A fair coin is spun 6 times and the random variable T represents the number of tails obtained.

(a) Give two reasons why a binomial model would be a suitable distribution for modelling T.

(2)

(b) Find $P(T = 5)$

(2)

(c) Find the probability of obtaining more tails than heads.

(2)

A second coin is biased such that the probability of obtaining a head is $\frac{1}{4}$

This second coin is spun 6 times.

(d) Find the probability that, for the second coin, the number of heads obtained is greater than or equal to the number of tails obtained.

(3)

A cadet fires shots at a target at distances ranging from 25 m to 90 m. The probability of hitting the target with a single shot is p. When firing from a distance d m, $p = \frac{3}{200}(90 - d)$.

Each shot is fired independently.

The cadet fires 10 shots from a distance of 40 m.

(a) (i) Find the probability that exactly 6 shots hit the target.

(ii) Find the probability that at least 8 shots hit the target.

(5)

A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.

(2)

(b) Find the probability that there are at least 2 defective components in the box.

(3)

2(a)	Only 2 outcomes **Heads** and **Tails oe**		
	Constant probability of **spinning** a **Head/Tail oe**		
	Coin is **spun** a fixed number of times **oe**		
	Each **spin** of the **coin** is independent **oe**	B1 B1	
			(2)
(b)	$T \sim B(6, 0.5)$		
	$P(T \le 5) - P(T \le 4) = 0.9844 - 0.8906$ or $6\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)$ **oe**	M1	
	$= 0.09375$ or $\frac{3}{32}$ oe awrt 0.0938	A1	
			(2)
(c)	$P(T = 4,5,6) = 1 - P(T \le 3)$	M1	
	$= 1 - 0.6563$		
	$= 0.3437$ or $\frac{11}{32}$ awrt 0.344	A1	
			(2)
(d)	$P(H = 3,4,5,6) = 1 - P(H \le 2)$	B1M1d	
	$= 1 - 0.8306$		
	$= 0.1694$ or $\frac{347}{2048}$ awrt 0.169	A1	
			(3)

4. (a)	X is the random variable the Number of successes, $X \sim B(10, 0.75)$	B1	
(i)	$P(X = 6) = (0.75)^6(0.25)^4\ {}^{10}C_6$ or $P(X \le 6) - P(X \le 5)$	M1	
	$= 0.145998$ awrt 0.146	A1	
(ii)	Using $X \sim B(10, 0.75)$		
	$P(X \ge 8) = P(X = 8) + P(X = 9) + P(X = 10)$	M1	
	$= (0.75)^8(0.25)^2\ {}^{10}C_8 + (0.75)^9(0.25)^1\ {}^{10}C_9 + (0.75)^{10}$		
	$= 0.52559$ awrt 0.526	A1	
	Or		
	Using $Y \sim B(10, 0.25)$ and $P(Y \le 2) = 0.5256$		(5)

5	(a)	X represents the number of defective components.		
		$P(X = 1) = (0.99)^9(0.01) \times 10 = 0.0914$	M1A1	
				(2)
	(b)	$P(X \ge 2) = 1 - P(X \le 1)$	M1	
		$= 1 - (p)^{10} - (a)$	A1√	
		$= 0.0043$	A1	
				(3)

In a certain country, 25 per cent of the adult population have blond hair.

(a) A random sample of 30 adults is selected.

Determine the probability that the number of adults with blond hair in the sample is:

(i) exactly 5 ;

[2 marks]

(ii) fewer than 10 ;

[1 mark]

(iii) at least 6 but at most 12 ;

[3 marks]

(iv) more than the mean of the distribution $B(30, 0.25)$.

[3 marks]

(b) The random variable Y has a binomial distribution with parameters n and p .

(i) Given that Y has a mean of 16 and a standard deviation of 2.4 , find values for n and p .

[5 marks]

(ii) Hence determine $P(Y = 20)$.

[2 marks]

Q	Solution	Marks	Total	Comments
7	**Accept 3 dp rounding of probabilities from tables**			**Accept the equivalent percentage answers with %-sign (see GN5)**
(a) (i)	$P(\text{Blond} = 5) = \binom{30}{5}(0.25)^5(0.75)^{25}$ $= 142506 \times 0.00097656 \times 0.00075254$ **or** $= 0.2026 - 0.0979$	M1		Correct expression Can be implied by a **correct** answer Ignore additional expressions
	$=$ **<u>0.104 to 0.105</u>**	A1	**2**	AWFW (0.104728 / 0.1047)
(ii)	$P(\text{Blond} < 10)$ $=$ **<u>0.803</u>**	B1	**1**	AWRT (0.8034)
(iii)	$P(6 \le \text{Blond} \le 12) =$ 0.9784 **or** 0.9493 (p_1)	M1		Seen as **first** term in a **subtraction**
	MINUS 0.2026 **or** 0.3481 (p_2)	M1		Seen as **second** term in a **subtraction**
	$=$ **<u>0.775 to 0.776</u>**	A1	**3**	AWFW (0.7758)
(iv)	Mean $= np = 7.5 \Rightarrow P(\text{Blond} \ge 8)$ $= 1 - 0.5143$	M2		
	$= (1 - 0.6736)$ **or** 0.3264 **or** $= 0.5143$ **or** $= (1 - 0.3481)$ **or** 0.6519	(M1)		
	$=$ **<u>0.485 to 0.486</u>**	A1	**3**	AWFW (0.4857)
(b) (i)	Mean $=$ **<u>$np = 16$</u>**	B1		Equating; seen or used
	$np(1-p)$ **or** npq **or** $\sqrt{np(1-p)}$ **or** $\sqrt{npq}$ $= 2.4^2$ **or** 5.76 **or** 2.4 **but not** $\sqrt{2.4}$	M1		Equating; seen or used
	$np(1-p)$ **or** npq $= 2.4^2$ **or** 5.76	A1		Equating; seen or used
	$p =$ <u>0.64</u> and $n =$ <u>25</u>	A1 A1	**5**	Each CAO
Notes	1 Equating npq to 2.4 (OE) then $\Rightarrow$ B1 M1 A0 A0 A0 (max) followed by M0 A0 in (ii) 2 For any method, answer of $p = 0.64$ (CAO) and $n = 25$ (CAO) $\Rightarrow$ 5 marks 3 For method of 'trial & improvement': B1 (equating/use of $np = 16$); M1 (at least one seen trial combination of **either** integer n **or** $0 < p < 1$); m1 (at least one seen attempt at evaluating npq with **both** integer n **and** $0 < p < 1$ but comparison with 5.76/2.4 **not** required); A1 ($p = 0.64$ CAO); A1 ($n = 25$ CAO)			
(ii)	$P(Y = 20) = \binom{25}{20}(0.64)^{20}(0.36)^5$ $= 53130 \times 0.00013292 \times 0.0060466$	M1		Correct expression Can be implied by a **correct** answer Do **not** ignore additional expressions
	$=$ **<u>0.0426 to 0.0428</u>**	A1	**2**	AWFW (0.042702)

Douglas plays darts, and the probability that he hits the number he is aiming at is 0.87 for any particular dart.

Douglas aims a set of three darts at the number 20; the number of times he is successful can be modelled by $B(3, 0.87)$.

(i) Calculate the probability that Douglas hits 20 twice. [3]

(ii) Douglas aims fifty sets of 3 darts at the number 20. Find the expected number of sets for which Douglas hits 20 twice. [1]

(iii) Douglas aims four sets of 3 darts at the number 20. Calculate the probability that he hits 20 twice for two sets out of the four. [2]

A telesales operator is selling a magazine. Each day he chooses a number of people to telephone. The probability that each person he telephones buys the magazine is 0.1

(a) Suggest a suitable distribution to model the number of people who buy the magazine from the telesales operator each day.

(1)

(b) On Monday, the telesales operator telephones 10 people. Find the probability that he sells at least 4 magazines.

(3)

(c) Calculate the least number of people he needs to telephone on Tuesday, so that the probability of selling at least 1 magazine, on that day, is greater than 0.95

(3)

A disease occurs in 3% of a population.

(a) State any assumptions that are required to model the number of people with the disease in a random sample of size n as a binomial distribution.

(2)

(b) Using this model, find the probability of exactly 2 people having the disease in a random sample of 10 people.

(3)

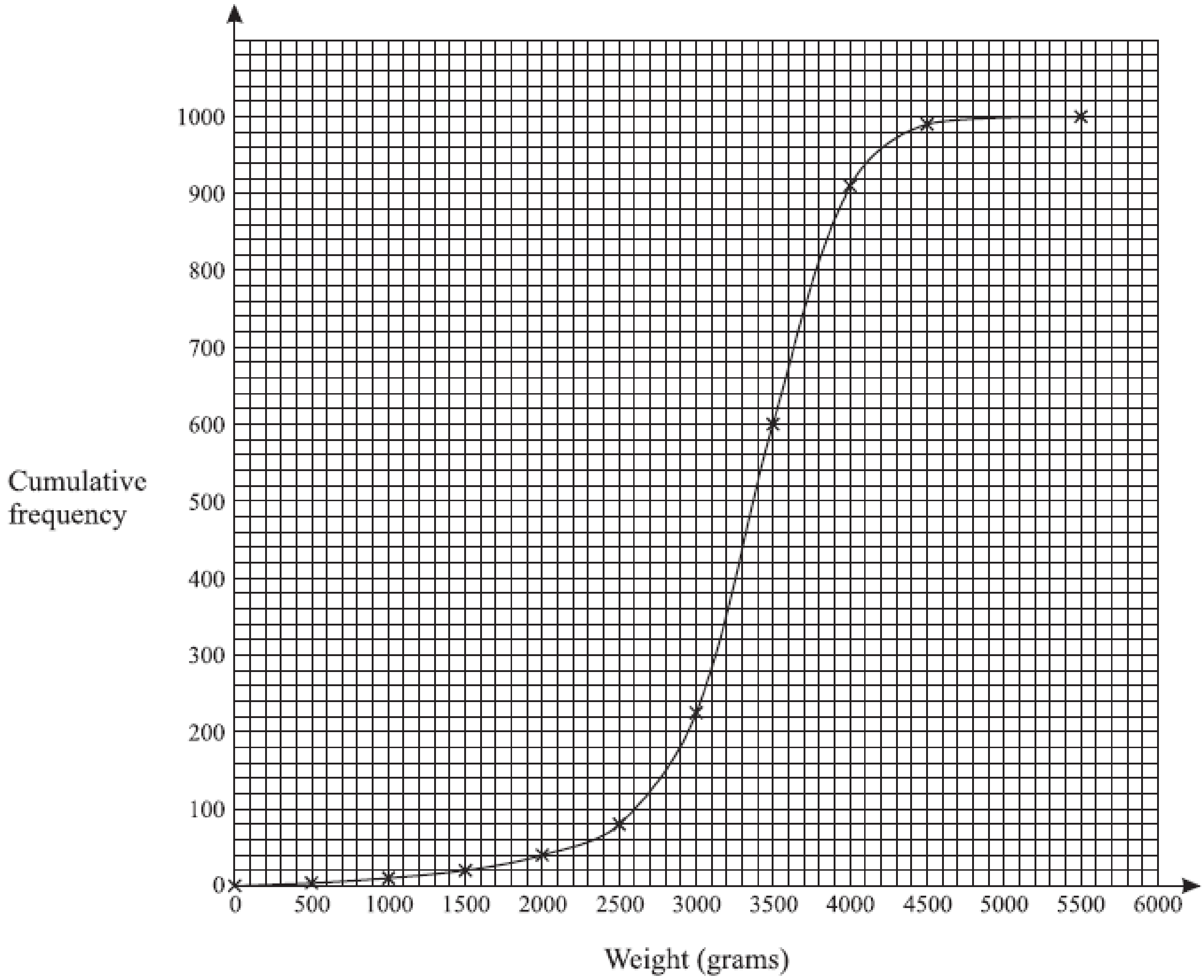
(c) Find the mean and variance of the number of people with the disease in a random sample of 100 people.

(2)

Q5 **(i)**	$P(X=2)=\binom{3}{2}\times 0.87^2\times 0.13=0.2952$	M1 0.87^2 x 0.13 M1 $\binom{3}{2}$ x p^2q with p+q=1 A1 CAO	**3**
(ii)	In 50 throws expect 50 (0.2952) = 14.76 times	B1 FT	**1**
(iii)	P (two 20's twice) = $\binom{4}{2}\times 0.2952^2\times 0.7048^2=0.2597$	M1 $0.2952^2\times 0.7048^2$ A1 FT their 0.2952	**2**
		TOTAL	**6**

7(a)	Distribution $X \sim B(n, 0.1)$	B1	(1)
7(b)	$Y \sim B(10, 0.1)$ $P(Y \geq 4) = 1 - P(Y \leq 3)$ $= 1 - 0.9872$ $= 0.0128$	B1 M1 A1	(3)
7(c)	$0.9^n < 0.05$ or $1-(0.9)^n > 0.95$ $n > 28.4$ $n = 29$ *alternative* B(28,0.1): P(0) = 0.0523 B(29,0.1): P(0) = 0.0471 $n = 29$	M1 A1 A1 M1 A1 A1cao	(3)

1. **(a)**	Occurrences of the disease are independent The probability of catching the disease remains constant.	B1 B1	(2)
(b)	$X \sim \text{Bin}(10, 0.03)$ $P(X=2)=\frac{10\times 9}{2}(0.03)^2(0.97)^8=0.0317$	B1 M1A1	(3)
(c)	$E(X)=100\times 0.03=3$ $\text{Var}(X)=100\times 0.03\times 0.97=2.91$	B1cao B1cao	(2)



(i) Use the diagram to estimate the median and interquartile range of the data. [3]

(ii) Use your answers to part **(i)** to estimate the number of outliers in the sample. [4]

(iii) Should these outliers be excluded from any further analysis? Briefly explain your answer. [2]

(iv) Any baby whose weight is below the 10th percentile is selected for careful monitoring. Use the diagram to determine the range of weights of the babies who are selected. [2]

12% of new-born babies require some form of special care. A maternity unit has 17 new-born babies. You may assume that these 17 babies form an independent random sample.

(v) Find the probability that

(A) exactly 2 of these 17 babies require special care, [3]

(B) more than 2 of the 17 babies require special care. [3]

(vi) On 100 independent occasions the unit has 17 babies. Find the expected number of occasions on which there would be more than 2 babies who require special care. [1]

Q 6 (i)	Median = 3370 $Q_1 = 3050 \quad Q_3 = 3700$ Inter-quartile range = 3700 – 3050 = 650	B1 B1 for Q_3 or Q_1 B1 for IQR	**3**
(ii)	Lower limit $3050 - 1.5 \times 650 = 2075$ Upper limit $3700 + 1.5 \times 650 = 4675$ Approx 40 babies below 2075 and 5 above 4675 so total 45	B1 B1 M1 (for either) A1	**4**
(iii)	Decision based on convincing argument: eg 'no, because there is nothing to suggest that they are not genuine data items and these data may influence health care provision'	E2 for convincing argument	**2**
(iv)	All babies below 2600 grams in weight	B2 CAO	**2**
(v)	(*A*) $X \sim B(17, 0.12)$ $P(X = 2) = \binom{17}{2} \times 0.12^2 \times 0.88^{15} = 0.2878$ (*B*) $P(X > 2)$ $= 1 - \left(0.2878 + \binom{17}{1} \times 0.12 \times 0.88^{16} + 0.88^{17}\right)$ $= 1 - (0.2878 + 0.2638 + 0.1138) = 0.335$	M1 $\binom{17}{2} \times p^2 \times q^{15}$ M1 indep $0.12^2 \times 0.88^{15}$ A1 CAO M1 for $P(X=1) + P(X=0)$ M1 for $1 - P(X \leq 2)$ A1 CAO	**3** **3**
(vi)	Expected number of occasions is 33.5	B1 FT	**1**
		TOTAL	**18**

In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that

(i) exactly 6 ask for water with their meal,

(ii) less than 9 ask for water with their meal.

(5)

A second random sample of 50 customers is selected.

(b) Find the smallest value of n such that

$$P(X < n) \geqslant 0.9$$

where the random variable X represents the number of these customers who ask for water.

(3)

The probability of a telesales representative making a sale on a customer call is 0.15

Find the probability that

(a) no sales are made in 10 calls,

(2)

(b) more than 3 sales are made in 20 calls.

(2)

Representatives are required to achieve a mean of at least 5 sales each day.

(c) Find the least number of calls each day a representative should make to achieve this requirement.

(2)

(d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95

(3)

8(a)	Let X be the random variable the number of customers asking for water.		
(i)	$X \sim B(10,0.6)$ | $Y \sim B(10,0.4)$	B1	
	$P(X=6) = (0.6)^6(0.4)^4 \frac{10!}{6!4!}$ | $P(Y=4) = (0.4)^4(0.6)^6 \frac{10!}{6!4!}$	M1	
	$= 0.2508\ldots$ | $= 0.2508$ | awrt 0.251	A1	
(ii)	$X \sim B(10,0.6)$ | $Y \sim B(10,0.4)$		
	$P(X<9) = 1 - (P(X=10) + P(X=9))$ $= 1 - (0.6)^{10} - (0.6)^9(0.4)^1 \frac{10!}{9!1!}$ | $P(X<9) = 1 - P(Y \le 1)$ $= 1 - 0.0464$	M1	
	$= 0.9536\ldots$ | $= 0.9536\ldots$ | awrt 0.954	A1	
			(5)
(b)	$X \sim B(50,0.6)$	M1	
	$Y \sim B(50,0.4)$		
	$P(X<n) \ge 0.9$		
	$P(Y>50-n) \ge 0.9$ or $P(X<34) = 0.8439$ awrt 0.844		
	$P(Y \le 50-n) \le 0.1$ $P(X<35) = 0.9045$ awrt 0.904/0.905	M1	
	$50-n \le 15$		
	$n \ge 35$		
	$n = 35$	A1	
			(3)
			Total 8

3 (a)	$P(X=0) = 0.85^{10}$ or from tables	M1	
	$= 0.1969$ awrt 0.197	A1	
			(2)
(b)	$P(X>3) = 1 - P(X \le 3)$	M1	
	$= 1 - 0.6477$		
	$= 0.3523$ awrt 0.352	A1	
			(2)
(c)	$n \times 0.15 = 5$	M1	
	$n = 33$ or 34	A1	
			(2)
(d)	$1 - P(X=0) > 0.95$	M1	
	$1 - (0.85)^n > 0.95.$	A1	
	$0.85^n < 0.05$		
	$n > 18.4$		
	$n = 19$	A1	
			(3)
			9

Bhim and Joe play each other at badminton and for each game, independently of all others, the probability that Bhim loses is 0.2

Find the probability that, in 9 games, Bhim loses

(a) exactly 3 of the games,

(3)

(b) fewer than half of the games.

(2)

Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05

Bhim and Joe agree to play a further 60 games.

(c) Calculate the mean and variance for the number of these 60 games that Bhim loses.

(2)

A manufacturer supplies DVD players to retailers in batches of 20. It has 5% of the players returned because they are faulty.

(a) Write down a suitable model for the distribution of the number of faulty DVD players in a batch.

(2)

Find the probability that a batch contains

(b) no faulty DVD players,

(2)

(c) more than 4 faulty DVD players.

(2)

(d) Find the mean and variance of the number of faulty DVD players in a batch.

(2)

Q2	(a)	Let X be the random variable the number of games Bhim loses. $X \sim B(9, 0.2)$	B1
		$P(X \leq 3) - P(X \leq 2) = 0.9144 - 0.7382$ or $(0.2)^3(0.8)^6\frac{9!}{3!6!}$	M1
		$= 0.1762$ $= 0.1762$ awrt 0.176	A1 (3)
	(b)	$P(X \leq 4) = 0.9804$ awrt 0.98	M1A1 (2)
	(c)	Mean = 3 variance $= 2.85, \frac{57}{20}$	B1 B1 (2)

Q1	(a)	$X \sim B(20, 0.05)$	B1 B1 (2)
	(b)	$P(X = 0) = 0.95^{20} = 0.3584859\ldots$ or 0.3585 using tables .	M1 A1 (2)
	(c)	$P(X > 4) = 1 - P(X \leq 4)$	M1
		$= 1 - 0.9974$	
		$= 0.0026$	A1 (2)
	(d)	Mean $= 20 \times 0.05 = 1$	B1
		Variance $= 20 \times 0.05 \times 0.95 = 0.95$	B1 (2)
			Total [8]

5 **(i)** 20% of people in the large town of Carnley support the Residents' Party. 12 people from Carnley are selected at random. Out of these 12 people, the number who support the Residents' Party is denoted by U.

Find

(a) $\mathrm{P}(U \leqslant 5)$, [2]

(b) $\mathrm{P}(U \geqslant 3)$. [3]

3 **(i)** A random variable X has the distribution $\mathrm{B}(8, 0.55)$. Find

(a) $\mathrm{P}(X < 7)$, [1]

(b) $\mathrm{P}(X = 5)$, [2]

(c) $\mathrm{P}(3 \leqslant X < 6)$. [3]

(ii) A random variable Y has the distribution $\mathrm{B}\left(10, \frac{5}{12}\right)$. Find

(a) $\mathrm{P}(Y = 2)$, [2]

(b) $\mathrm{Var}(Y)$. [1]

7 At a factory that makes crockery the quality control department has found that 10% of plates have minor faults. These are classed as 'seconds'. Plates are stored in batches of 12. The number of seconds in a batch is denoted by X.

(i) State an appropriate distribution with which to model X. Give the value(s) of any parameter(s) and state any assumptions required for the model to be valid. **[4]**

Assume now that your model is valid.

(ii) Find

(a) $\mathrm{P}(X = 3)$, **[2]**

(b) $\mathrm{P}(X \geqslant 1)$. **[2]**

(iii) A random sample of 4 batches is selected. Find the probability that the number of these batches that contain at least 1 second is fewer than 3. **[4]**

5ia	Binomial stated or implied 0.9806	B1 B1 2	by use of tables or $0.2^a \times 0.8^b$, $a+b = 12$
b	0.5583 seen 1 – 0.5583 = 0.442 (3 sfs)	M1 M1 A1 3	add 10 corr terms or 1-(add 3 corr terms): M2 or 1– 0.7946 or 0.205 or 1-0.6774 or 0.323 or 1-0.3907 or 0.609 or add 9 terms or 1-(add 2 or 4 terms): M1
ii	${}^{15}C_4 \times 0.3^4 \times 0.7^{11}$ = 0.219 (3 sfs)	M2 A1 3	${}^{15}C_4 \times 0.3^{11} \times 0.7^4$: M1
Total		**8**	

3(i)(a)	0.9368 or 0.937	B1 1	
(b)	0.7799 – 0.5230 or ${}^8C_5 \times 0.45^3 \times 0.55^5$ = 0.2569 or 0.2568 or 0.257	M1 A1 2	Allow 0.9368 – 0.7799
(c)	0.7799 seen – 0.0885 (not 1 – 0.0885) = 0.691 (3 sfs)	M1 M1 A1 3	${}^8C_5 \times 0.45^3 \times 0.55^5 + {}^8C_4 \times 0.45^4 \times 0.55^4 + {}^8C_3 \times 0.45^5 \times 0.55^3$: M2 1 term omitted or wrong or extra: M1
(ii)(a)	${}^{10}C_2 \times (^7/_{12})^8 \times (^5/_{12})^2$ seen = 0.105 (3 sfs)	M1 A1 2	or 0.105 seen, but not ISW for A1
(b)	$2\,^{31}/_{72}$ or $^{175}/_{72}$ or 2.43 (3 sfs)	B1 1	NB $^{12}/_5 = 2.4$: B0
Total		**9**	

7 (i)	Binomial $n = 12$, $p = 0.1$ Plates (or seconds) independent oe Prob of fault same for each plate oe	B1 B1 B1 B1 4	 B(12, 0.1) : B2 NOT: batches indep Comments must be in context Ignore incorrect or irrelevant
(ii)(a)	0.9744 – 0.8891 or ${}^{12}C_3 \times 0.9^9 \times 0.1^3$ = 0.0852 or 0.0853 (3 sfs)	M1 A1 2	
(b)	1 – 0.2824 or $1 - 0.9^{12}$ =0.718 (3 sfs)	M1 A1 2	allow 1 – 0.6590 or $1 - 0.9^{11}$
(iii)	"0.718" and 1 – "0.718" used $(1\text{-}0.718)^4 + 4(1\text{-}0.718)^3 \times 0.718 + {}^4C_2(1\text{-}0.718)^2 \times 0.718^2$ = 0.317 (3 sfs)	B1 M2 A1 4	ft (b) for B1M1M1 M1 for any one term correct (eg opp tail or no coeffs) 1 – P(3 or 4) follow similar scheme M2 or M1 1 – correct wking (= 0.623) B1M2 cao
Total		**12**	

1 20% of packets of a certain kind of cereal contain a free gift. Jane buys one packet a week for 8 weeks. The number of free gifts that Jane receives is denoted by X. Assuming that Jane's 8 packets can be regarded as a random sample, find

(i) $P(X = 3)$, **[3]**

(ii) $P(X \geqslant 3)$, **[2]**

(iii) $E(X)$. **[2]**

(a) For **each** of the three variables described below, state whether the distribution $B(n, p)$ is an appropriate model.
If such a model is appropriate, give values for n and p.
If such a model is **not** appropriate, give a reason why.

(i) Variable U denotes the number of scores of 'five or six' when an unbiased six-sided die is rolled 20 times.

(ii) Variable V denotes the number of tosses of an unbiased coin until exactly 5 heads have been observed.

(iii) Variable W denotes the number of yellow highlighter pens in a random sample of 5 pens, selected without replacement from a box containing 50 highlighter pens, of which exactly 10 are yellow.

[5 marks]

(b) On a particular island, with an adult population of many thousands, 15 per cent of men and 10 per cent of women are left-handed.

Use an appropriate binomial distribution in each case to estimate the probability that:

(i) a sample of 25 men contains at most 3 who are left-handed;

(ii) a sample of 40 women contains at least 2 but at most 6 who are left-handed;

(iii) a sample of 50 women contains more than 40 who are **not** left-handed.

[7 marks]

1			Q1: if consistent "0.8" incorrect or $^1/_8$, $^7/_8$ or 0.02 allow M marks in ii , iii & 1st M1 in i
i	Binomial stated 0.9437 – 0.7969 or $^8C_3 \times 0.2^3 \times 0.8^5$ = 0.147 (3 sfs)	M1 M1 A1 3	or implied by use of tables or $^8C_3$ or $0.2^a \times 0.8^b$ $(a+b = 8)$
ii	1– 0.7969 = 0.203 (3 sf)	M1 A1 2	allow 1– 0.9437 or 0.056(3) or equiv using formula
iii	8 × 0.2 oe 1.6	M1 A1 2	8 × 0.2 = 2 M1A0 1.6 ÷ 8 or $^1/_{1.6}$ M0A0
Total		7	

(a)	(i) Appropriate or Yes (ii) Not appropriate or No (iii) Not appropriate or No	B1		All 3 stated correctly Cannot be implied
(i)	n = <u>20</u>	B1		CAO B(20, 0.33 (OE)) ⇒ B2
	p = <u>2/6 or 1/3 or 0.3r or 0.33 or 33%</u>	B1		CAO
(ii)	**Number of trials or tosses or n is not fixed**	B1		No other alternatives
(iii)	**P(yellow highlighter pen) or p is not constant/not fixed/variable/changes/varies or selection of pens or events is/are dependent/not independent**	B1	**5**	No other alternatives
(b) (i)	$P(M_{LH} \le 3)$ = <u>0.471</u>	B1	**(1)**	AWRT (0.471121)
(ii)	$P(2 \le W_{LH} \le 6)$ = 0.9005 or 0.7937 (p_1)	M1		Seen as **first term** in a **subtraction**
	minus 0.0805 or 0.2228 (p_2)	M1		Seen as **second term** in a **subtraction**
	= <u>0.82</u>	A1	**(3)**	AWRT (0.820001)
(iii)	Use of **B(50, 0.10)** $P(W_{NLH} > 40) = P(W_{LH} \le 9)$ or $P(W_{LH} < 10)$	B1		Seen or used; can be implied by either 0.9755 or 0.9906 seen
	= 0.9755 or 0.9906	M1		
	= <u>0.975 to 0.976</u> **or**	A1		AWFW (0.975462)

The proportions of different colours of loom bands in a box of 10 000 loom bands are given in the table.

Colour	**Blue**	**Green**	**Red**	**Orange**	**Yellow**	**White**
Proportion	0.25	0.25	0.18	0.12	0.15	0.05

(a) A sample of 50 loom bands is selected at random from the box.

Use a binomial distribution with $n = 50$, together with relevant information from the table, to estimate the probability that this sample contains:

(i) exactly 4 **red** loom bands;

[2 marks]

(ii) at most 10 **yellow** loom bands;

[1 mark]

(iii) at least 30 **blue or green** loom bands;

[3 marks]

(iv) more than 35 but fewer than 45 loom bands that are **neither yellow nor white**.

[4 marks]

(b) The random variable R denotes the number of **red** loom bands in a random sample of **300** loom bands selected from the box.

Estimate values for the mean and the variance of R.

[2 marks]

5 **(i)** A random variable X has the distribution B(25, 0.6). Find

(a) $P(X \leqslant 14)$, **[1]**

(b) $P(X = 14)$, **[2]**

(c) $\text{Var}(X)$. **[2]**

(ii) A random variable Y has the distribution B(24, 0.3). Write down an expression for $P(Y = y)$ and evaluate this probability in the case where $y = 8$. **[2]**

(iii) A random variable Z has the distribution B(2, 0.2). Find the probability that two randomly chosen values of Z are equal. **[3]**

Question	Solution	Marks	Total	Comments
6(a)	Accept 3 dp rounding of probabilities from tables — Accept the equivalent percentage answers with %-sign (see GN5)			
(i)	$P(\text{Red} = 4) = \binom{50}{4}(0.18)^4(0.82)^{46}$	M1		Correct expression Can be implied by a correct answer Ignore additional expressions
	$= 230300 \times 0.00104976 \times 0.000108502$			
	$=$ **0.026 to 0.027**	A1	2	AWFW (0.02623)
(ii)	$P(\text{Yellow} \le 10) =$ **0.88**	B1	1	AWRT (0.8801)
(iii)	P(Blue or Green) = **0.5**	B1		CAO; indicated as a value of p or implied by any one of the probabilities opposite
	$P(\text{Blue or Green} \ge 30)$ $= 1 - 0.8987$	M1		
	$=$ **0.101 to 0.102**	A1		AWFW (0.1013)
	$= 1 - 0.9405$ or 0.059 to 0.06	(M1)	3	
(iv)	Using $p =$ **0.2** gives — Using $p =$ **0.8** gives	B1		Either CAO; indicated as a value of p or implied by any one of the probabilities opposite
	0.9393 or 0.9692 (p_1) — **0.9520 or 0.9815**	M1		One of either pair
	MINUS — **MINUS**			
	0.0480 or 0.0185 (p_2) — **0.0607 or 0.0308**	M1		One of matching pair from above
	$=$ **0.89 to 0.892**	A1	4	AWFW (0.8913)
Notes	1 For calculation of individual terms or no method: award **B4** for 0.89 to 0.892 (AWFW); **B3** for 0.92 to 0.922 (AWFW); **B3** for 0.95 to 0.952 (AWFW) 2 Answers involving $(1-p_2) - (1-p_1) \Rightarrow$ (B1) M1 M1 A1 or (B1) M1 M1 or (B1) M1 3 Answers involving $1 - (p_1 - p_2)$ even after $(p_1 - p_2)$ (eg $1 - (0.9393 - 0.0480) = 0.1087$) $\Rightarrow$ max of B1			
(b)	Mean $= 300 \times 0.18 =$ **54**	B1		CAO
	Variance $= 300 \times 0.18 \times 0.82$ $=$ **44.2 to 44.3**	B1	2	AWFW (44.28)

5	i	a	0.414(2)	B1 [1]		
	i	b	0.4142 – 0.2677 = 0.1465 or 0.147 (3 sf) allow 0.146	M1 A1 [2]	${}^{25}C_{14} \times 0.4^{11} \times 0.6^{14}$	or their (i) - 0.2677, dep +ve result: M1
	i	c	25 × 0.6 × 0.4 or 15 - 9 (ie from $np(1-p)$) = 6	M1 A1 [2]	Allow √(25 × 0.6 × 0.4) or 2.45 for M1	
	ii		${}^{24}C_y \times 0.7^{24-y} \times 0.3^y$ oe (${}^{24}C_8 \times 0.7^{16} \times 0.3^8 =$) 0.160 (3 sf)	B1 B1 [2]	Allow other letters for y Allow 0.16	**NB Must see this for 1st B1** **0.16(0) scores only the second B1** **No M-mark for the correct express'n**
	iii		$(0.8^2)^2 + (2\times0.8\times0.2)^2 + (0.2^2)^2$ oe $= \frac{321}{625}$ or 0.5136 or 0.514	M2 A1 [3]	or $0.64^2+0.32^2+0.04^2$ or $\frac{256}{625}+\frac{64}{625}+\frac{1}{625}$ oe	M1 for any correct term or value of term
Total				10		

(a) In a particular country, 35 per cent of the population is estimated to have at least one mobile phone.

A sample of 40 people is selected from the population.

Use the distribution $B(40, 0.35)$ to estimate the probability that the number of people in the sample that have at least one mobile phone is:

(i) at most 15;

(ii) more than 10;

(iii) more than 12 but fewer than 18;

(iv) exactly equal to the mean of the distribution.

[9 marks]

(b) In the same country, 70 per cent of households have a landline telephone connection.

A sample of 50 households is selected from all households in the country.

Stating a necessary condition regarding this selection, estimate the probability that fewer than 30 households have a landline telephone connection.

[4 marks]

6	**Accept 3 dp rounding of probabilities from tables**			**Accept percentage equivalent answers in (a) & (b) but see GN4**
(a) (i)	$P(X \le 15)$ = <u>0.694 to 0.695</u>	B1	(1)	AWFW (0.6946)
(ii)	$P(X > 10)$ = 1 − 0.1215 = <u>0.878 to 0.879</u> = 1 − 0.0644 or 0.935 to 0.936	M1 A1 (M1)	(2)	 AWFW (0.8785)
Note	1 For calculation of individual terms or no method: award **B2** for 0.878 to 0.879 (AWFW); **B1** for 0.935 to 0.936 (AWFW)			
(iii)	$P(12 < X < 18)$ (p_1) (p_2) = 0.8761 **or** 0.9301 **MINUS** 0.3143 **or** 0.2053 = <u>0.561 to 0.562</u>	 M1 M1 A1	(3)	 AWFW (0.5618)
(iv)	Mean of distribution = 40×0.35 = <u>14</u> $P(X=14)$ $= \binom{40}{14} 0.35^{14} 0.65^{26}$ **or** = 0.5721 − 0.4408 = <u>0.131 to 0.132</u>	B1 M1 A1	(3)	CAO; can be implied **Fully correct expression** Can be implied **Correct difference** AWFW (0.1313)
			9	
(b)	Selection is at **random** $P(Y < 30 \mid B(50, 0.7))$ = 1 − 0.9522 = <u>0.047 to 0.048</u> = 1 − 0.9152 or 0.084 to 0.085 = 1 − 0.9749 or 0.025 to 0.026 = 0.952 to 0.953	B1 M2 A1 (M2) (M2) (M1)	4	Statement must include word "random" AWFW (0.0478)

An analysis of the number of vehicles registered by each household within a city resulted in the following information.

Number of vehicles registered	0	1	2	$\geqslant 3$
Percentage of households	18	47	25	10

(a) A random sample of 30 households within the city is selected.

Use a binomial distribution with $n = 30$, together with relevant information from the table in each case, to find the probability that the sample contains:

(i) exactly 3 households with **no** registered vehicles;

[3 marks]

(ii) at most 5 households with **three or more** registered vehicles;

[2 marks]

(iii) more than 10 households with **at least two** registered vehicles;

[3 marks]

(iv) more than 5 households but fewer than 10 households with **exactly two** registered vehicles.

[3 marks]

(b) If a random sample of **150** households within the city were to be selected, estimate the mean and the variance for the number of households in the sample that would have **either one or two** registered vehicles.

[2 marks]

5	No MR or MC in this question			Accept percentage equivalents in (a)
(a) (i)	$p(0) = \underline{0.18}$	B1		CAO; can be implied from working or **correct answer**
	$P(H = 3) = \binom{30}{3}(p)^3(1-p)^{27}$	M1		Correct expression using p = 0.18, 0.47, 0.25 or 0.10 Can be implied by **correct answer** Ignore extra terms
	= $\underline{0.111 \text{ to } 0.112}$	A1	3	AWFW (0.11151)
(ii)	$p(\geq 3) = \underline{0.1}$	B1		CAO; can be implied from working or **correct answer**
	$P(H \leq 5) = \underline{0.926 \text{ to } 0.927}$	B1	2	AWFW (0.9268)
(iii)	$p(\geq 2) = \underline{0.35}$	B1		CAO; can be implied from 0.5078 or 0.3575 (accept 3dp rounding) or **correct answer**
	$P(H > 10) = \underline{1 - (0.5078 \text{ or } 0.3575)}$	M1		Requires "1 – either probability" Accept 3 dp rounding Can be implied by (0.492) but **not** by (0.642 to 0.643)
	= $\underline{0.492}$	A1	3	AWRT (0.4922)
SC	For calculation of individual terms: award B1 B2 for 0.492 (AWRT); award B1 for 0.642 to 0.643 (AWFW)			
(iv)	$\underline{p(=2) = 0.25}$ $P(5 < H < 10)$ = 0.8034 or 0.8943 (p_1)	M1		Accept 3 dp rounding Can be implied by **correct answer**
	MINUS 0.2026 or 0.0979 (p_2)	M1		Accept 3 dp rounding Can be implied by **correct answer**
	= $\underline{0.6 \text{ to } 0.601}$	A1	3	AWFW (0.6008)
(b)	Mean $(\mu \text{ or } \bar{x}) = \underline{108}$	B1		CAO; B(150, 0.72)
	Variance $(\sigma^2 \text{ or } s^2) = \underline{30.2 \text{ to } 30.3}$	B1	2	AWFW (30.24)

An auction house offers items of jewellery for sale at its public auctions. Each item has a reserve price which is less than the lower price estimate which, in turn, is less than the upper price estimate. The outcome for any item is independent of the outcomes for all other items.

The auction house has found, from past records, the following probabilities for the outcomes of items of jewellery offered for sale.

Outcome	**Probability**
Item does not achieve its reserve price	0.15
Item achieves at least its reserve price	0.85
Item achieves at least its lower price estimate	0.50
Item achieves at least its upper price estimate	0.175

For example, the probability that an item achieves at least its lower price estimate but not its upper price estimate is 0.325.

A particular auction includes exactly 40 items of jewellery that may be assumed to be a random sample of such items.

(a) Use binomial distributions to find the probability that:

(i) at most 10 items do not achieve their reserve prices; *(1 mark)*

(ii) 25 or more items achieve at least their lower price estimates; *(2 marks)*

(iii) exactly 2 items achieve at least their upper price estimates; *(2 marks)*

(iv) more than 10 items but fewer than 15 items achieve at least their reserve prices but not their lower price estimates. *(4 marks)*

(b) How many of the 40 items of jewellery would you expect to achieve at least their reserve prices but not their upper price estimates? *(2 marks)*

				for 27
3(a)(i)	$\underline{O \sim \text{B}(40, p)}$			
	$\text{P}(NS \le 10) = \underline{\mathbf{0.97}}$	B1	1	AWRT (0.9701)
(ii)	$\text{P}(LPE \ge 25) = \underline{\mathbf{1 - (0.9231\ \text{or}\ 0.9597)}}$	M1		Requires '1 –' Accept 3 dp rounding Can be implied by (0.0769 to 0.077) but **not** by (0.04 to 0.0403)
	$= \underline{\mathbf{0.077}}$	A1	2	AWRT (0.0769)
(iii)	$\text{P}(UPE = 2) = \binom{40}{2}(0.175)^2(0.825)^{38}$	M1		Correct expression; may be implied by a **correct** answer Ignore extra terms
	$= \underline{\mathbf{0.016}}$	A1	2	AWRT (0.0160)
(iv)	$p = 0.85 - 0.50 = \underline{\mathbf{0.35}}$	B1		CAO; award on value only May be implied by any of four probabilities below or by a **correct** answer
	$\text{P}(10 < X < 15) = \mathbf{0.5721}\ \text{or}\ \mathbf{0.6946}\ (p_1)$	M1		Accept 3 dp rounding May be implied by a **correct** answer
	MINUS **0.1215 or 0.0644** (p_2)	M1		Accept 3 dp rounding May be implied by a **correct** answer
	$= \underline{\mathbf{0.45\ \text{to}\ 0.451}}$	A1	4	AWFW (0.4506)
(b)	$p = 0.85 - 0.175 = \underline{\mathbf{0.675}}$ **or** $p' = \underline{\mathbf{0.325}}$	B1		CAO; may be implied by 27 Each can be found in several ways CAO; may be implied by 13 or 27
	Number $= 40 \times 0.675 = \mathbf{27}$	B1	2	CAO; can be found in several ways

Stopoff owns a chain of hotels. Guests are presented with the bills for their stays when they check out.

(a) Assume that the number of bills that contain errors may be modelled by a binomial distribution with parameters n and p, where $p = 0.30$.

Determine the probability that, in a random sample of 40 bills:

(i) at most 10 bills contain errors;

(ii) at least 15 bills contain errors;

(iii) exactly 12 bills contain errors. *(6 marks)*

(b) Calculate the mean and the variance for **each** of the distributions B(16, 0.20) and B(16, 0.125). *(3 marks)*

(c) Stan, who is a travelling salesperson, always uses *Stopoff* hotels. He holds one of its diamond customer cards and so should qualify for special customer care. However, he regularly finds errors in his bills when he checks out.

Each month, during a 12-month period, Stan stayed in *Stopoff* hotels on exactly 16 occasions. He recorded, each month, the number of occasions on which his bill contained errors. His recorded values were as follows.

2 1 4 3 1 3 0 3 1 0 5 1

(i) Calculate the mean and the variance of these 12 values. *(2 marks)*

(ii) Hence state with reasons which, if either, of the distributions B(16, 0.20) and B(16, 0.125) is likely to provide a satisfactory model for these 12 values. *(3 marks)*

3 (a) (i)	$\underline{E \sim B(40,\ 0.30)}$	M1		Used anywhere in (a) even only by implication from a correct value
	$P(E \le 10) = \underline{\mathbf{0.308\ to\ 0.309}}$	A1	(2)	AWFW (0.3087)
SC	For calc^n of individual terms: award B2 for answer within above range; award B1 for answer within range 0.3 to 0.32			
(ii)	$P(E \ge 15) = \underline{1 - (0.8074 \text{ or } 0.8849)}$	M1		Requires '1 –' Accept 3 dp rounding or truncation Can be implied by 0.192 to 0.193 but not by 0.115 to 0.116
	$= \underline{\mathbf{0.192\ to\ 0.193}}$	A1	(2)	AWFW (0.1926)
SC	For calc^n of individual terms: award B2 for answer within above range; award B1 for answer within range 0.18 to 0.2			
(iii)	$P(E \le 12) = \mathbf{0.5772 - 0.4406}$ or $P(E \le 12) = \binom{40}{12}0.3^{12}0.7^{28}$	M1		Accept 3 dp rounding or truncation Correct expression; may be implied by a correct answer
	$= \underline{\mathbf{0.136\ to\ 0.138}}$	A1	(2)	AWFW (0.1366)
			6	
(b)	Means = $\underline{\mathbf{3.2\ and\ 2}}$	B1		CAO both values; ignore notation *If not labelled, assume order in question*
	Variances = $\underline{\mathbf{2.56\ and\ 1.75}}$	B1 B1	3	CAO each value; ignore notation ISW all subsequent working
(c)(i)	Mean = $\underline{\mathbf{2}}$	B1		CAO value; ignore notation
	Variance = $\underline{\mathbf{2.54\ to\ 2.55\ \ or\ \ 2.33\ to\ 2.34}}$ (SD = 1.59 to 1.6 or 1.52 to 1.53)	B1	2	Any value within either range; ignore notation ISW all subsequent working
(ii)	<u>B(16, 0.20) or eg "One dist^n"</u> **Different/larger** mean **Similar/same** variance or standard deviation	Bdep1		Identification of distribution not required Both; dep on 3.2, 2.56 /1.6 & (c)(i)
	<u>B(16, 0.125) or eg "Other dist^n"</u> **Equal/same** mean **Different/smaller** variance or standard deviation	Bdep1		Identification of distribution not required Both; dep on 2, 1.75/1.3 & (c)(i)
	Neither likely to provide satisfactory model	Bdep1	3	Dep on Bdep1 and on Bdep1

A bin contains a very large number of paper clips of different colours. The proportion of each colour is shown in the table.

Colour	White	Yellow	Green	Blue	Red	Purple
Proportion	0.15	0.15	0.20	0.15	0.22	0.13

(a) Packets are filled from the bin. Each filled packet contains exactly 30 paper clips which may be considered to be a random sample.

Use binomial distributions to determine the probability that a filled packet contains:

(i) exactly 2 purple paper clips; *(3 marks)*

(ii) a **total** of more than 10 red or purple paper clips; *(3 marks)*

(iii) at least 5 but at most 10 green paper clips. *(3 marks)*

(b) Jumbo packets are also filled from the bin. Each filled jumbo packet contains exactly 100 paper clips.

(i) Assuming that the number of paper clips in a jumbo packet may be considered to be a random sample, calculate the mean and the variance of the number of **red** paper clips in a filled jumbo packet. *(2 marks)*

(ii) It is claimed that the proportion of red paper clips in the bin is greater than 0.22 and that jumbo packets do not contain random samples of paper clips.

An analysis of the number of red paper clips in each of a random sample of 50 filled jumbo packets resulted in a mean of 22.1 and a standard deviation of 4.17.

Comment, with numerical justification, on **each** of the two claims. *(3 marks)*

Q	Solution	Marks	Total	Comments
6 (a)(i)	<u>$U \sim \text{B}(30,\ 0.13,\ 0.35 \text{ or } 0.20)$</u>	M1		Used correctly anywhere in (a)
	$\text{P}(P=2) = \binom{30}{2}(0.13)^2(0.87)^{28}$	A1		Can be implied by a correct answer
	$= \underline{0.148 \text{ to } 0.15}$	A1	3	AWFW (0.1489)
(ii)	$p = \underline{0.35}$	B1		CAO
	$\text{P}(R \cup P > 10) = \underline{1 - (0.5078 \text{ or } 0.3575)}$	M1		Requires '1 –' Accept 3 dp rounding or truncation Can be implied by 0.49 to 0.493 but **not** by 0.642 to 0.643
	$= \underline{0.49 \text{ to } 0.493}$	A1	3	AWFW (0.4922)
(iii)	$\text{P}(5 \le G \le 10) = 0.9744 \text{ or } 0.9389 \quad (p_1)$	M1		Accept 3 dp rounding or truncation
	MINUS $0.2552 \text{ or } 0.4275 \quad (p_2)$	M1		Accept 3 dp rounding or truncation
	$= \underline{0.719 \text{ to } 0.72} \quad (p_3)$	A1	3	AWFW (0.7192)
Notes	**1** $p_3 \le 0$ or $p_3 \ge 1 \Rightarrow$ M0 M0 A0 **2** $p_2 - p_1 \Rightarrow$ M0 M0 A0 **3** $(1-p_2) - p_1 \Rightarrow$ M0 M0 A0			**4** $p_1 - (1-p_2) \Rightarrow$ M1 M0 A0 **5** $p_1 \times p_2 \Rightarrow$ M1 M0 A0 **6** $(1-p_2) - (1-p_1) \Rightarrow$ M1 M1 (A1)
(b)(i)	Mean or $\mu = 100 \times 0.22 \quad = \underline{22}$	B1		CAO
	Variance or $\sigma^2 = 100 \times 0.22 \times 0.78$ $= \underline{17.1 \text{ to } 17.2}$	B1	2	AWFW (ignore notation) (17.16) ISW all subsequent working
(ii)	22.1 ≈/= 22 or means similar/equal **or** 0.221 ≈/= 0.22 or proportions similar/equal so **reject claim** (that $p > 0.22$) **or** **accept that** $p = 0.22$	B1		Dependent on 22 seen in (b)(i) or (ii) Accept diff = 0.1 CAO **Correct** (numerical) comparison with **correct** conclusion (even if at end and stated as 'reject (both) claims')
	$\sqrt{17.1 \text{ to } 17.2} = \underline{4.13 \text{ to } 4.15 \approx/= 4.17}$ **or** $\underline{17.1 \text{ to } 17.2 \approx/= 17.3 \text{ to } 17.4}$	B1		Comparison using two values **or** one value + diff (0.02 to 0.04 AWFW) Comparison using two values **or** one value + diff (0.1 to 0.3 AWFW)
	so **reject claim that not random samples** **or** **accept that are random samples**	Bdep1	3	Dependent on previous B1 **Correct** conclusion regarding randomness of sample

The records at a passport office show that, on average, 15 per cent of photographs that accompany applications for passport renewals are unusable.

Assume that exactly one photograph accompanies each application.

(a) Determine the probability that in a random sample of 40 applications:

(i) exactly 6 photographs are unusable;

(ii) at most 5 photographs are unusable;

(iii) more than 5 but fewer than 10 photographs are unusable. *(7 marks)*

(b) Calculate the mean and the standard deviation for the number of photographs that are unusable in a random sample of **32** applications. *(3 marks)*

(c) Mr Stickler processes 32 applications each day. His records for the previous 10 days show that the numbers of photographs that he deemed unusable were

8	6	10	7	9	7	8	9	6	7

By calculating the mean and the standard deviation of these values, comment, with reasons, on the suitability of the $B(32, 0.15)$ model for the number of photographs deemed unusable each day by Mr Stickler. *(4 marks)*

4(a)	$U \sim B(40,\ 0.15)$	M1		Used somewhere in (a)
(i)	$P(U = 6) = 0.6067 - 0.4325$ or $= \binom{40}{6}(0.15)^6(0.85)^{34}$ $= 0.174$	M1 A1	3	Accept 3 dp rounding or truncation Can be implied by a correct answer AWRT (0.1742)
(ii)	$P(U \le 5)$ $= 0.432$ to 0.433	B1	1	AWFW (0.4325)
(iii)	**See supplementary sheet for individual probabilities** $P(5 < U < 10) = 0.9328$ or 0.9701 (p_1) MINUS 0.4325 or 0.2633 (p_2) $= 0.5(00)$ to 0.501	M1 M1 A1	3	Accept 3 dp rounding or truncation but allow 0.97 $p_2 - p_1 \Rightarrow$ M0 M0 A0 $(1-p_2) - p_1 \Rightarrow$ M0 M0 A0 $p_1 - (1-p_2) \Rightarrow$ M1 M0 A0 $(1-p_2) - (1-p_1) \Rightarrow$ M1 M1 (A1) only providing result > 0 Accept 3 dp rounding or truncation AWFW (0.5003)
(b)	Mean or $\mu = 32 \times 0.15$ $= 4.8$ (V or $\sigma^2 =$) $32 \times 0.15 \times 0.85$ or (SD or $\sigma =$) $\sqrt{32 \times 0.15 \times 0.85}$ (SD or σ) $= 2.02$	B1 M1 A1	3	CAO Either numerical expression; ignore terminology May be implied by 4.08 CAO seen or 2.02 AWRT seen AWRT (2.0199) Do not award if labelled V or σ^2
(c)	Mean $= 7.7$ SD $= 1.26$ to 1.34 (Sample) mean is bigger / greater / different or $7.7/32 = 0.24 > 0.15$ and (Sample) SD is smaller / less / different So model appears unsuitable	B1 B1 Bdep1 Bdep1	4	CAO $\left(\sum x = 77\right)$ AWFW $\left(\sum x^2 = 609\right)$ Both; dependent on all previous 5 marks of B1 M1 A1 B1 B1 Can be scored for incorrect (b) re-done correctly in (c) Means & SDs different $\Rightarrow$ Bdep0 OE; dependent on Bdep1