$.
Sub '$z = 180 – b$' into '$2x = 180 – z$' to get
$2x = 180 – (180 – b)$ which simplifies to $2x = b$.

If we add the two equations '$2y = a$' and '$2x = b$' we get $2y + 2x = a + b$.

This simplifies to $2(x + y) = a + b$. This proves that the angle '$x + y$' is twice the size of the angle '$a + b$'.

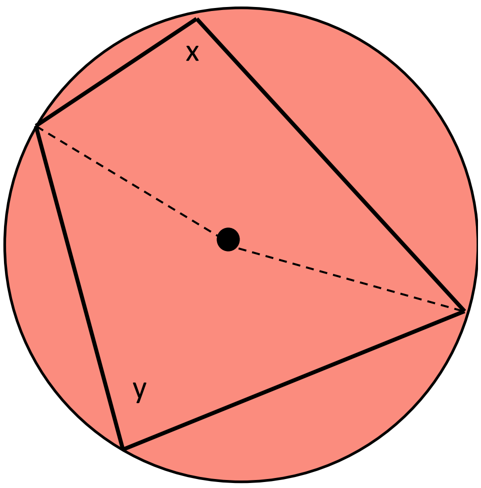
We have proved the theorem 'angles at the centre are twice the size of angles at the circumference'.

Opposite angles in a cyclic quadrilateral sum to 180°



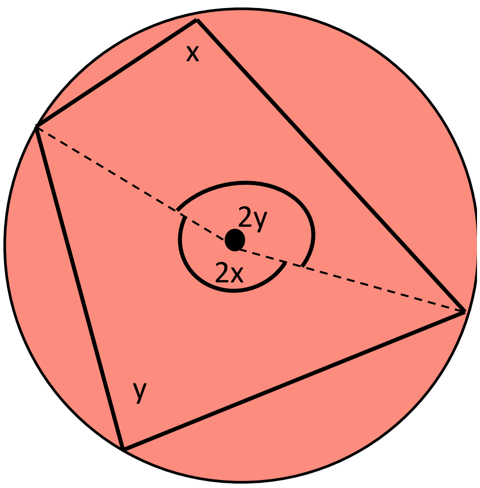
Aim:

Show that x + y = 180°



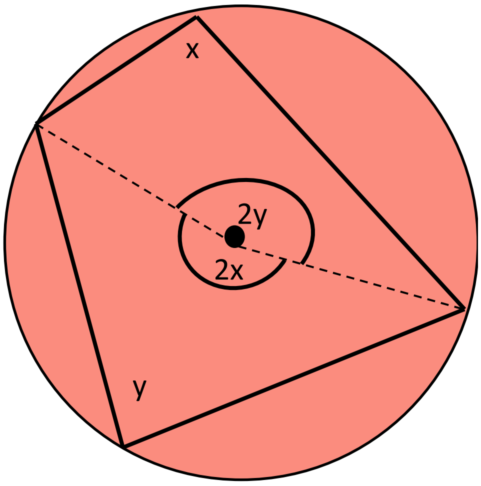
Step One:

Draw two lines from the quadrilateral corners to the centre as shown in the diagram.



Step Two

By drawing the lines, we have created the circle theorem 'angles at the centre are twice the size of angles at the circumference'. We can label the angles as shown.



Step Three

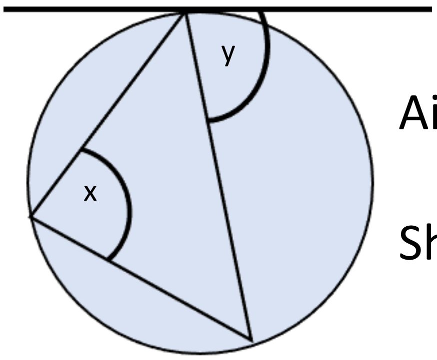
We can now derive the following equations:

$2y + 2x = 360^{\circ}$ because angles around a point sum to 360°.

Therefore $x + y = 180°$.

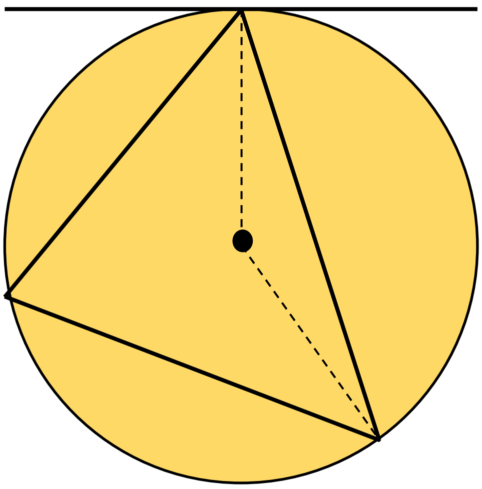
We have proved the circle theorem ‘opposite angles in a cyclic quadrilateral sum to 180°’.

Angles in alternate segments are equal



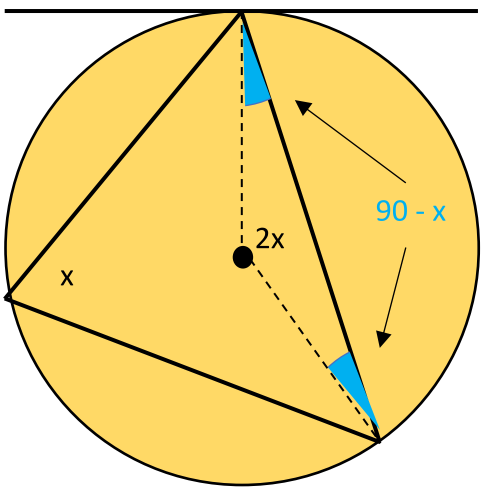
Aim:

Show that x = y



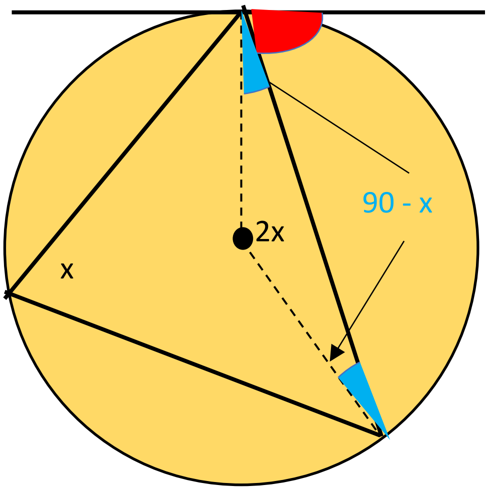
Step One:

Draw two lines to the centre of the circle, as show in the diagram.



Step Two:

We can use the 'angles at the centre are double the size of angles at the circumference' to label the angles on the diagram. We also know that angles in a triangle sum to 180˚ so we can label more angles.

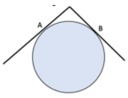


Step Three:

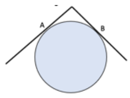
We know that a right angle is formed when a radius meets a tangent. Therefore, we can say that the angle between the triangle and the tangent equals 90 – (90 – x).

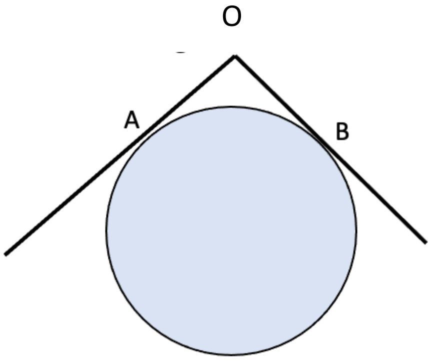
When we simplify this, we are left with x.

Therefore, the angle between the tangent and the triangle equals x and the angle already labelled equals x. We have proved the circle theorem 'angles in alternate segments are equal'.



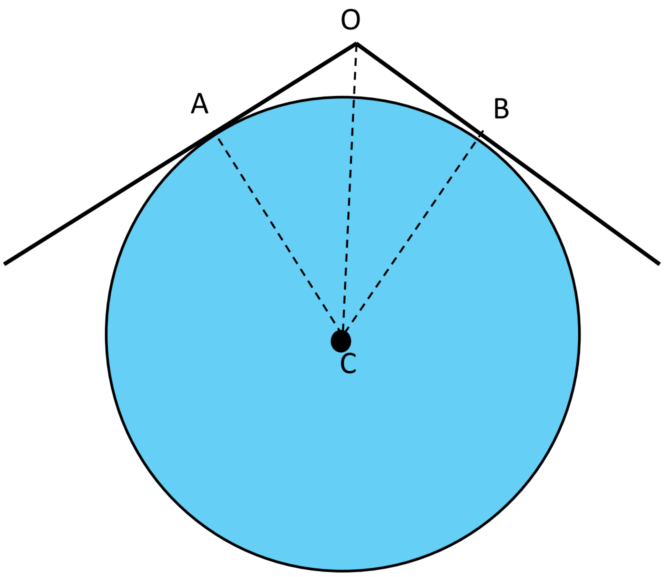
Tangents to a circle that meet at a point are of equal length





Aim:

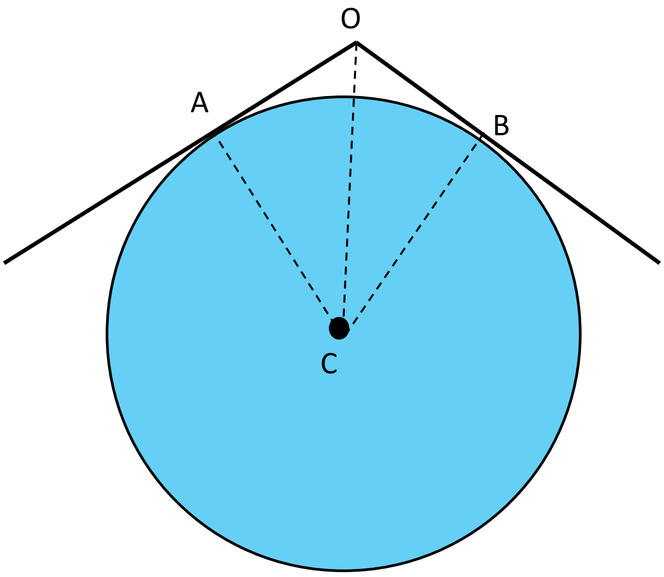
Show that AO = BO



Step One:

Draw a line from the point where the tangents meet to the centre of the circle.

Draw another line from where the tangent touches the circle to the centre of the circle, C.



Step Two:

We now have two triangles. To show that AO = BO, all we need to do is show that the two triangles, AOC and BOC, are congruent.

We are going to use two sides and an angle to prove this.

Length OC is present in both triangles, therefore, we have a side that is the same for both triangles.

Length AC = Length BC because both are radius to the circle, so they are both the same length. This means we have another side the same length for both triangles

Angle CAO = Angle CBO = 90° because a right angle is formed where a radius meets a tangent. We now have an angle that is the same for both triangles.

Putting all these together, we have proved congruency between triangles AOC and BOC. This consequently means that sides AO and BO are of equal length. We have now proved the circle theorem ‘tangents to a circle that meet at a point are the same length’.

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