Question pack for Algebra, GCSE Exam

Included:

- 50 Normal Questions, in no particular order of difficulty.
- 15 Challenge Questions
- Exam Tips throughout
- All worked solutions are at the end of the booklet, highlighted in blue.

PLS Suggestion:

- We've included tonnes of questions to help you with your exams. It is completely up to you how you approach them, however, we would suggest doing 4-6 normal questions and one challenge question every time you revise this topic.
- Make a note of your mistakes and go back to the ones you got wrong at the end.
- Good Luck!

Important Information:

- All normal questions vary in difficulty form grade 3 up to grade 7. Challenge questions extend up to grade 9.

- Questions that are in red are non-calculator questions
  For example:

  Question One: Would be a non-calculator question
  Question One: Would be a calculator question

Algebra	Question No.
Algebra Basics	1, 7, 18, 28, 33, 35, 45, 47,
Algebraic Fractions	8, 42, C13
Completing The Square	10, 48, 49, C2, C3,
Expanding Brackets	2, 15,
Factorising	3, 8, 16, 22, 29, 31, 41, 44, C4, C12, C13,
Formulas	18, 19, 38,
Functions	13, 21, 24, 26, 34, 37, C8, C10
Inequalities	11, C14, 23,
Iteration	17, C7
Powers & Roots	4, 36,
Proof	9, 12, 27, 39, C1, C5, C15,
Quadratic Formula	40, C9,
Rearranging Equations	19, 38, 42, 14
Sequences	5, 20, 30, 32, C6,
Simultaneous Equations	6, 14, 43, 50, C11,
Surds	25, 46,

Normal Difficulty Questions

Question 1: One slab is A cm wide and B cm long. 35 slabs completely cover a patio. Work out the area of the patio in terms of A and B.

Question 2: Expand the brackets and simplify as much as possible:

a) 7y(10 - 3y)
b) - 2y(11- 5y)
c) (7y + 5)(6y – 5)

Question 3: Factorise $3y^2 – 15y$.

Question 4: Calculate $\mathbf{27}^{\frac{2}{3}}$.

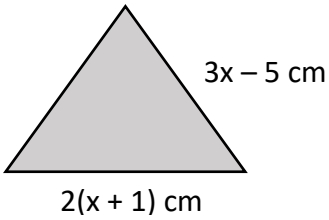
Question 5: Find the n^{th} term of the following sequence and explain why 55 cannot be a term in the sequence.

-12, -6, 0, 6, 12

Question 6: Solve the pair of simultaneous equations:

$$5x + 2y = 13$$
$$x + 2y = 9$$

Question 7: The diagram below is an equilateral triangle. Find the length of one of the sides.



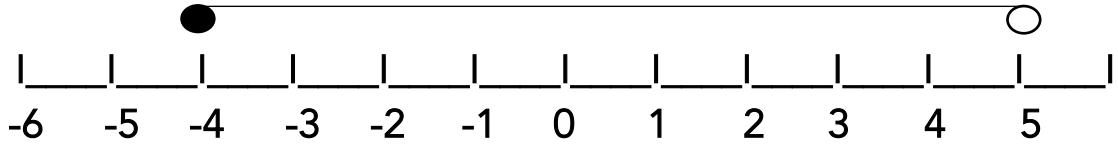
Question 8: Simplify $\frac{y^2-16}{y-4}$.

Question 9: Jamie claims that if you square a number and add 1, the result is a prime number. Find a counter example to prove his statement is not true.

Question 10: Solve $x^2 + 6x - 4 = 0$ by completing the square.

Exam Tip - When inequalities are expressed on a number line, a shaded circle means that number is included and a non-shaded number means that number is not included. See question 11.

Question 11: An inequality is shown on the number line below:



Write down what inequality is being represented.

Question 12: Prove that the sum of the interior angles in a triangle is 180°.

Question 13: A function, f, is such that f(x) = 7x – 11. Find f(12).

Question 14: Solve the simultaneous equations below:

$$x + 3y = 27$$
$$xy = 24$$

Question 15: Expand and simplify (7x – 3)(6x – 5)(x + 2).

Question 16: By factorising, solve the equation $3x^2 - 17x = 28$.

Question 17: Starting with $x_0 = 0$, use the iterative formula below to find an estimate for the solution to $x^3 + 4x - 1 = 0$, up to and including x_3. Round your answer for x_3 to 3.d.p

$$x_{n+1} = \frac{1 - {x_n}^3}{4}$$

Question 18: Solve the equation $4x^2 = 484$.

Question 19: The equation used to find the current in an electrical circuit is:

$$I = \frac{V}{R}$$

Where I = Current, V = Voltage and R = Resistance.

Find the voltage, in Volts, in a circuit that has a current of 10 amps and a resistance of 1.5 ohms.

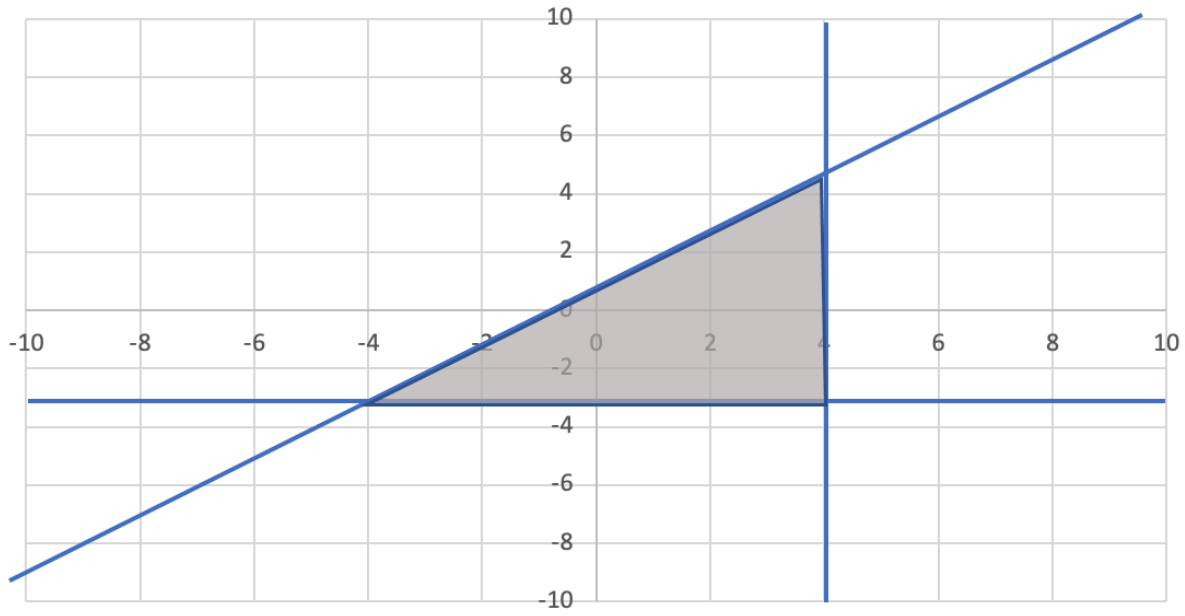
Question 20: The first four terms of a sequence are 4, 9, 14, 19…. Find the n^{th} term of the sequence.

Exam Tip - If you have completed the square and are left with $(x - 5)^2 - 12$, then the minimum point of this graph would be (5, -12).

Question 21: A function, f, is such that f(y) = $\frac{2y-5}{5}$. Find f(15).

Question 22: Factorise fully $x^2 - 25$.

Question 23: On the graph below, the shaded region is bounded by the lines x = 4, y = -3 and y = x + 1. Write down the inequalities that define the shaded region.



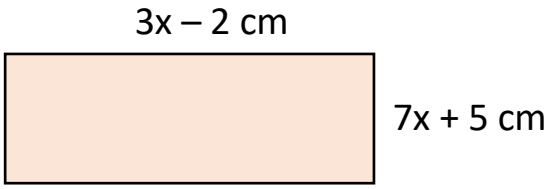
Question 24: f(x) = 2x – 1, g(x) = 7x + 12. Find fg(x).

Question 25: Write $(5 + \sqrt{5})(6 - \sqrt{5})$ in the form $a + b\sqrt{5}$.

Question 26: g(x) = $\frac{x}{6}$ – 1. Find $g^{-1}(x)$.

Question 27: Prove that the sum of two consecutive numbers is odd.

Question 28: Find the area of the rectangle below:



Question 29: Solve $x^2 - x - 6 < 0$.

Question 30: Find the n^{th} term of the following sequence:

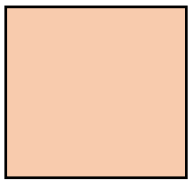
$$\sqrt{3}, \quad 3, \quad 3\sqrt{3}, \quad 9$$

Exam Tip - When solving quadratic inequalities, it is easier to change the <> signs to an = sign while rearranging and solving the equation. Once you have found your x values, don't forget to take the <> signs into consideration! See question 31.

Question 31: Solve $x^2 + x < 20$.

Question 32: The first four terms of a sequence are 13, 19, 25, 31... Find the n^{th} term of the sequence.

Question 33: The diagram below is a square. Find the perimeter of the square in its simplest form.



Question 34: $f(x) = 5x + 3$. Find $f^{-1}(x)$.

Question 35: You are told that $x^2 + 2x - 10 = x^2 + \underline{a}x + \underline{b}$
Find the values of $\underline{a}$ and $\underline{b}$.

Question 36: Write the value of 6^{-3}.

Question 37: A function machine takes a number, x, as an input. It triples it and then subtracts 7. Write this function in algebraic terms in the form $g(x) = ax + b$.

Question 38: Force (F) = Mass (M) x Acceleration (A)
Find A when Force = 108N and Mass = 3kg.

Question 39: Prove that if you sum the squares of two consecutive even numbers, the answer is always a multiple of four.

Question 40: Using the quadratic formula, solve the equation $2x^2 - 2x - 24 = 0$.

Exam Tip - When dealing with quadratic simultaneous equations, rearrange one of the equations to make x or y the subject. Now sub this into the quadratic equation. See question 43.

Question 41: Solve $x^2 - 3x - 10 < 0$.

Question 42: Simplify the equation:

$$\frac{6 - x}{9} + \frac{5}{8 + x} = 2$$

Question 43: Solve the following simultaneous equations:

$$x^2 + 2y = 1$$
$$y = x + 1$$

Question 44: Solve the quadratic equation $x^2 = 4x + 21$

Question 45: Find the largest three consecutive odd numbers that sum to less than 1000.

Question 46: Express $\frac{7+\sqrt{3}}{2-\sqrt{3}}$ in the form a + b$\sqrt{3}$.

Question 47: Alex, Billy and Connor sell cans between them. Alex sells twice as many as Billy and Billy sells 100 more than Connor. 600 are sold altogether, how many do each of them sell?

Question 48: The equation below is in complete the square form. Find the original equation:

$$(x - 1)^2 + 16$$

Question 49: Write the equation below in complete the square form.

$$4x^2 + 8x + 13$$

Question 50: Solve the simultaneous equations:

$$2x^2 + y^2 = 51$$
$$y = x + 6$$

Challenge Questions

Challenge 1: Show that $2^{32} - 1$ is not prime.

Challenge 2: By completing the square, find the coordinates of the minimum point of the graph of $2x^2 - 8x + 19$.

Challenge 3: The expression $x^2 + 6x + 1$ can be written in the form $(x + p)^2 + q$. Find the values of p and q.

Next, use your answer to solve the equation $x^2 + 6x + 1 = 0$, giving your answers in the form $a + \sqrt{b}$.

Challenge 4: Solve $x^2 + 4 < x + 10$.

Challenge 5: Prove that, for any positive whole number n, $(n + 2)^2 - (n - 2)^2$ is divisible by 8.

Challenge 6: A sequence is as follows:

5, 16, 33, 56

Find the n^{th} term of the sequence.

Challenge 7: Explain the relationship between x_1, x_2, and x_3 and the equation $x^3 + 2x^2 + 4 = 0$.

Challenge 8: f(x) = 11x - 4, g(x) = x + 6. Find gf(x).

Challenge 9: $x^2 + 4x - 96 = 0$. Use the quadratic formula to find x.

Challenge 10: $f(x) = 3x^2 - 8$. Find $f^{-1}(x)$.

Challenge 11: Solve the simultaneous equations:

$$x^2 + y^2 = 25$$
$$y = x + 5$$

Challenge 12: Solve $2x^2 - 3 > 5x$.

Challenge 13: Simplify the algebraic fraction below as much as possible:

$$\frac{x^2 - 9x + 20}{x^2 - 11x + 28}.$$

Challenge 14: Solve the inequality:

$$5x^2 < 80$$

Challenge 15: The equation $-2x^3 + 3x = -5$ has one solution. Show that this solution lies in the interval $1.5 < x < 2$.

Answers

(1) $A \times B = AB\text{cm}^2$

$35 \times AB = 35AB\text{cm}^2$

(2) a) $7y(10-3y) = 70y - 21y^2$

b) $-2y(11-5y) = -22y + 10y^2$

c) $(7y+5)(6y-5)$

$= 42y^2 - 35y + 30y - 25$

$= 42y^2 - 5y - 25$

(3) $3y^2 - 15y$

$= 3y(y-5)$

(4) $\sqrt[3]{27} = 3$

$3^2 = 9$

(5) $-12, -6, 0, 6, 12$

$+6 \quad +6 \quad +6 \quad +6$

Difference between terms = +6

$6n$

$6n$:	6	12	18	24	30
Original :	-12	-6	0	6	12
ifference :	-18	-18	-18	-18	-18

To go from $6n \rightarrow$ original, -18

$6n - 18$

$6n - 18 = 55$

$6n = 73$ → 73 not divisible by 6

(6)

$5x + 2y = 13$ (1)

$x + 2y = 9$ (2)

(1) − (2)

$4x = 4$

$x = 1$

Sub $x = 1$ into $x + 2y = 9$

$1 + 2y = 9$

$2y = 8$

$y = 4$

(7) "Equilateral triangle" so we know all sides are the same length

$3x - 5 = 2(x + 1)$

$3x - 5 = 2x + 2$

$x - 5 = 2$

$x = 7$

Sub $x = 7$ into $3x - 5$

$3(7) - 5 =$ 16 cm

(8)

$$\frac{y^2 - 16}{y - 4}$$

$$= \frac{(y + 4)(y - 4)}{(y - 4)}$$

$(\div (y-4))$

$$= \frac{y + 4}{1}$$

$$= y + 4$$

(9) $1^2 + 1 = 1 + 1 = 2$ = prime

$2^2 + 1 = 4 + 1 = 5$ = prime

$3^2 + 1 = 9 + 1 = 10$ not prime

(10) $x^2 + 6x - 4 = 0$

$(x+3)^2 - (3)^2 - 4 = 0$

$(x+3)^2 - 9 - 4 = 0$

$(x+3)^2 - 13 = 0$

$(x+3)^2 = 13$

$x + 3 = \pm\sqrt{13}$

$x = -3 \pm \sqrt{13}$

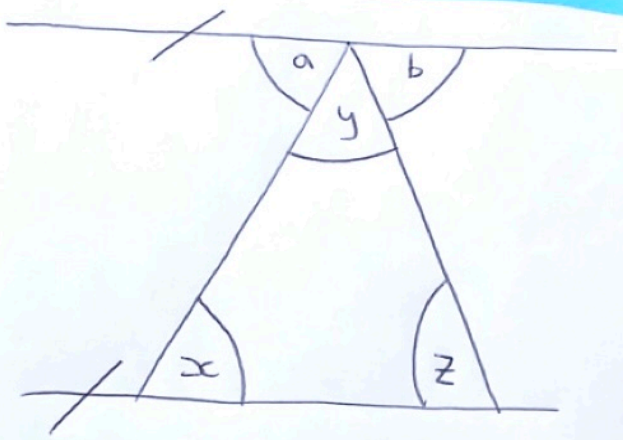
(11)

● = $\leq$ or $\geq$

○ = $<$ or $>$

$-4 \leq x < 5$

(12)



$\angle a + \angle y + \angle b = 180^\circ$ because angles on a straight line sum to 180°

$\angle a = \angle x$ because alterate angles are equal

$\angle b = \angle z$ because alternate angles are equal

Therefore $\angle a + \angle y + \angle b = 180^\circ$ can be changed to $\angle x + \angle y + \angle z = 180^\circ$

13

$f(x) = 7x - 11$

$f(12) = 7(12) - 11$

$= 84 - 11$

$= 73$

(14) $x + 3y = 27$ ①

$xy = 24$ ②

Rearrange ①:

$x = 27 - 3y$

Sub ① into ②

$(27 - 3y)y = 24$

$27y - 3y^2 = 24$

$3y^2 - 27y + 24 = 0$ $(\div 3)$

$y^2 - 9y + 8 = 0$

$-1 + -8 = -9$

$-1 \times -8 = 8$

$(y - 1)(y - 8) = 0$

$y = 1$ or $y = 8$

$x + 3(1) = 27$

$x = 24$

$x + 3(8) = 27$

$x = 3$

$y = 1, x = 24$

$y = 8, x = 3$

15

$$(7x-3)(6x-5)(x+2)$$

$$=(42x^2-35x-18x+15)(x+2)$$

$$=(42x^2-53x+15)(x+2)$$

$$=42x^3+84x^2-53x^2-106x+15x+30$$

$$=42x^3+31x^2-91x+30$$

(16) $3x^2 - 17x = 28$

$3x^2 - 17x - 28 = 0$

$3 \times -28 = -84$

$-4 + 21 = -17$

$-4 \times 21 = -84$

$3x^2 + 4x \mid -21x - 28 = 0$

$x(3x + 4) \quad -7(3x + 4)$

$(3x + 4)(x - 7) = 0$

$x = -\frac{4}{3}$ or $x = 7$

(17)

$x_0 = 0$

$x_1 = \dfrac{1 - (0)^3}{4} = \dfrac{1}{4}$

$x_2 = \dfrac{1 - \left(\frac{1}{4}\right)^3}{4} = \dfrac{63}{256}$

$x_3 = \dfrac{1 - \left(\frac{63}{256}\right)^3}{4} = 0.246 \text{ (3.d.p)}$

(18) $4x^2 = 484$

$x^2 = 121$

$x = \pm 11$

(19) $I = \frac{V}{R}$

$10 = \frac{V}{1.5}$

$V = 15$ volts

(20) 4 9 14 19

+5 +5 +5

Difference between terms = +5

5n

5n's	5	10	15	20
Original 's	4	9	14	19
5n → orig	-1	-1	-1	-1

5n - 1

(21) $f(y) = \frac{2y - 5}{5}$

$f(15) = \frac{2(15) - 5}{5}$

$= \frac{25}{5}$

$= 5$

(22) $x^2 - 25$

$= (x + 5)(x - 5)$

(23)

$y \geq -3$

$x \leq 4$

$y \leq x + 1$

(24) $f(x) = 2x - 1$

$g(x) = 7x + 12$

$fg(x)$

$= f(g(x))$

$= 2(7x + 12) - 1$

$= 14x + 24 - 1$

$= 14x + 23$

25

$$(5+\sqrt{5})(6-\sqrt{5})$$

$$= 30 - 5\sqrt{5} + 6\sqrt{5} - 5$$

$$= 25 + \sqrt{5}$$

26

$$g(x) = \frac{x}{6} - 1$$

$$y = \frac{x}{6} - 1$$

$$y + 1 = \frac{x}{6}$$

$$6(y+1) = x$$

$$x = 6y + 6$$

$$g^{-1}(x) = 6x + 6$$

(27)

$(n) + (n+1)$

$= 2n+1$

$2n+1$ is definition of odd number

(28)

$(3x-2)(7x+5)$

$= 21x^2 + 15x - 14x - 10$

$= 21x^2 + x - 10$ cm^2

(29)

$x^2 - x - 6 < 0$

$2 + -3 = -1$

$2 \times -3 = -6$

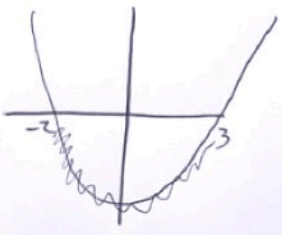
$x^2 + 2x \mid -3x - 6 = 0$

$x(x+2) \mid -3(x+2)$

$(x+2)(x-3) = 0$

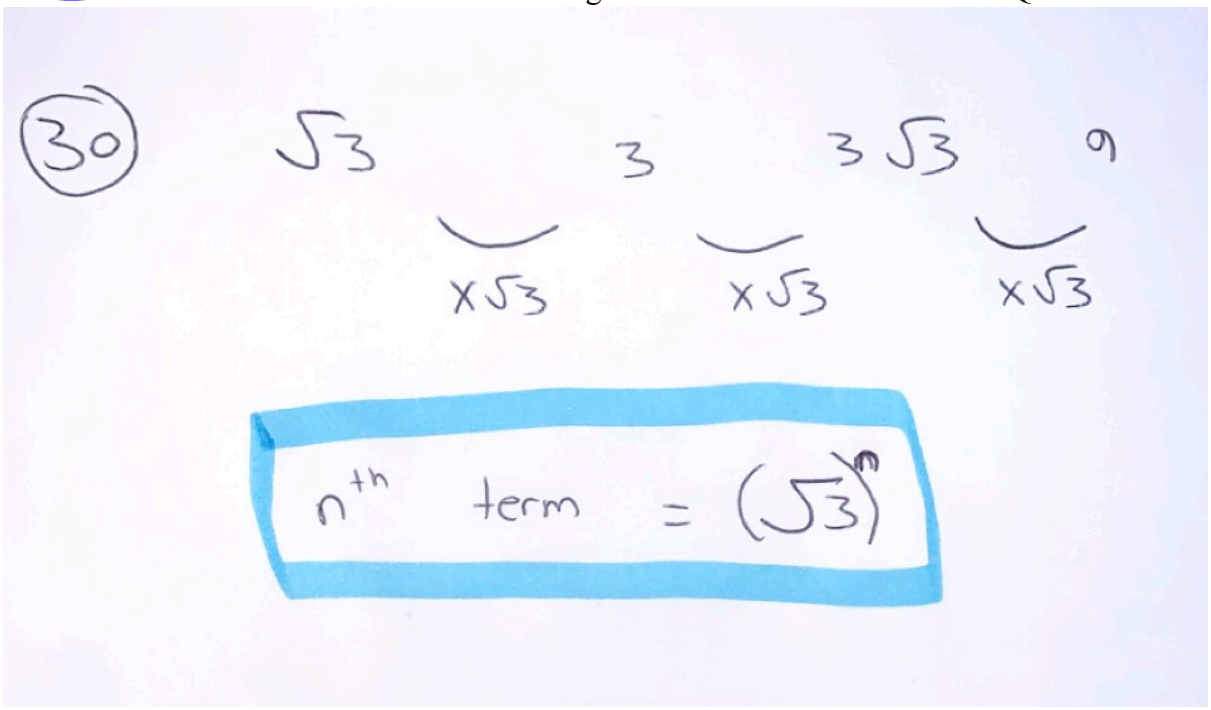
$x = -2$

$x = 3$



Dealing with <0 so shade in below x axis

$-2 < x < 3$


30
√3
3
3√3
9
×√3
×√3
×√3
nth term = (√3)^n

(31)

$x^2 + x < 20$

$x^2 + x - 20 < 0$

$x^2 + x - 20 = 0$

$-4 + 5 = 1$

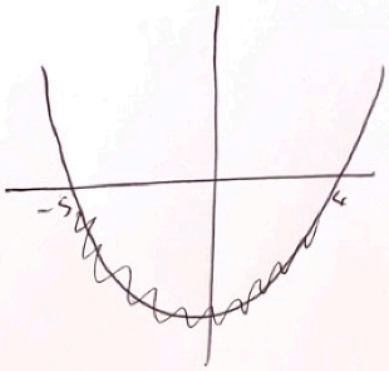
$-4 \times 5 = -20$

$x^2 - 4x \mid +5x - 20 = 0$

$x(x-4) \quad 5(x-4)$

$(x+5)(x-4) < 0$

$x = -5 \qquad x = 4$



Dealing with < 0 so shade in bit below x axis

$-5 < x < 4$

(32)

13 19 25 31

+6 +6 +6

6n	6	12	18	24
original	13	19	25	31
n → original	+7	+7	+7	+7

6n +7

(33) All sides on a square are same length

$5x - 3 = 4x + 8$

$x - 3 = 8$

$x = 11$

$4x + 8$

$4(11) + 8 = 52$

$52 \times 4 =$ 208 cm perimeter

(34) $f(x) = 5x + 3$

$y = 5x + 3$

$y - 3 = 5x$

$\frac{y-3}{5} = x$

$f^{(-1)}(x) = \frac{x-3}{5}$

(35) $a = 2, \quad b = -10$

(36) $6^{-3} = \frac{1}{6^3}$

$= \frac{1}{216}$

(37) $g(x) = 3x - 7$

(38) $F = ma$

$108 = 3a$

$a = 36 ms^{-2}$

(39)

$2n$, $2n+2$

$(2n)^2 + (2n+2)^2$

$= 4n^2 + 4n^2 + 8n + 4$

$= 8n^2 + 8n + 4$

$= 4(2n^2 + 2n + 1)$

↑ multiple of four

40

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$a = 1,\ b = -1,\ c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{--1 \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 - -48}}{2}$$

$$x = \frac{1 \pm \sqrt{49}}{2}$$

$$x = \frac{1+7}{2}, \quad x = \frac{1-7}{2}$$

$$x = \frac{8}{2} \qquad x = \frac{-6}{2}$$

$$x = 4 \qquad x = -3$$

(41)

$x^2 - 3x - 10 < 0$

$2 + -5 = -3$

$2 \times -5 = -10$

$(x+2)(x-5)$

$x = -2, \; x = 5$

Dealing with < 0 so highlight below x axis

$-2 < x < 5$

(42)

$$\frac{6-x}{9} + \frac{5}{8+x} = 2$$

$(\times 9)$

$$6-x + \frac{5(9)}{8+x} = 2(9)$$

$(\times (8+x))$

$$(6-x)(8+x) + 45 = 18(8+x)$$

$$48+6x-8x-x^2+45 = 144+18x$$

$$-x^2-2x+93 = 144+18x$$

$$-x^2-20x-51 = 0$$

$(\times -1)$

$$x^2+20x+51 = 0$$

$$3 + 17 = 20$$

$$3 \times 17 = 51$$

$$(x+3)(x+17)$$

$x = -3, \quad x = -17$

(43) $x^2 + 2y = 1$ (1)

$y = x + 1$ (2)

Sub (2) into (1)

$x^2 + 2(x+1) = 1$

$x^2 + 2x + 2 = 1$

$x^2 + 2x + 1 = 0$

$\frac{1}{-} + \frac{1}{-} = 2$

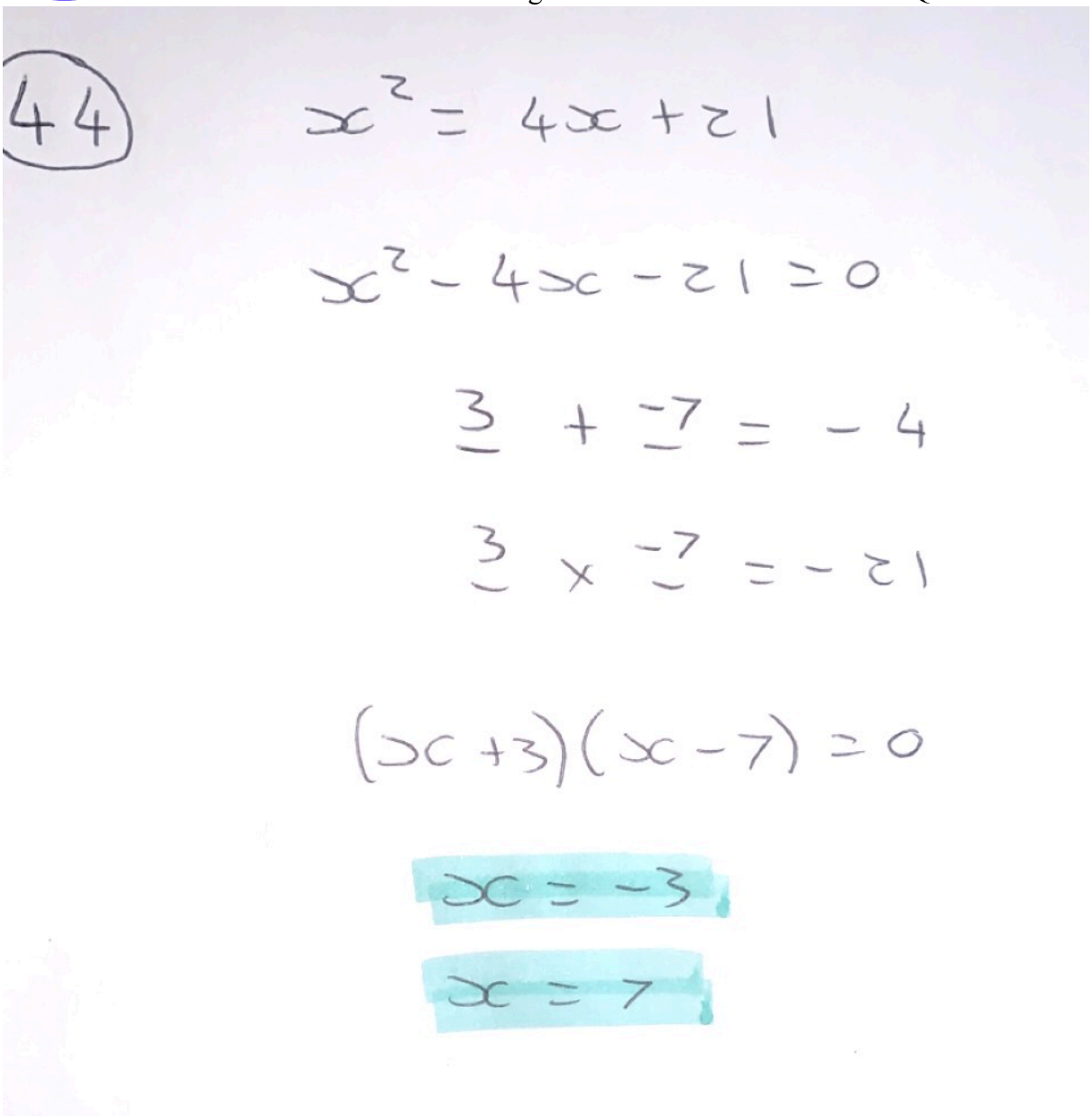
$\frac{1}{-} \times \frac{1}{-} = 1$

$(x+1)(x+1)$

$x = -1$

$y = (-1) + 1$

$y = 0$


44
$x^2 = 4x + 21$
$x^2 - 4x - 21 = 0$
$3 + -7 = -4$
$3 \times -7 = -21$
$(x+3)(x-7) = 0$
$x = -3$
$x = 7$

(45) $(2n+1)+(2n+3)+(2n+5)$

$= 6n+9$

$6n+9 < 1000$

$6n < 991$

$n < 165.167$

try $n = 165$

$2(165)+1 = 331$

$2(165)+3 = 333$

$2(165)+5 = 335$

$331+333+335 = 999$

Largest three are 331, 333, 335

(46)

$$\frac{7+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{14+7\sqrt{3}+2\sqrt{3}+3}{4+2\sqrt{3}-2\sqrt{3}-3}$$

$$= \frac{17+9\sqrt{3}}{1}$$

$$= 17+9\sqrt{3}$$

(47)

Alex = $2x$

Billy = x

Connor = $x-100$

$$2x+x+x-100 = 4x-100$$

$$4x-100 = 600$$

$$4x = 700, \quad x = 175$$

Alex = 350
Billy = 175
Connor = 75

48 $(x-1)^2 + 16$

$(x-1)^2 - (-1)^2 + c$

$\left[-(-1)^2 + c\right] = 16$

$-1 + c = 16$

$c = 17$

$x^2 - 2x + 17$

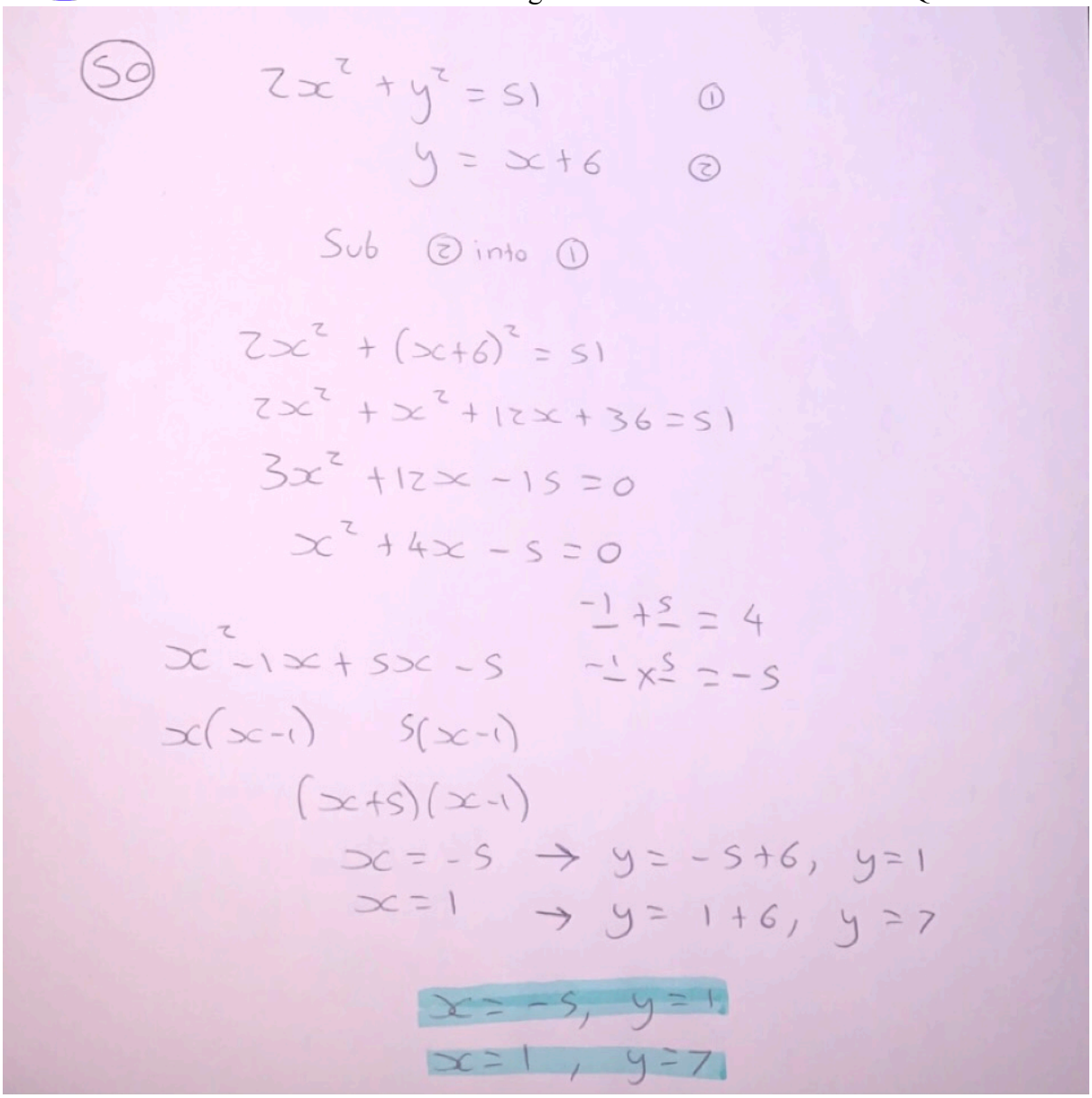
49 $4x^2 + 8x + 13$

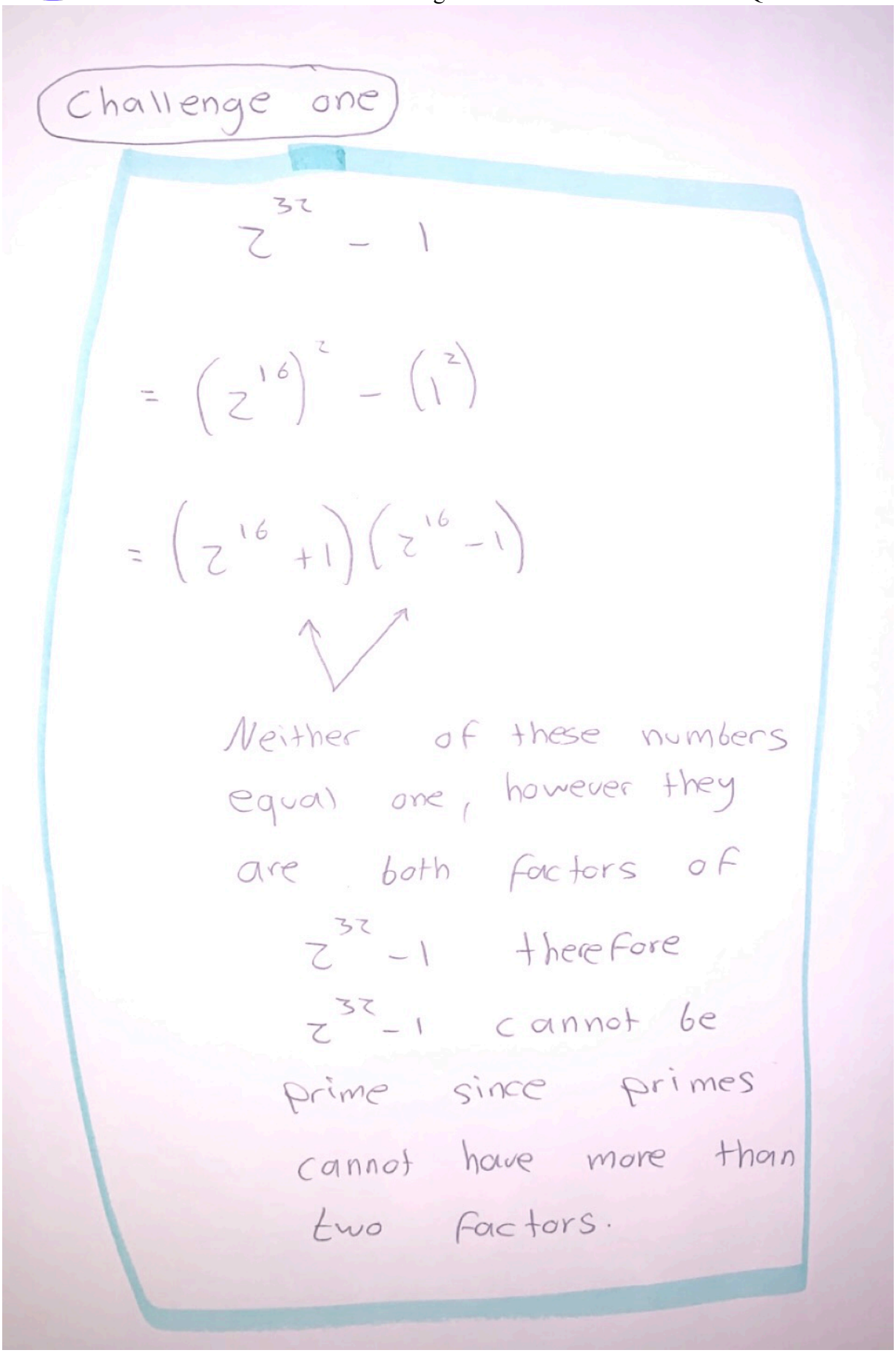
$= 4(x^2 + 2x) + 13$

$= 4((x+1)^2 - (1)^2) + 13$

$= 4(x+1)^2 - 4 + 13$

$= 4(x+1)^2 + 9$


(50)
$2x^2 + y^2 = 51$ ①
$y = x + 6$ ②
Sub ② into ①
$2x^2 + (x+6)^2 = 51$
$2x^2 + x^2 + 12x + 36 = 51$
$3x^2 + 12x - 15 = 0$
$x^2 + 4x - 5 = 0$
$-1 + 5 = 4$
$-1 \times 5 = -5$
$x^2 - 1x + 5x - 5$
$x(x-1)$ $5(x-1)$
$(x+5)(x-1)$
$x = -5 \rightarrow y = -5 + 6, \; y = 1$
$x = 1 \rightarrow y = 1 + 6, \; y = 7$
$x = -5, \; y = 1$
$x = 1, \; y = 7$


Challenge one
$z^{32} - 1$
$= (z^{16})^2 - (1^2)$
$= (z^{16} + 1)(z^{16} - 1)$
Neither of these numbers equal one, however they are both factors of $z^{32} - 1$ therefore $z^{32} - 1$ cannot be prime since primes cannot have more than two factors.

Challenge two

$$2x^2 - 8x + 19$$

$$2\left(x^2 - 4x\right) + 19$$

$$2\left((x-2)^2 - (-2)^2\right) + 19$$

$$2(x-2)^2 - 8 + 19$$

$$2(x-2)^2 + 11$$

Minimum point $= (2, 11)$

Challenge three

$$x^2 + 6x + 1$$

$$= (x+3)^2 - (3)^2 + 1$$

$$= (x+3)^2 - 8$$

$p = 3, \quad q = -8$

$$(x+3)^2 - 8 = 0$$

$$(x+3)^2 = 8$$

$$x + 3 = \pm\sqrt{8}$$

$$x = -3 \pm \sqrt{8}$$

Challenge four

$x^2 + 4 < x + 10$

$x^2 - x - 6 < 0$

$x^2 - x - 6 = 0$

$\frac{2}{-} + \frac{-3}{-} = -1$

$\frac{2}{-} \times \frac{-3}{-} = -6$

$(x + 2)(x - 3) = 0$

$x = -2$

$x = 3$

Dealing with < 0 so highlight below x axis

$-2 < x < 3$

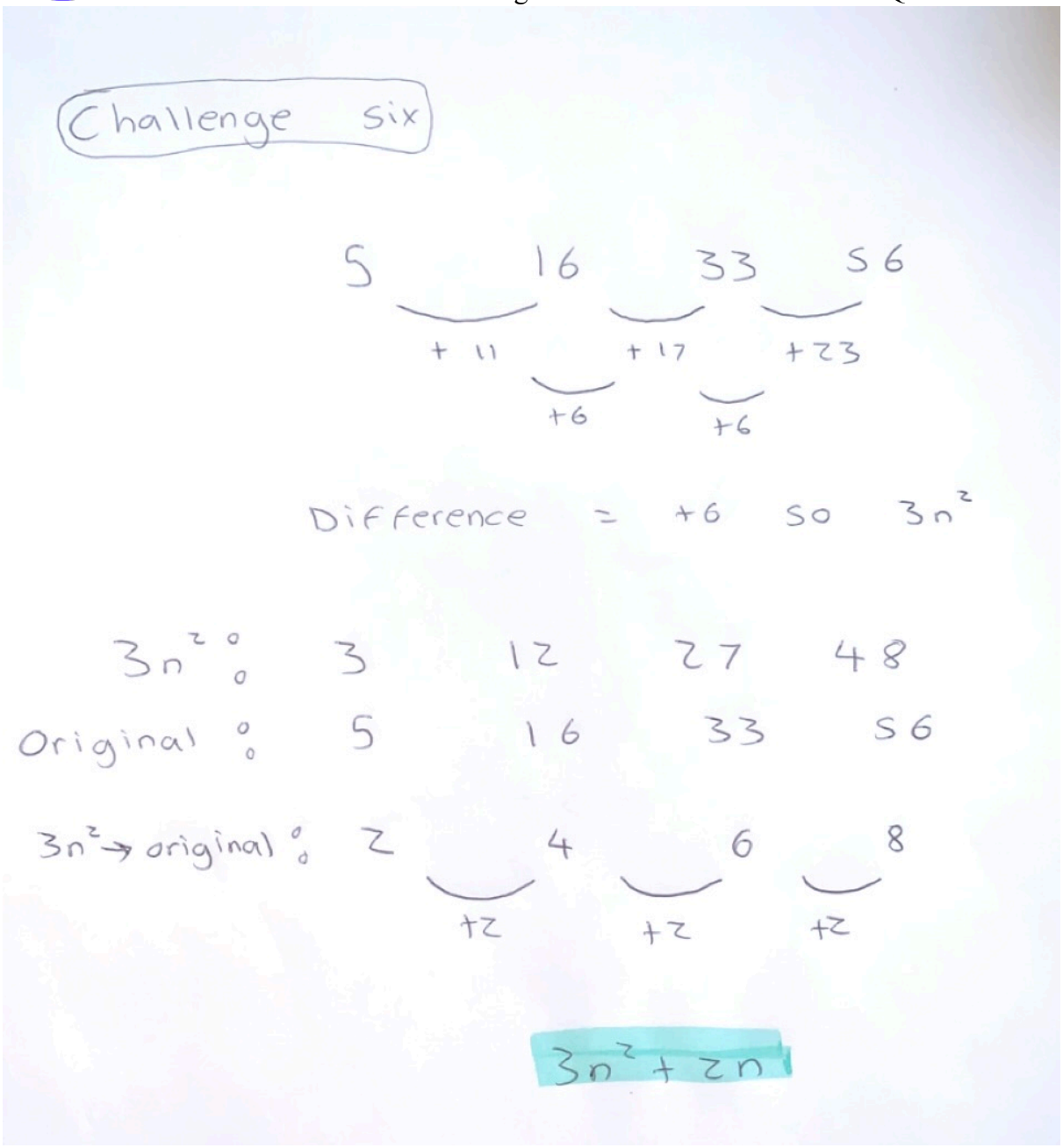
Challenge five

$$(n+2)^2 - (n-2)^2$$

$$= (n^2 + 4n + 4) - (n^2 - 4n + 4)$$

$$= 8n$$

$$= 8(n) \rightarrow \text{divisible by } 8$$


Challenge six
5 16 33 56
+11 +17 +23
+6 +6
Difference = +6 so 3n²
3n² : 3 12 27 48
Original : 5 16 33 56
3n² → original : 2 4 6 8
+2 +2 +2
3n² + 2n

challenge seven

x_1, x_2, x_3 are all estimates to the solution of the equation, $x^3 + 2x^2 + 4 = 0$

Each one is progressively more accurate.

Challenge eight

$f(x) = 11x - 4$

$g(x) = x + 6$

$gf(x)$

$= g(f(x))$

$= (11x - 4) + 6$

$= 11x + 2$

Challenge nine

$$x^2 + 4x - 96 = 0$$

$$a = 1, \quad b = 4, \quad c = -96$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-96)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 384}}{2}$$

$$x = \frac{-4 \pm \sqrt{400}}{2}$$

$$x = \frac{-4 \pm 20}{2} = -2 \pm 10$$

$$x = 8, \quad x = -12$$

Challenge ten

$$f(x) = 3x^2 - 8$$

$$y = 3x^2 - 8$$

$$y + 8 = 3x^2$$

$$\frac{y+8}{3} = x^2$$

$$x = \sqrt{\frac{y+8}{3}}$$

$$f^{-1}(x) = \sqrt{\frac{x+8}{3}}$$

Challenge eleven

$x^2 + y^2 = 25$ ①

$y = x + 5$ ②

Sub ② into ①

$x^2 + (x+5)^2 = 25$

$x^2 + (x^2 + 10x + 25) = 25$

$2x^2 + 10x = 0$

$x^2 + 5x = 0$

$x(x+5) = 0$

$x = 0$, $x = -5$

$y = 0 + 5$ $\quad$ $y = -5 + 5$

$y = 5$ $\quad$ $y = 0$

$x = 0, \ y = 5$

$x = -5, \ y = 0$

Challenge twelve

$2x^2 - 3 > 5x$

$2x^2 - 5x - 3 > 0$

$2x - 3 = -6$

$-1 + -6 = -5$

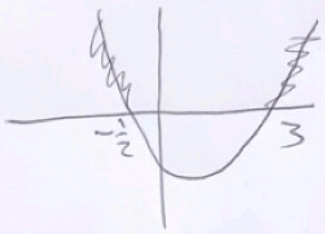
$-1 \times -6 = -6$

$2x^2 + x - 6x - 3 = 0$

$x(2x+1) \quad -3(2x+1)$

$(x-3)(2x+1) = 0$

$x = 3, \quad x = -\frac{1}{2}$



Dealing with > 0 so highlight above x axis

$x < -\frac{1}{2}$

$x > 3$

Challenge thirteen

$$\frac{x^2 - 9x + 20}{x^2 - 11x + 28}$$

$x^2 - 9x + 20$

$-4 + -5 = -9$

$-4 \times -5 = 20$

$(x-4)(x-5)$

$x^2 - 11x + 28$

$-4 + -7 = -11$

$-4 \times -7 = 28$

$(x-4)(x-7)$

$$\frac{(x-4)(x-5)}{(x-4)(x-7)}$$

$(x-4)$ on both so these cancel out

$$\frac{(x-5)}{(x-7)} = \frac{x-5}{x-7}$$

Challenge fourteen

$5x^2 < 80$

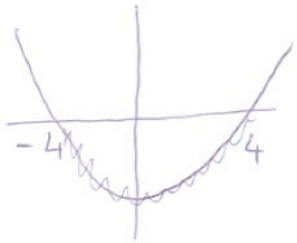
$5x^2 - 80 < 0$

$x^2 - 16 < 0$

$(x + 4)(x - 4) < 0$

$x = -4$

$x = 4$



Dealing with < 0 so highlight below x axis

$-4 < x < 4$

Challenge fifteen

To prove this, we need to show that each interval boundary (1.5 and 2) is on opposite sides of −5.

Try $x = 1.5$

$$-2(1.5)^3 + 3(1.5)$$

$$= -2.25 \rightarrow \text{Answer} > -5$$

Try $x = 2$

$$-2(2)^3 + 3(2)$$

$$= -10 \rightarrow \text{Answer} < -5$$

Therefore $1.5 < x < 2$ because $x = 1.5$ is > -5 whereas $x = 2$ is < -5.

We hope this question pack was helpful. We opted for handwritten worked solutions as a pose to standard mark-scheme type answers found elsewhere. If you're still struggling, you can find in-depth video walkthrough solutions for every question in this pack on our website as well as lots more question packs for other GCSE topics.

Also, challenge papers can be found on our website too if you're feeling especially confident with the content.

Thank you
The PLS Tutors

http://www.plstutors.co.uk/

© 2023, The PLS Tutors Ltd