Pure Sector 1: Graph Sketching

Aims:

- understand and use language and symbols associated with set theory.
- find the domain and range of a function and be able to use correct language and notation to describe functions accurately.
- understand the graphs, symmetry and periodicity of the sine, cosine and tangent functions including transformations.
- sketch and use transformations of the functions a^x, e^x and $\ln x$.
- understand and use graphs of the functions $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ and as well as transformations.
- understand and be able to use the graph of $y = |x|$and combinations of transformations of this graph, points of intersection and solutions of equations and inequalities.

Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\mathbb{N}$	the set of natural numbers $\{1, 2, 3, \dots\}$
$\{x_1, x_2, \dots\}$	the set with the elements $x_1, x_2, \dots$
$\{x: \dots\}$	the set of all x such that...
$\mathbb{Z}$	the set of integers $\{0, \pm1, \pm2, \pm3, \dots\}$
$\mathbb{Z}^+$	the set of positive integers $\{1, 2, 3, \dots\}$
$\mathbb{Z}_0^+$	the set of non-negative integers $\{0, 1, 2, 3, \dots\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{Q}$	the set of rational numbers $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$

Range and Domain

The set of all possible inputs of a mapping or function is called the **domain** (x values).

The set of outputs for a particular set of inputs for a mapping or function is called the **range** (y values).

Example 1

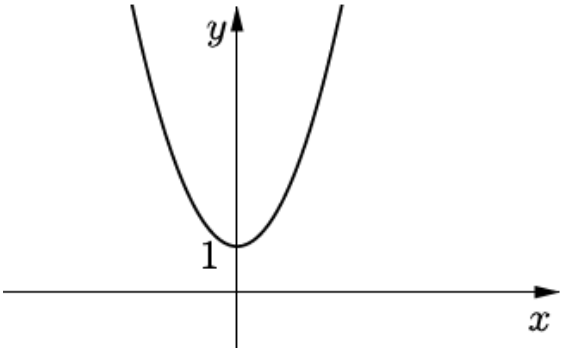
The function h has domain $\{-2, -1, 0, 3, 7\}$ and is defined as $h(x) = (x-3)^2 + 2$. Find the range of h.

If any outputs are repeated we only write the value once in the range.

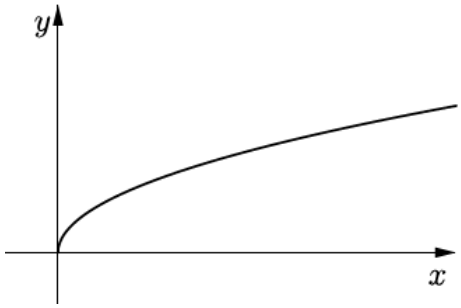
Example 2

Using set notation state range of the following functions:

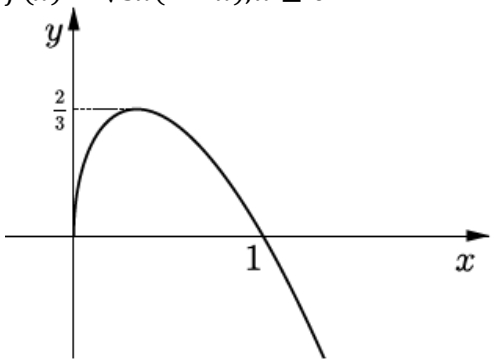
a) $f(x) = x^2 + 1, x \in \mathbb{R}$



b) $f: x \to \sqrt{x},\ x \geq 0$



c) $f(x) = \sqrt{3x}(1-x), x \geq 0$

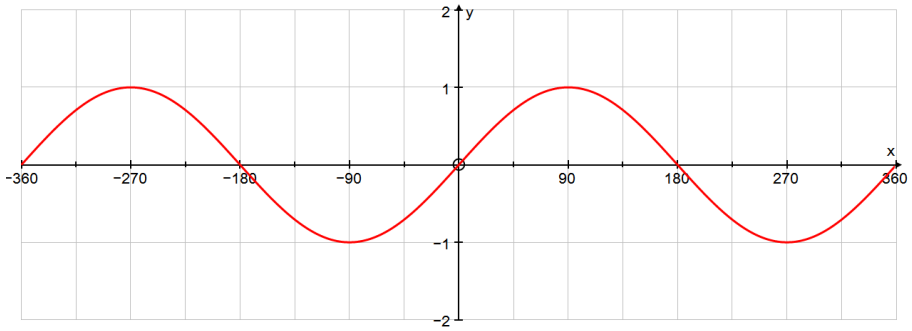


Example 3

Function and Domain	Graph	Range
$f(x) = x^2$ $x \in \mathbb{R}$		
$f(x) = 4 - x^2$ $x \in \mathbb{R}$		
$f(x) = 4 - x^2$ $x \in \mathbb{R}, x > 3$		
$f(x) = \sqrt{2x-1}$ $x \geq 0.5$		
$f(x) = \frac{x^2 - 2}{5}$ $x \in \mathbb{R}, x \leq 0$		

Trigonometric Graphs

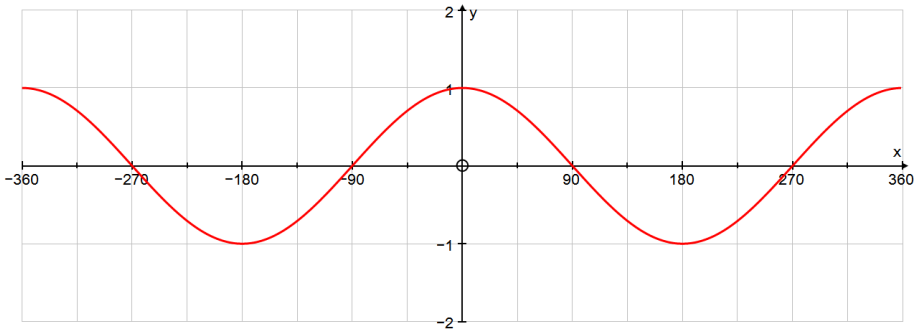
$f(x) = \sin x$



Domain:

Range:

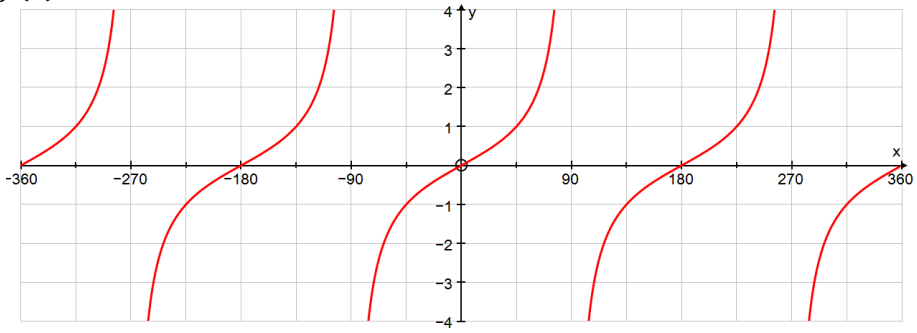
$f(x) = \cos x$



Domain:

Range:

$f(x) = \tan x$



Domain:

Range:

Example 4

Sketch each function and state its range.

a) $f(x) = 2\sin x$

b) $f(x) = \cos(3x)$

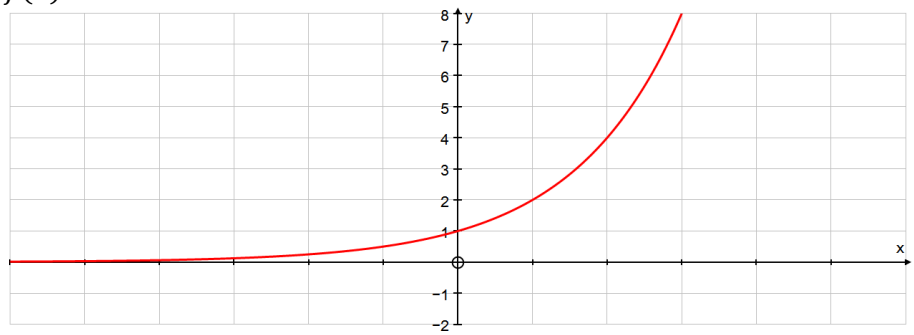
c) $f(x) = \tan(x + 45)$

d) $f(x) = 3\cos x + 1$

e) $f(x) = \sin(x - 10) + 5$

Exponential Graphs

$f(x) = a^x$



Domain:

Range:

The constant e is the base of a natural logarithm written as $\ln x$ or $\log_e x$ (we study this in more detail later in Pure Sector 1). e is an irrational number and has the value $2.718\ldots$

Example 5

Sketch the graph of $f(x) = e^x$. State the domain and range.

Example 6

Sketch the graph of $f(x) = lnx$. State the domain and range.

Example 7

Sketch each function and state its range.

a) $f(x) = e^{-x}, x \in \mathbb{R}$

b) $f(x) = e^{3x} + 2, x \in \mathbb{R}$

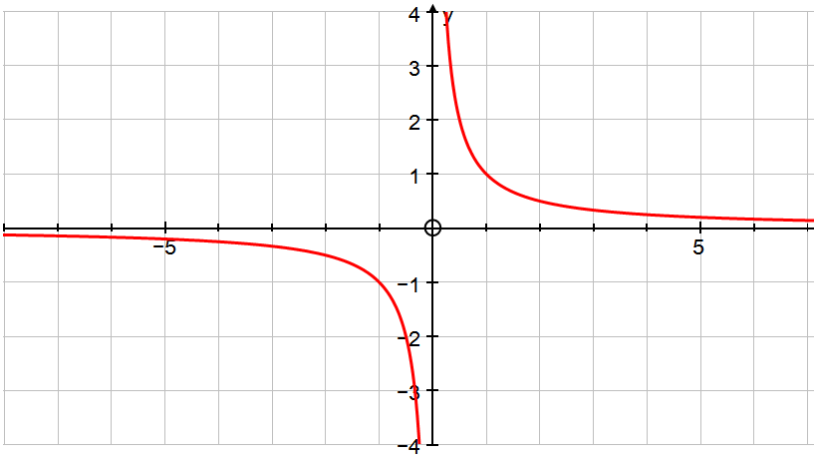
c) $f(x) = \ln 3x, x > 0$

d) $f(x) = 3 - e^{2x}, x \in \mathbb{R}$

e) $f(x) = 3\ln(x - 2),\ x > 2$

Reciprocal Graphs

$f(x) = \frac{1}{x}$



An asymptote is a line which the graph approaches but never reaches.

Domain:

Range:

Asymptotes:

Example 8

On the same axes, sketch $y = \frac{1}{x}$ and $y = \frac{3}{x}$

Example 9

Sketch the graph of $y = \frac{1}{x^2}$

Example 10

Sketch each function and state any asymptotes.

a) $f(x) = -\frac{b}{x},\ x \in \mathbb{R}, x \neq 0$

b) $f(x) = \frac{4}{x^2}, x \in \mathbb{R}, x \neq 0$

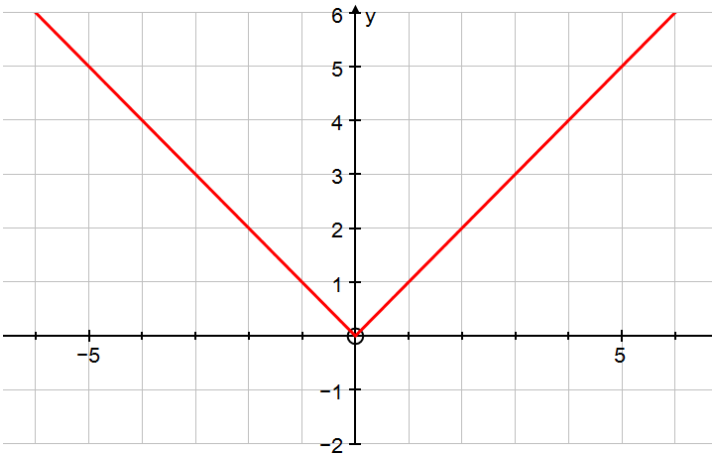
c) $f(x) = \frac{2}{x+1}, x \in \mathbb{R}, x \neq -1$

d) $f(x) = \frac{2}{x} + 1, x \in \mathbb{R}, x \neq 0$

e) $f(x) = 3 - \frac{2}{x^2}, x \in \mathbb{R}, x \neq 0$

Modulus

$f(x) = |x|$



The modulus sign | |means you use the positive value of whatever is inside. Using your graphics calculator, this is abs ( ).

So: $|-4| =$

$|2.5| =$

Domain:

Range:

Example 11

Sketch each function and state the range.

a) $f(x) = |3x - 2|$

b) $f(x) = |5 - 2x|$

c) $f(x) = |2x| + 4$

Example 12

Solve the equation $|3x - 4| = 5$

Example 13

Solve the equation $|x + 1| = |2x - 1|$

Example 14

Solve each equation:

a) $|x - 5| = 3$

b) $|2x - 7| = 4$

c) $|x - 2| = |x + 4|$

d) $|2x + 1| = |9 - 2x|$

e) $|3x - 4| = |2x + 3|$

Example 15

a) Solve the equation $|3x + 2| = |3 - 2x|$
b) Hence solve the inequality $|3x + 2| > |3 - 2x|$

Example 16

a) $|x - 3| < 8$

b) $|2x - 11| \leq 5$

c) $|x + 4| \leq |x + 1|$

d) $|x + 2| > |2x - 5|$

e) $|2x - 1| < |3x + 2|$

Exam Questions

4 (a) Sketch the graph of $y = |8 - 2x|$. *(2 marks)*

(b) Solve the equation $|8 - 2x| = 4$. *(2 marks)*

(c) Solve the inequality $|8 - 2x| > 4$. *(2 marks)*

7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*