5 (a) The first four terms in the binomial expansion of $(1+2x)^9$ in ascending powers of x are $1+ax+bx^2+cx^3$. Find the values of the integers a, b and c.

[3 marks]

(b) Hence find the value of the coefficient of x^3 in the expansion of $\left(3-\frac{1}{2}x\right)^2(1+2x)^9$.

[4 marks]

7 (a) The expression $(1-2x)^5$ can be written in the form

$$1+px+qx^2+rx^3+80x^4-32x^5$$

By using the binomial expansion, or otherwise, find the values of the coefficients p, q and r.

[3 marks]

(b) Find the value of the coefficient of x^{10} in the expansion of $(1-2x)^5(2+x)^7$.

[5 marks]

Q	Solution	Mark	Total	Comment
5(a)	$a = 9(2) = 18$	B1		Condone $18x$
	$b = \frac{9(8)(2)^2}{2} = 144$	B1		Condone $144x^2$
	$c = \frac{9(8)(7)(2)^3}{2(3)} = 672$	B1	3	Condone $672x^3$
(b)	$\left(3 - \frac{1}{2}x\right)^2 = 9 - 3x + \frac{1}{4}x^2$	B1		
	x^3 terms from expn of $\left(3 - \frac{1}{2}x\right)^2 (1+2x)^9$ are '9' cx^3 and '−3' bx^3 and '$\frac{1}{4}$' ax^3	M1		Any **two** of the three, **ft** from products of non-zero terms using c's two expansions. May just see the coefficients without the x^3
	'9' cx^3 + '−3' bx^3 + '$\frac{1}{4}$' ax^3	A1F		Relevant products identified, with or without 'x^3', ft on <u>at most two errors</u> in values for a, b, c, $9, -3, \frac{1}{4}$.
	$9 \times 672 - 3 \times 144 + 0.25 \times 18$ (coefficient of x^3 =) 5620.5	A1	4	OE Condone $5620.5x^3$ OE
	Total		**7**	

Q7	Solution	Mark	Total	Comment
(a)	$p = -10$; $q = 40$; $r = -80$	B1 B1 B1	3	Accept correct embedded values for p, q and r within the expansion
(b)	$(2+x)^7 = \ldots\ldots\ldots + mx^5 + nx^6 + x^7$	M1		Attempting to find **at least two of** x^5 term, x^6 term, x^7 term in the expansion of $(2+x)^7$
	$m = 84$, $n = 14$	A1		Either correct. (M1 must be scored). PI by later correct work
	Coefficients of x^{10} terms in expansion of $(1-2x)^5(2+x)^7$ are $-32m + 80n + r$	m1		Identifying at least two of the three products $-32m$, $80n$, r that give x^{10} terms
	Coeff. of $x^{10} = (-32)(84) + (80)(14) + r$ $= -2688 + 1120 + r \quad = -1568 + r$	A1F		**Only** ft c's value of r in **(a)**. If not shown in any of these forms, can be implied by final answer which matches correct evaluation of $(-1568 + \text{c's } r)$
	Coeff. of $x^{10} = -1648$	A1	5	-1648 or left as '$-1648x^{10}$'. Ignore other powers of x terms
	Total		**8**	

4 (a) Find the binomial expansion of $(1-2x)^{\frac{1}{4}}$ up to and including the term in x^2.

[2 marks]

(b) (i) Find the binomial expansion of $(81-16x)^{\frac{1}{4}}$ up to and including the term in x^2.

[3 marks]

(ii) Use your expansion from part **(b)(i)** to find an approximation for $\sqrt[4]{76}$, giving your answer to five decimal places.

[2 marks]

3 (a) Find the binomial expansion of $(1-9x)^{\frac{2}{3}}$ up to and including the term in x^2.

[2 marks]

(b) (i) Find the binomial expansion of $(64-9x)^{\frac{2}{3}}$ up to and including the term in x^2.

[3 marks]

(ii) Use your expansion from part **(b)(i)** to find an estimate for $67^{\frac{2}{3}}$, giving your answer in the form $p+\frac{q}{r}$ where p, q and r are positive integers with $q<r$.

[2 marks]

Q 4	Solution	Mark	Total	Comment
(a)	$(1-2x)^{\frac{1}{4}} = 1 - \frac{1}{2}x + kx^2$	M1		where $k \neq 0$.
	$= 1 - \frac{1}{2}x - \frac{3}{8}x^2$	A1	2	Ignore any higher powers of x.
	Allow ACF for the coefficients - e.g. $1 - \frac{2}{4}x - \frac{12}{32}x^2$ would be **M1 A1** but signs **must** be simplified			
(b)(i)	$(81-16x)^{\frac{1}{4}} = 81^{\frac{1}{4}}\left(1 - \frac{16}{81}x\right)^{\frac{1}{4}}$	B1		**ACF** - e.g. $3\left(1 - \frac{16}{81}x\right)^{\frac{1}{4}}$.
	$\left(1 - \frac{16}{81}x\right)^{\frac{1}{4}}$ $= 1 + \left(\frac{1}{4}\right)\left(-\frac{16}{81}\right)x$ $+ \left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)\left((-)\frac{16}{81}x\right)^2\left(\frac{1}{2!}\right)$	M1		Condone missing brackets if recovered For their $(1+kx)^{\frac{1}{4}}$
	$(81-16x)^{\frac{1}{4}} = 3\left(1 - \frac{4}{81}x - \frac{8}{2187}x^2\right)$	A1	3	ACF: e.g. $3 - \frac{4}{27}x - \frac{8}{729}x^2$ (3 **NOT** $81^{\frac{1}{4}}$)
(ii)	For $\sqrt[4]{76}$ we need $81 - 16x = 76 \rightarrow x = \frac{5}{16}$			or $x = 0.3125$
	Substituting $x = \frac{5}{16}$ in their (b)(i)	M1		**Either** seen or implied by correct answer
	$\sqrt[4]{76} = 2.95263$	A1	2	**CAO - Must** be **correct** to **5 d.p.**
	Beware calculator answer is 2.95259			

(a)	$(1-9x)^{\frac{2}{3}} = 1 + \frac{2}{3}(-9x) + kx^2$	M1		or better with $k \neq 0$
	$= 1 - 6x - 9x^2$	A1	2	Coefficients must be simplified.
(b)(i)	$(64-9x)^{\frac{2}{3}} = 64^{\frac{2}{3}}\left(1 - \frac{9}{64}x\right)^{\frac{2}{3}}$	B1		ACF – e.g. $16\left(1 - \frac{9}{64}x\right)^{\frac{2}{3}}$
	$\left(1 - \frac{9}{64}x\right)^{\frac{2}{3}} = 1 + \frac{2}{3}\times\left(-\frac{9}{64}x\right) + \frac{2}{3}\times\frac{-1}{3}\left(-\frac{9}{64}x\right)^2 \times \frac{1}{2}$	M1		OE – condone missing brackets.
	$(64-9x)^{\frac{2}{3}} = 16 - \frac{3}{2}x - \frac{9}{256}x^2$	A1	3	Accept $16\left(1 - \frac{3}{32}x - \frac{9}{4096}x^2\right)$
(b)(ii)	Substituting $x = -\frac{1}{3}$ (OE) into their (b)(i)	M1		$64 - 9x = 67 \Rightarrow x = -\frac{1}{3}$ or $-\frac{3}{9}$
	$\left(67^{\frac{2}{3}}\right) = 16 + \frac{127}{256}$	A1	2	Must have **correct** expansion in (b)(i) **and** use $x = -\frac{1}{3}$. OE for $\frac{127}{256}$ - e.g. $\frac{254}{512}$ etc. **NMS** is **0/2** since 'Hence'...

4 (a) Find the binomial expansion of $(1-4x)^{-\frac{1}{2}}$ up to and including the term in x^2.

[2 marks]

(b) Find the binomial expansion of $(16+4x)^{\frac{3}{4}}$ up to and including the term in x^2.

[3 marks]

(c) Hence find the expansion of $\sqrt{\dfrac{(16+4x)^{\frac{3}{2}}}{(1-4x)}}$ in ascending powers of x up to and including the term in x^2.

[2 marks]

4 (a) Find the binomial expansion of $(1+5x)^{\frac{1}{5}}$ up to and including the term in x^2.

[2 marks]

(b) (i) Find the binomial expansion of $(8+3x)^{-\frac{2}{3}}$ up to and including the term in x^2.

[3 marks]

(ii) Use your expansion from part **(b)(i)** to find an estimate for $\sqrt[3]{\dfrac{1}{81}}$, giving your answer to four decimal places.

[2 marks]

Q4	Solution	Mark	Total	Comment
(a)	$(1-4x)^{-\frac{1}{2}} = 1 + 2x + kx^2$	M1		$k \neq 0$
	$= 1 + 2x + 6x^2$	A1	2	
(b)	$(16+4x)^{\frac{3}{4}} = 16^{\frac{3}{4}}\left(1+\frac{4x}{16}\right)^{\frac{3}{4}}$	B1		OE e.g. $8\left(1+\frac{x}{4}\right)^{\frac{3}{4}}$
	$\left(1+\frac{4}{16}x\right)^{\frac{3}{4}} = 1 + \frac{3}{4}.\left(\frac{4x}{16}\right) + \frac{3}{4}.-\frac{1}{4}\left(\frac{4x}{16}\right)^2.\frac{1}{2}\ \ldots$	M1		Condone missing / poor use of brackets
	$(16+4x)^{\frac{3}{4}} =$ $8\left(1+\frac{3}{16}x-\frac{3}{512}x^2\right)$ or $8+\frac{3}{2}x-\frac{3}{64}x^2$	A1	3	Must be 8 not $16^{3/4}$
	or $(16+4x)^{\frac{3}{4}} = 16^{\frac{3}{4}} + \frac{3}{4}(16)^{-\frac{1}{4}}(4x) + \frac{3}{4}.-\frac{1}{4}(16)^{-\frac{5}{4}}(4x)^2.\frac{1}{2}\ldots$ **M1** $= 8+\frac{3}{2}x-\frac{3}{64}x^2$ **A2**			
(c)	$(1+2x+6x^2)\left(8+\frac{3}{2}x-\frac{3}{64}x^2\right)$	M1		Setting up the product of their two expansions - be convinced
	$= 8 + \frac{35}{2}x + \frac{3261}{64}x^2$	A1	2	CAO

Q4	Solution	Mark	Total	Comment
(a)	$1+\frac{1}{5}\times 5x + kx^2$	M1		k any non-zero numerical expression
	$1+x-2x^2$	A1	2	Simplified to this
(b) (i)	$(8+3x)^{-\frac{2}{3}} = 8^{-\frac{2}{3}}\left(1+\frac{3}{8}x\right)^{-\frac{2}{3}}$	B1		ACF for $8^{-\frac{2}{3}} = \frac{1}{4}$
	$\left(1+\frac{3}{8}x\right)^{-\frac{2}{3}}$ $=1+\left(-\frac{2}{3}\right)\left(\frac{3}{8}x\right)+\frac{1}{2}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)\left(\frac{3}{8}x\right)^2$	M1		Expand correctly using their $\frac{3}{8}x$ Condone poor use of or missing brackets
	$\frac{1}{4}-\frac{1}{16}x+\frac{5}{256}x^2$	A1	3	Accept $=\frac{1}{4}\left(1-\frac{1}{4}x+\frac{5}{64}x^2\right)$
(ii)	$x=\frac{1}{3}$	M1		$x=\frac{1}{3}$ **used** in their expansion from (b)(i)
	0.2313 (4dp)	A1	2	Note **3** in **4th** decimal place
	Total		7	

3. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a positive constant. Give each term in its simplest form.

(3)

Given that, in this expansion, the coefficients of x and x^3 are equal,

(b) find the exact value of k,

(3)

(c) find the coefficient of x^2

(1)

2. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a non-zero constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^3 in this expansion is 1890

(b) find the value of k.

(3)

1. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(4)

	$\left(1+kx\right)^{10}$	
3. (a)	$1+{}^{10}C_1(kx)+{}^{10}C_2(kx)^2+{}^{10}C_3(kx)^3\ldots$	
	$1+\left({}^{10}C_1\times\ldots\times x\right)+\left({}^{10}C_2\times\ldots\times x^2\right)+\left({}^{10}C_3\times\ldots\times x^3\right)\ldots$	M1
	$=1+10kx,+45k^2x^2+120k^3x^3..$	**B**1, A1
		(3)
(b)	$120k^3=10k$	M1
	$k^2=\dfrac{1}{12}\ so\ k=\ldots$	M1
	$k=\dfrac{\sqrt{3}}{6}$ o.e	A1
		(3)
(c)	$\dfrac{15}{4}$ o.e.	B1
		(1)
		(7 marks)

	$\left(2+kx\right)^7$	
2. (a)	$2^7+{}^7C_1 2^6(kx)+{}^7C_2 2^5(kx)^2+{}^7C_3 2^4(kx)^3\ldots$	
	First term of 128	B1
	$\left({}^7C_1\times\ldots\times x\right)+\left({}^7C_2\times\ldots\times x^2\right)+\left({}^7C_3\times\ldots\times x^3\right)\ldots$	M1
	$=(128..)+448kx+672k^2x^2+560k^3x^3..$	A1, A1
		(4)
(b)	$560k^3=1890$	M1
	$k^3=\dfrac{1890}{560}\ so\ k=$	dM1
	$k=1.5$ o.e.	A1
		(3)
		(7marks)

	$\left(3-\frac{1}{3}x\right)^5$ -	
1.	$3^5+{}^5C_1 3^4(-\frac{1}{3}x)+{}^5C_2 3^3(-\frac{1}{3}x)^2+{}^5C_3 3^2(-\frac{1}{3}x)^3\ldots$	
	First term of 243	B1
	$\left({}^5C_1\times\ldots\times x\right)+\left({}^5C_2\times\ldots\times x^2\right)+\left({}^5C_3\times\ldots\times x^3\right)\ldots$	M1
	$=(243..)-\dfrac{405}{3}x+\dfrac{270}{9}x^2-\dfrac{90}{27}x^3\ldots$	A1
	$=(243..)-135x+30x^2-\dfrac{10}{3}x^3..$	A1
		(4)
		[4]

5. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of f(x) up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

5.	(a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$		
(a) **Way 1**	First term of 16 in their final series		B1
	At least one of $({}^4C_1 \times ... \times x)$ or $({}^4C_2 \times ... \times x^2)$		M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 3**	$\{(2-9x)^4 =\}\ 2^4\left(1-\frac{9}{2}x\right)^4$	First term of 16 in final series	B1
	$= 2^4\left(1+4\left(-\frac{9}{2}x\right)+\frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2+...\right)$	At least one of $(4\times...\times x)$ or $\left(\frac{4(3)}{2}\times...\times x^2\right)$	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
	Parts (b), (c) and (d) may be marked together		
(b)	$A = "16"$	Follow through their value from (a)	B1ft
			[1]
(c)	$\{(1+kx)(2-9x)^4\} = (1+kx)(16 - 288x + \{1944x^2 + ...\})$	*May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).*	M1
	x terms: $-288x + 16kx = -232x$		
	giving, $16k = 56 \Rightarrow k = \frac{7}{2}$	$k = \frac{7}{2}$	A1
			[2]
(d)	x^2 terms: $1944x^2 - 288kx^2$		
	So, $B = 1944 - 288\left(\frac{7}{2}\right)$; $= 1944 - 1008 = 936$	See notes	M1
		936	A1
			[2]
			9

1.	$\left(2-\frac{x}{4}\right)^{10}$		
Way 1	$2^{10} + \binom{10}{1}2^9\left(-\frac{1}{4}x\right) + \binom{10}{2}2^8\left(-\frac{1}{4}x\right)^2 + ...$	For **either** the x term **or** the x^2 term including a correct binomial coefficient with a correct power of x	M1
		First term of 1024	B1
	$= 1024 - 1280x + 720x^2$	**Either** $-1280x$ **or** $720x^2$ (Allow +-1280x here)	A1
		Both $-1280x$ and $720x^2$ (Do not allow +-1280x here)	A1
			[4]
Way 2	$\left(2-\frac{x}{4}\right)^{10} = 2^k\left(1-10\times\frac{x}{8}+\frac{10\times9}{2}\left(-\frac{x}{8}\right)^2\right)$		M1
	$1024(1 \pm ....)$		
	$= 1024 - 1280x + 720x^2$		B1A1 A1
			[4]

1. The binomial series expansion of

$$(1 + ax)^{\frac{2}{3}} \qquad |ax| < 1$$

up to and including the term in x^2 is

$$1 + \frac{1}{2}x + kx^2$$

where a and k are constants.

(a) Find the value of a.

(2)

(b) Find the value of k, giving your answer in its simplest form.

(2)

(c) Hence find the numerical coefficient of x^2 in the series expansion of

$$(4 - 9x)(1 + ax)^{\frac{2}{3}} \qquad |ax| < 1$$

(2)

1. (a) Find the binomial series expansion of

$$\sqrt{4 - 9x}, \quad |x| < \frac{4}{9}$$

in ascending powers of x, up to and including the term in x^2
Give each coefficient in its simplest form.

(5)

(b) Use the expansion from part (a), with a suitable value of x, to find an approximate value for $\sqrt{310}$
Show all your working and give your answer to 3 decimal places.

(3)

1.	$(1+ax)^{\frac{2}{3}} \approx 1+\frac{1}{2}x+kx^2$; $\mathrm{f}(x)=(4-9x)(1+ax)^{\frac{2}{3}}$, $\lvert ax\rvert<1$		
	$\left\{(1+ax)^{\frac{2}{3}} \approx 1+\left(\frac{2}{3}\right)(ax)+\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2+\ldots=1+\frac{2}{3}ax-\frac{1}{9}a^2x^2+\ldots\right\}$		
(a)	$\frac{2}{3}a=\frac{1}{2}$	**see notes**	M1
	$a=\frac{3}{4}$	$a=\frac{3}{4}$ or 0.75	A1 o.e.
			(2)
(b)	$\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(a)^2$	**Either** $\frac{(\frac{2}{3})(\frac{2}{3}-1)}{2!}(a)^2$ **or** $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(a)^2$ **or** $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2$ **or** $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(\text{their } a)^2$ **or** $-\frac{1}{9}a^2$	M1
	$\left\{k=\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}\left(\frac{3}{4}\right)^2\right\} \Rightarrow k=-\frac{1}{16}$	$k=-\frac{1}{16}$ or -0.0625	A1
			(2)
(c)	$\left\{(4-9x)\left(1+\frac{1}{2}x-\frac{1}{16}x^2\right)\right\}$		
	$\{x^2:\}\ -\frac{1}{4}-\frac{9}{2};\ =-\frac{19}{4}$ or -4.75	Either $4(\text{their } k)-\frac{9}{2}$ or $4(\text{their } k)x^2-\frac{9}{2}x^2$	M1
		$-\frac{19}{4}$ or -4.75	A1
			(2)

1. (a)	$\sqrt{(4-9x)}=(4-9x)^{\frac{1}{2}}=\underline{(4)^{\frac{1}{2}}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}=\underline{2}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	<u>B1</u>
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$		
	$=2\left[1-\frac{9}{8}x-\frac{81}{128}x^2+\ldots\right]$	see notes	
	$=2-\frac{9}{4}x;-\frac{81}{64}x^2+\ldots$	isw	A1; A1
			[5]
(b)	$\sqrt{310}=10\sqrt{3.1}=10\sqrt{(4-9(0.1))}$, so $x=0.1$	E.g. For $10\sqrt{3.1}$ (can be implied by later working) **and** $x=0.1$ (or uses $x=0.1$) **Note:** $\sqrt{(100)(3.1)}$ by itself is B0	B1
	When $x=0.1$ $\sqrt{(4-9x)}\approx 2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2+\ldots$	Substitutes their x, where $\lvert x\rvert<\frac{4}{9}$ into all three terms of their binomial expansion	M1
	$=2-0.225-0.01265625=1.76234375$		
	So, $\sqrt{310}\approx 17.6234375=\underline{17.623}$ (3 dp)	17.623 **cao**	A1 **cao**
	Note: the calculator value of $\sqrt{310}$ is 17.60681686 which is 17.607 to 3 decimal places		[3]

2. $$\mathrm{f}(x) = (2 + kx)^{-3}, \quad |kx| < 2, \text{ where } k \text{ is a positive constant}$$

The binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2 is

$$A + Bx + \frac{243}{16}x^2$$

where A and B are constants.

(a) Write down the value of A.

(1)

(b) Find the value of k.

(3)

(c) Find the value of B.

(2)

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3}, \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a fraction in its simplest form.

(6)

(a)	$\{A=\}\ \frac{1}{8}$	$\frac{1}{8}$ or 2^{-3} or 0.125, clearly identified as A **or** as their answer to part (a)	B1 **cao**
			[1]
(b)	$\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$	Uses the x^2 term of the binomial expansion to give	
		either $\frac{(-3)(-4)}{2!}$ **or** $\left(\frac{k}{2}\right)^2$ **or** $\left(\frac{kx}{2}\right)^2$ **or** $\frac{(-3)(-4)}{2}$ **or** 6	M1
		either (their A)$\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$ **or** (their A)$\frac{(-3)(-4)}{2!}\left(\frac{kx}{2}\right)^2$, where (their A) □1, **or** $\frac{3}{16}k^2$ **or** $\frac{3}{16}k^2x^2$ **or** $(2^{-5})\frac{(-3)(-4)}{2!}(kx)^2$ **or** $(2^{-5})\frac{(-3)(-4)}{2!}(k)^2$	M1 o.e.
	$\left\{\text{So, } \left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2 = \frac{243}{16} \Rightarrow \frac{3}{16}k^2 = \frac{243}{16} \Rightarrow k^2 = 81\right\}$		
	So, $k=9$	$k=9$ cao	A1 **cso**
	Note: $k=\pm 9$ with no reference to $k=9$ only is A0		**[3]**
(c)	$"\left(\frac{1}{8}\right)"(-3)\left(\frac{k}{2}\right)$	Uses the x term of the binomial expansion to give **either** (their A)$(-3)\left(\frac{k}{2}\right)$ **or** (their A)$(-3)\left(\frac{kx}{2}\right)$; where (their A) □1, **or** $(2)^{-4}(-3)(k)$ **or** $(2)^{-4}(-3)(kx)$ **or** $-\frac{3k}{16}$	M1
	$\left\{\text{So, } B=\left(\frac{1}{8}\right)(-3)\left(\frac{9}{2}\right) \Rightarrow\right\}\ \underline{B=-\frac{27}{16}}$	$-\frac{27}{16}$ or $-1\frac{11}{16}$ or -1.6875	A1 **cso**
			[2]
			6

1. Way 1	$\left\{\frac{1}{(2+5x)^3}=\right\}(2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$=\underline{(2)^{-3}}\left(1+\frac{5x}{2}\right)^{-3}=\underline{\frac{1}{8}}\left(1+\frac{5x}{2}\right)^{-3}$	$\underline{2^{-3}}$ or $\underline{\frac{1}{8}}$	<u>B1</u>
	$=\left\{\frac{1}{8}\right\}\left[1+(-3)(kx)+\frac{(-3)(-4)}{2!}(kx)^2+\frac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$	see notes	M1 A1
	$=\left\{\frac{1}{8}\right\}\left[1+(-3)\left(\frac{5x}{2}\right)+\frac{(-3)(-4)}{2!}\left(\frac{5x}{2}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(\frac{5x}{2}\right)^3+\ldots\right]$		
	$=\frac{1}{8}\left[1-\frac{15}{2}x+\frac{75}{2}x^2-\frac{625}{4}x^3+\ldots\right]$		
	$=\frac{1}{8}\left[1-7.5x+37.5x^2-156.25x^3+\ldots\right]$		
	$=\frac{1}{8}-\frac{15}{16}x;+\frac{75}{16}x^2-\frac{625}{32}x^3+\ldots$ **or** $\frac{1}{8}-\frac{15}{16}x;+4\frac{11}{16}x^2-19\frac{17}{32}x^3+\ldots$		A1; A1
			[6]

1. (a) Find the binomial expansion of

$$(4 + 5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient in its simplest form.

(5)

(b) Find the exact value of $(4 + 5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(1)

(c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a) and hence find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

(2)

1. (a)	$(4+5x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	<u>B1</u>
	$= \{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
	$= \{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$ $= 2\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\ldots\right]$	See notes below!	
	$= 2+\frac{5}{4}x;-\frac{25}{64}x^2+\ldots$	isw	A1; A1
			[5]
(b)	$\left\{x=\frac{1}{10} \Rightarrow (4+5(0.1))^{\frac{1}{2}} = \sqrt{4.5} = \sqrt{\frac{9}{2}} = \underline{\underline{\frac{3}{\sqrt{2}}}} = \frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}\right\}$		
	$= \frac{3}{2}\sqrt{2}$	$\frac{3}{2}\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e.	B1
			[1]
(c)	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots\ \{=2.121\ldots\}$	See notes	M1
	So, $\frac{3}{2}\sqrt{2} = \frac{543}{256}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}} = \frac{543}{256}$		
	yields, $\sqrt{2} = \frac{181}{128}$ or $\sqrt{2} = \frac{256}{181}$	$\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ **or** $\frac{256}{181}$ etc.	A1 oe