# Pure Sector 1: Indices (and Surds)

**Aims**

- Use the rules of indices with whole number indices
- Understand and use negative indices
- Understand and use fraction indices
- Solve equations in index form

The number being raised to the power is called the **base**

The power the base is raised to is called the **index** or **exponent**

$$x^m$$

This is called **index** or **exponential form**

**The laws of indices**

- $x^m \times x^n = x^{m+n}$
- $x^m \div x^n = x^{m-n}$
- $(x^m)^n = x^{mn}$

- $x^0 = 1$
- $\frac{1}{x^m} = x^{-m}$
- $\sqrt[n]{x} = x^{\frac{1}{n}}$

<u>Example 1</u>

Simplify the following expressions:

a) $\mathbf{3^2 \times 3^3} =$

b) $\mathbf{4^7 \div 4^5} =$

c) $\frac{\mathbf{5^4}}{\mathbf{5^2}} =$

d) $\mathbf{(2^3)^5} =$

e) $2x^4y^3 \times 4x \times (x^2y^3)^4 =$

f) $\frac{(3s^3t^4)^2 \times s^2t^5}{(s^2t^3)^4} =$

<u>Example 2</u>

Express the following expressions in index form:

a) $\frac{\mathbf{1}}{\boldsymbol{x^3}} =$

b) $\frac{\mathbf{4}}{\boldsymbol{y^3}} =$

c) $\mathbf{3}\sqrt{\boldsymbol{x}} =$

d) $\sqrt[3]{\boldsymbol{x}} =$

e) $\sqrt[4]{a^3} =$

f) $\frac{\boldsymbol{x^4 - 17x^2 + 6}}{\boldsymbol{x^2}}$

**Exam question**

A curve is defined, for $x > 0$, by the equation $y = \mathrm{f}(x)$, where

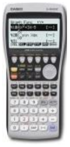
$$\mathrm{f}(x) = \frac{x^8 - 1}{x^3}$$

(a) Express $\frac{x^8 - 1}{x^3}$ in the form $x^p - x^q$, where $p$ and $q$ are integers. *(2 marks)*

**Solving equations using indices**

Example 3

Solve the following equations for $x$:



a) $x^{\frac{1}{2}} = 6$

b) $6x^{\frac{1}{3}} + 1 = 0$

*c)* $x^{\frac{2}{3}} = 16$

*d)* $\frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} = 0$

Example 4

Solve the following equations for $x$:

a) $4^{x+1} = 8$

*b)* $3^{x+1} = 27^{x-3}$

**Exam questions**

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

**2**

(a) (i) Find the value of $p$ for which $\sqrt{125} = 5^p$. *(2 marks)*

(ii) Hence solve the equation $5^{2x} = \sqrt{125}$. *(1 mark)*

3 (a) Write down the values of $p$, $q$ and $r$ given that:

(i) $64 = 8^p$;

(ii) $\frac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of $x$ for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$ *(2 marks)*

## Surds

0.5 = $\frac{1}{2}$ is rational

0.666… = $\frac{2}{3}$ is rational

### Aims

- To be able to simplify surds
- To be able to put numbers in surd form
- To be able to rationalise the denominators of fractions

### Definitions

- A number is said to be **rational** if it can be written as a fraction.
  - All recurring and terminating decimals are rational
- A number is said to be **irrational** if it cannot be written as a fraction.
  - π, $\sqrt{\mathbf{2}}$, e, $\sqrt[3]{\mathbf{5}}$ are examples of irrational numbers.
- Numbers that involve the 'root' $\sqrt{\ }$ symbol are called 'surds'. This includes cube roots, 4th roots and higher.

**Exact**

When an exam question asks for an answer in 'exact' form, leave it as a surd.

$\sqrt{2}$ ✓ 1.414… ✗

1.414… is an **approximation** of $\sqrt{2}$, no matter how many decimal places you use.

### Surd Rules

$$\sqrt{\mathbf{ab}} = \sqrt{\mathbf{a}} \times \sqrt{\mathbf{b}}$$

$$\sqrt{\frac{\mathbf{a}}{\mathbf{b}}} = \frac{\sqrt{\mathbf{a}}}{\sqrt{\mathbf{b}}}$$

$$\sqrt{\mathbf{a}} \times \sqrt{\mathbf{a}} = \mathbf{a}$$

When you are asked to simplify a surd, or write a number in surd form, you need to write the number as a product of two numbers, one of which is a square number.

<u>Example 1</u>
Write these numbers in surd form

$\sqrt{\mathbf{72}}$ =

$\sqrt{\mathbf{24}}$ =

Example 2

Simplify these numbers:

a) $\sqrt{72} + 3\sqrt{2} =$

b) $\sqrt{12} + \sqrt{75} =$

Example 3

Expand and simplify

a) $\sqrt{5}(2 + \sqrt{15}) =$

b) $(3 - 2\sqrt{5})(4 + \sqrt{5})$=

**Rationalising the denominator**

Fractions with surds in can be easier to deal with if their denominator is rational.
When we see a fraction with a surd on the bottom, we must ‘Rationalise the denominator’.

Example 4

| If it looks like this: | Do this | We get |
|---|---|---|
| $\frac{3}{\sqrt{3}}$ | $\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ | |
| $\frac{5}{2\sqrt{7}}$ | $\frac{5}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ | |
| $\frac{3 + \sqrt{8}}{\sqrt{2}}$ | $\frac{3 + \sqrt{8}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ | |

**We can do this, because we’re only multiplying by one.**

| If it looks like this: | Do this | We get |
|---|---|---|
| $\frac{3}{7-\sqrt{3}}$ | $\frac{3}{7-\sqrt{3}} \times \frac{7+\sqrt{3}}{7+\sqrt{3}}$ | |
| $\frac{7+\sqrt{5}}{3+\sqrt{5}}$ | $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ | |
| $\frac{2\sqrt{7}-1}{2\sqrt{7}+5}$ | $\frac{2\sqrt{7}-1}{2\sqrt{7}+5} \times \frac{2\sqrt{7}-5}{2\sqrt{7}-5}$ | |
| $\frac{4\sqrt{5}-7\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$ | $\frac{4\sqrt{5}-7\sqrt{2}}{2\sqrt{5}+\sqrt{2}} \times \frac{2\sqrt{5}-\sqrt{2}}{2\sqrt{5}-\sqrt{2}}$ | |

**Exam questions**

1 Express each of the following in the form $m + n\sqrt{3}$, where $m$ and $n$ are integers:

(a) $(\sqrt{3}+1)^2$; *(2 marks)*

(b) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$. *(3 marks)*

2 (a) Simplify $(\sqrt{12}+2)(\sqrt{12}-2)$. *(2 marks)*

(b) Express $\sqrt{12}$ in the form $m\sqrt{3}$, where $m$ is an integer. *(1 mark)*

(c) Express $\dfrac{\sqrt{12}+2}{\sqrt{12}-2}$ in the form $a + b\sqrt{3}$, where $a$ and $b$ are integers. *(4 marks)*

3(a) Express $(3+\sqrt{2})^2$ in the form $p+q\sqrt{2}$. *(2 marks)*

(b) Hence express $\dfrac{98}{(3+\sqrt{2})^2}$ in the form $m+n\sqrt{2}$, where $m$ and $n$ are integers. *(3 marks)*

4 (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where $n$ is an integer. *(2 marks)*