Indices

Aims

1. Use the rules of indices with whole number indices
2. Understand and use negative indices
3. Understand and use fraction indices
4. Solve equations in index form

The number being raised to the power is called the **base**

The power the base is raised to is called the **index** or **exponent**

$$x^m$$

This is called **index** or **exponential form**

The laws of indices

- $x^m \times x^n = x^{m+n}$
- $x^m \div x^n = x^{m-n}$
- $(x^m)^n = x^{mn}$

- $x^0 = 1$
- $\frac{1}{x^m} = x^{-m}$
- $\sqrt[n]{x} = x^{\frac{1}{n}}$

Example 1

Simplify the following expressions:

a. $3^2 \times 3^3 =$ 3^5

b. $4^7 \div 4^5 =$ 4^2

c. $\frac{5^4}{5^2} =$ 5^2

d. $(2^3)^5 =$ 2^{15}

e. $2x^4y^3 \times 4x \times (x^2y^3)^4 =$

$2x^4y^3 4x(x^8y^{12}) = 8x^{13}y^{15}$

f. $\frac{(3s^3t^4)^2 \times s^2t^5}{(s^2t^3)^4} =$

$\frac{(9s^6t^8)s^2t^5}{s^8t^{12}} = \frac{9s^8t^{13}}{s^8t^{12}} = 9t$

Example 2

Express the following expressions in index form

a. $\frac{1}{x^3} =$ x^{-3}

b. $\frac{4}{y^3} =$ $4y^{-3}$

c. $3\sqrt{x} =$ $3x^{\frac{1}{2}}$

d. $\sqrt[3]{x} =$ $x^{\frac{1}{3}}$

e. $\sqrt[4]{a^3} =$ $a^{\frac{3}{4}}$

f. $\frac{x^4-17x^2+6}{x^2}$

$(x^4-17x^2+6)x^{-2}$

$=x^2-17+6x^{-2}$

Exam question

A curve is defined, for $x > 0$, by the equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{x^8 - 1}{x^3}$$

(a) Express $\frac{x^8 - 1}{x^3}$ in the form $x^p - x^q$, where p and q are integers. *(2 marks)*

$x^5 - x^{-3}$

Solving equations using indices

Example 3

Solve the following equations for x:

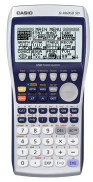
a) $x^{\frac{1}{2}} = 6$

$(x^{\frac{1}{2}})^2 = 6^2$

$x = 36$

b) $6x^{\frac{1}{3}} + 1 = 0$

$6x^{\frac{1}{3}} = -1$



$(x^{\frac{1}{3}})^3 = (-\frac{1}{6})^3$

$x = -\frac{1}{216}$

c) $x^{\frac{2}{3}} = 16$

$(x^{\frac{2}{3}})^{\frac{3}{2}} = 16^{\frac{3}{2}}$

$x = \pm 64$

d) $\frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} = 0$

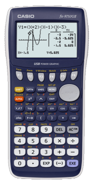
$45x^{\frac{1}{2}} - 5x^{\frac{3}{2}} = 0$

$9x^{\frac{1}{2}} - x^{\frac{3}{2}} = 0$

$x^{\frac{1}{2}}(9 - x^{\frac{2}{2}}) = 0$

$(x^{\frac{1}{2}})(x^{\frac{2}{2}}) = x^{\frac{3}{2}}$

$x^{\frac{1}{2}} = 0 \qquad x = 0,\ x = 9$



Example 4

Solve the following equations for x:

a) $4^{x+1} = 8$

$(2^2)^{x+1} = 2^3$

$2^{2(x+1)} = 2^3$

$2(x+1) = 3$

$2x + 2 = 3$

$x = \frac{1}{2}$

b) $3^{x+1} = 27^{x-3}$

$3^{x+1} = (3^3)^{x-3}$

$3^{x+1} = 3^{3(x-3)}$

$x + 1 = 3(x-3)$

$x + 1 = 3x - 9$

$10 = 2x$

$x = 5$

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

1(a)(i)	x^2	B1	1	
(ii)	$x^{\frac{1}{2}} = \sqrt{x}$	B1	1	Accept either form
(iii)	x^3	B1	1	

Exam question

(a) (i) Find the value of p for which $\sqrt{125} = 5^p$. *(2 marks)*

(ii) Hence solve the equation $5^{2x} = \sqrt{125}$. *(1 mark)*

$\sqrt{5^3} = 5^p$ $\quad$ $5^{\frac{3}{2}} = 5^p$ $\quad$ $p = \frac{3}{2}$

3 (a) Write down the values of p, q and r given that:

(i) $64 = 8^p$;

(ii) $\dfrac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of x for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$ *(2 marks)*

3(a)(i)	$\{p =\}\ 2$	B1		Condone '$64=8^2$'
(ii)	$\{q =\}\ -2$	B1ft		Ft on '$-p$' if q not correct
(iii)	$\{r =\}\ 0.5$	B1	3	Condone '$\sqrt{8} = 8^{0.5}$'
(b)	$\dfrac{8^x}{8^{0.5}} = 8^{-2} \Rightarrow 8^{x-0.5} = 8^{-2}$ OE	M1		Using parts (a) & valid index law to stage $8^c=8^d$ (PI)
	$\Rightarrow x - 0.5 = -2 \quad \Rightarrow x = -1.5$	A1ft	2	Ft on c's $(q + r)$ if not correct (Accept correct answer without working)
	ALT: $\log 8^x = \log k$, $x\log 8 = \log k$; $x = -1.5$			(M1 A1)
	Total		**5**	

Surds

Aims

1. To be able to simplify surds
2. To be able to put numbers in surd form
3. To be able to rationalise the denominators of fractions

$0.5 = \frac{1}{2}$ is rational

$0.666... = \frac{2}{3}$ is rational

$0.2323... = \frac{23}{99}$ is rational

Definitions

- A number is said to be **rational** if it can be written as a fraction.
- All recurring and terminating decimals are rational.
- A number is said to be **irrational** if it cannot be written as a fraction.
- $\pi, \sqrt{2}, e, \sqrt{5}$ are examples of irrational numbers.
- Numbers that involve the 'root' √ symbol are called 'surds'. This includes cube roots, 4th roots and higher.

Exact

When an exam question asks for an answer in 'exact' form, leave it as a surd.

✓ $\sqrt{2}$ ✗ 1.414…

1.414… is an **approximation** of $\sqrt{2}$, no matter how many decimal places you use.

Surd Rules

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a} \times \sqrt{a} = a$$

Similar rules do not apply to addition/subtraction!

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \text{ and } \sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$$

When you are asked to simplify a surd, or write a number in surd form, you need to write the number as a product of two numbers, one of which is a square number.

<u>Example 1</u>

Write these numbers in surd form

Look for "*square factors*"

$\sqrt{72} =$ $\sqrt{36\times2} = 6\sqrt{2}$

$\sqrt{24} =$ $\sqrt{4\times6} = 2\sqrt{6}$

<u>Example 2</u>

Simplify these numbers:

a) $\sqrt{72} + 3\sqrt{2} =$ $6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$

b) $\sqrt{12} + \sqrt{75} =$ $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

<u>Example 3</u>

Expand and simplify:

a) $\sqrt{5}(2 + \sqrt{15}) =$ $2\sqrt{5} + \sqrt{75} = 2\sqrt{5} + 5\sqrt{3}$

b) $(3 - 2\sqrt{5})(4 + \sqrt{5}) =$ $12 + 3\sqrt{5} - 8\sqrt{5} - 2\sqrt{25}$

$= 12 - 5\sqrt{5} - 10 = 2 - 5\sqrt{5}$

Rationalising the denominator

Fractions with surds in can be easier to deal with if their denominator is rational.
When we see a fraction with a surd on the bottom, we must ‘Rationalise the denominator’.

<u>Example 4</u>

If it looks like this:	Do this:	We get:
$\frac{3}{\sqrt{3}}$	$\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	$\frac{3\sqrt{3}}{3} = \sqrt{3}$
$\frac{5}{2\sqrt{7}}$	$\frac{5}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$	$\frac{5\sqrt{7}}{14}$
$\frac{3+\sqrt{8}}{\sqrt{2}}$	$\frac{3+\sqrt{8}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$	$\frac{3\sqrt{2}+\sqrt{16}}{2} = \frac{3\sqrt{2}+4}{2} = \frac{3\sqrt{2}}{2} + 2$

If it looks like this:	Do this:	We get:
$\frac{3}{7-\sqrt{3}}$	$\frac{3}{7-\sqrt{3}} \times \frac{7+\sqrt{3}}{7+\sqrt{3}}$	$\frac{21+3\sqrt{3}}{49-7\sqrt{3}+7\sqrt{3}-\sqrt{9}}$ $= \frac{21+3\sqrt{3}}{49-3} = \frac{21+3\sqrt{3}}{46}$
$\frac{7+\sqrt{5}}{3+\sqrt{5}}$	$\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$	$\frac{21-7\sqrt{5}+3\sqrt{5}-5}{9+3\sqrt{5}-3\sqrt{5}-\sqrt{25}}$ $= \frac{16-4\sqrt{5}}{9-5} = \frac{16-4\sqrt{5}}{4} = 4-\sqrt{5}$

If it looks like this:	Do this:	We get:
$\frac{2\sqrt{7}-1}{2\sqrt{7}+5}$	$\frac{2\sqrt{7}-1}{2\sqrt{7}+5} \times \frac{2\sqrt{7}-5}{2\sqrt{7}-5}$ $(2\sqrt{7})(2\sqrt{7})$ $=4\sqrt{49}=28$	$\frac{28-10\sqrt{7}-2\sqrt{7}+5}{28-25}$ $=\frac{33-12\sqrt{7}}{3} = 11-4\sqrt{7}$
$\frac{4\sqrt{5}-7\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$	$\frac{4\sqrt{5}-7\sqrt{2}}{2\sqrt{5}+\sqrt{2}} \times \frac{2\sqrt{5}-\sqrt{2}}{2\sqrt{5}-\sqrt{2}}$ $(2\sqrt{5})(2\sqrt{5})$ $=4\sqrt{25}=20$	$\frac{40-4\sqrt{10}-14\sqrt{10}+14}{20-2}$ $=\frac{54-18\sqrt{10}}{18} = 3-\sqrt{10}$

1 Express each of the following in the form $m+n\sqrt{3}$, where m and n are integers:

(a) $(\sqrt{3}+1)^2$; $1+3+2\sqrt{3}=4+2\sqrt{3}$ *(2 marks)*

(b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$. $\times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4+2\sqrt{3}}{3-1} = 2+\sqrt{3}$ *(3 marks)*

2(a) Simplify $(\sqrt{12}+2)(\sqrt{12}-2)$. $12-4=8$ *(2 marks)*

(b) Express $\sqrt{12}$ in the form $m\sqrt{3}$, where m is an integer. $2\sqrt{3}$ *(1 mark)*

(c) Express $\frac{\sqrt{12}+2}{\sqrt{12}-2}$ in the form $a+b\sqrt{3}$, where a and b are integers. *(4 marks)*

$\times \frac{\sqrt{12}+2}{\sqrt{12}+2} = \frac{16+4\sqrt{12}}{8} = 2+\sqrt{3}$

3(a) Express $(3+\sqrt{2})^2$ in the form $p+q\sqrt{2}$. $11+6\sqrt{2}$ *(2 marks)*

(b) Hence express $\frac{98}{(3+\sqrt{2})^2}$ in the form $m+n\sqrt{2}$, where m and n are integers. *(3 marks)*

$\frac{98}{11+6\sqrt{2}} \times \frac{11-6\sqrt{2}}{11-6\sqrt{2}} = \frac{98(11-6\sqrt{2})}{121-72} = \frac{98(11-6\sqrt{2})}{49} = 22-12\sqrt{2}$

4 (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

1(a)	$\left(\sqrt{5}\right)^2+2\sqrt{5}-2\sqrt{5}-4=1$	M1		Multiplying out or difference of two squares attempted
		A1	2	Full marks for correct answer /no working
(b)	$\sqrt{8}=2\sqrt{2}$; $\sqrt{18}=3\sqrt{2}$	M1		Either correct
	Answer $=5\sqrt{2}$	A1	2	Full marks for correct answer /no working

Attachments

C2 Laws of Logs RallyCoach.docx

Basic Logs and Exp matching.docx

LogarithmsHexJig.pdf

Simplifying Logarithmic Expressions.pdf

True or False.pdf

Target Board - Logs and Indices.docx

Target Board - Logs and Indices Solutions.docx

01. Indices odd one out.docx

Indices problem set.pdf

C1_-_Indices_-_Quiz.pptx

Surds Starter True or False.docx