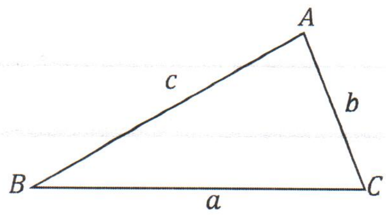
Pure Sector 2: Trigonometry 1

Aims:

- To be able to use the sine and cosine rule to find missing angles and lengths in triangles.
- To be able to find the area of triangles using trigonometry.
- To convert angles between degree and radian measure.
- To be able to find the length of an arc and the area of a sector.

To find missing angles or lengths in triangles that are **not** right angled we use the sine and cosine rules. The triangle ABC below has sides of length a, b and c. The angle A is opposite a, angle B is opposite b and angle C is opposite c. Make sure you label your diagram carefully!



Remember:

- Acute angles are between $0°$ and $90°$.
- Obtuse angles are between $90°$ and $180°$.
- Reflex angles are between $180°$ and $360°$.

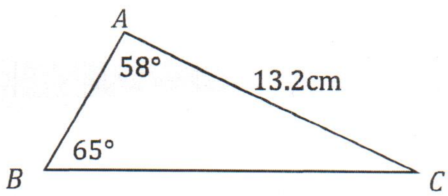
Sine Rule

We can use the sine rule to find a missing angle or length when one side and its opposite angle are known.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1

The diagram shows the triangle ABC. The length of AC is 13.2cm, and the sizes of angles ABC and BAC are $65°$ and $58°$ respectively. Show that the length of $BC = 12.4$cm, correct to the nearest 0.1cm.



$$\frac{BC}{\sin 58} = \frac{13.2}{\sin 65}$$

$$BC = \frac{13.2 \sin 58}{\sin 65}$$

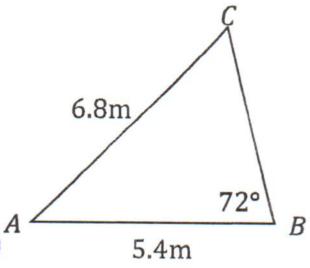
$$= 12.35147\ldots$$

$$= 12.4 \text{ cm} \quad (1 \text{ d.p.})$$

When answering a show that **question you must always show the** full unrounded version before the required rounded version otherwise you will lose marks!

Example 2

The size of angle B is 72°, and the lengths AB and AC are 5.4m and 6.8m respectively. Find the size of the angle ACB, give your answer to 3sf.



$$\frac{\sin C}{5.4} = \frac{\sin 72}{6.8}$$

$$\sin C = \frac{5.4 \sin 72}{6.8}$$

$$\sin C = 0.755\ldots$$

$$C = \sin^{-1}(0.755\ldots)$$

$$= 49.047\ldots$$

$$= 49.0° \quad (3 \text{ s.f.})$$

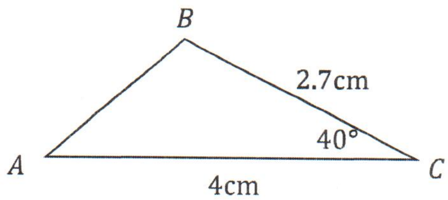
Cosine Rule

We can use the cosine rule to find a missing angle if we are given all three sides or a missing length if we are given two sides and the angle opposite the missing side. You need to remember this formula.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Example 3

The diagram show the triangle ABC. The lengths of AC and BC are 4cm and 2.7cm respectively. The size of angle BCA is 40°. Calculate the length of AB, giving your answer to 2 significant figures.



$$AB^2 = 4^2 + 2.7^2 - 2(4)(2.7)\cos 40$$

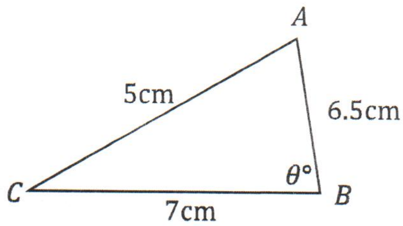
$$= 6.743\ldots$$

$$AB = 2.5968\ldots$$

$$= 2.6 \quad (2 \text{ s.f.})$$

Example 4

The triangle ABC, shown in the diagram, is such that $AB = 6.5$cm, $AC = 5$cm, $BC = 7$cm and angle $ABC = \theta$. Show that $\theta = 43.3°$, correct to the nearest $0.1°$.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\therefore \cos\theta = \frac{6.5^2 + 7^2 - 5^2}{2(6.5)(7)}$$

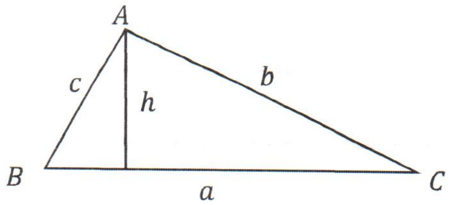
$$= 0.72802\ldots$$

$$\theta = \cos^{-1}(0.728\ldots)$$

$$= 43.2791\ldots$$

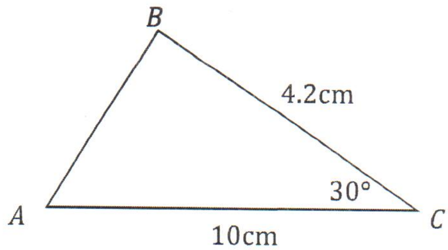
$$= 43.3° \quad (1 \text{ d.p.})$$

Area of a Triangle



Area of triangle $= \frac{1}{2}ab\sin C$

Example 5

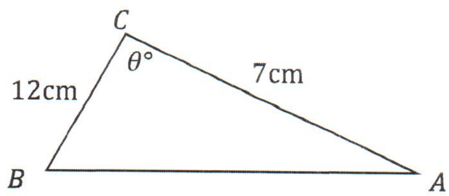


The diagram show the triangle ABC. The lengths of AC and BC are 10cm and 4.2cm respectively. The size of angle BCA is $30°$. Calculate the area of the triangle ABC.

$$\text{AREA} = \frac{1}{2}(4.2)(10)\sin 30$$

$$= 10.5 \text{ cm}^2$$

Example 6



The triangle ABC is such that $AC = 7$cm, $BC = 12$cm and the angle $ACB = \theta°$. The area of the triangle is 32cm^2. Show that the value of $\theta = 49.6$ correct to three significant figures.

$$\frac{1}{2}(7)(12)\sin\theta = 32$$

$$\sin\theta = \frac{32 \times 2}{7 \times 12}$$

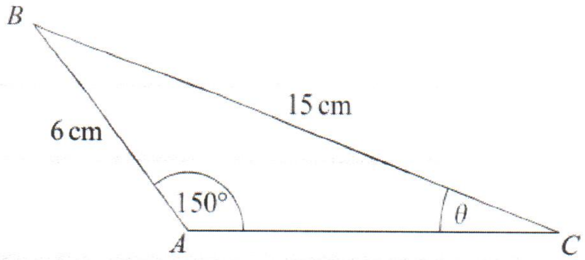
$$= 0.761904\ldots$$

$$\theta = 49.6324\ldots$$

$$= 49.6° \quad (3\text{ s.f.})$$

Exam Question

The triangle ABC, shown in the diagram, is such that $AB = 6\,\text{cm}$, $BC = 15\,\text{cm}$, angle $BAC = 150°$ and angle $ACB = \theta$.



(a) Show that $\theta = 11.5°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

a) $$\frac{\sin\theta}{6} = \frac{\sin 150}{15}$$

$$\sin\theta = \frac{6\sin 150}{15}$$

$$= 0.2$$

$$\theta = 11.5369\ldots$$

$$= 11.5° \quad (1\text{ d.p.})$$

b) $$\hat{ABC} = 180 - (150 + \theta)$$

$$= 18.4630\ldots$$

$$\text{AREA} = \tfrac{1}{2}(6)(15)\sin(18.46\ldots)$$

$$= 14.2511\ldots$$

$$= 14.3\text{ cm}^2 \quad (3\text{ s.f.})$$

Degrees and Radians
Angles can be measured in degrees or in radians where $360° = 2\pi$ rads. 1 radian can be written as 1 rads or 1^c.

Converting degrees to radians	Converting radians to degrees
$360° = 2\pi$ $180° = \pi$ $1° = \frac{\pi}{180}$	$2\pi = 360°$ $\pi = 180°$ $1^c = \frac{180}{\pi}$
$\theta \times \frac{\pi}{180}$	$\theta \times \frac{180}{\pi}$

Example 7

Convert the following angles from degrees into radians:

a) $20°$ $\times \frac{\pi}{180} = \frac{\pi}{9}^c$

b) $495°$ $\times \frac{\pi}{180} = \frac{11\pi}{4}^c$

Example 8

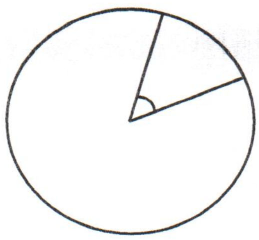
Convert the following angles from radians into degrees:

a) $\frac{3\pi}{5}$ $\times \frac{180}{\pi} = 108°$

b) $\frac{7\pi}{3}$ $\times \frac{180}{\pi} = 420°$

Check your calculator is in the correct mode!

Arc Length
A sector of a circle is the region bounded by two radii and an arc. The larger region is called the major sector and the smaller region is called the minor sector.



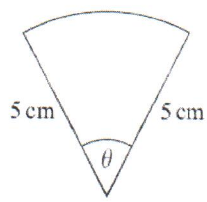
The length l of an arc of a circle is given by:

$$l = r\theta$$

where r is the radius and θ is the angle, in radians.

Example 9

The diagram shows the sector of a circle of radius 5cm and angle 0.6^c. Find the perimeter of the sector.

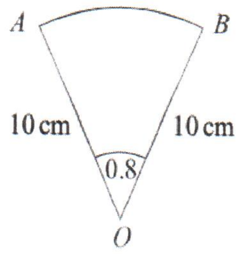


$P = 2\times 5 + 5(0.6)$

$= 13$ cm

Example 10

The diagram shows a sector OAB of a circle with centre O and radius 10cm. The perimeter of the sector OAB is equal to the perimeter of a square. Find the area of the square.



$P = 2 \times 10 + 10(0.8) = 28$

AREA OF SQUARE $= \left(\frac{28}{4}\right)^2 = 49 \text{ cm}^2$

Area of a Sector

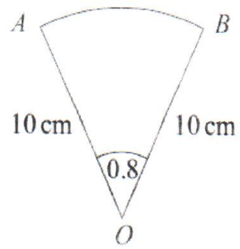
The area A of a sector of a circle is given by:

$$A = \frac{1}{2}r^2\theta$$

where r is the radius and θ is the angle, in radians.

Example 11

The diagram shows a sector OAB of a circle with centre O and radius 10cm. Find the area of the sector.

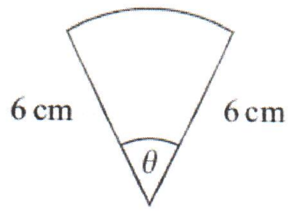


$A = \frac{1}{2}(10)^2(0.8)$

$= 40 \text{ cm}^2$

Example 12

The diagram shows a sector of a circle of radius 6cm and angle θ radians. The area of the rectangle, length 6cm and width 3cm, is twice the area of the sector. Show that $\theta = 0.5$



$$6 \times 3 = 2\left(\frac{1}{2}\right)(6)^2\theta$$

$$\theta = \frac{6 \times 3}{6^2}$$

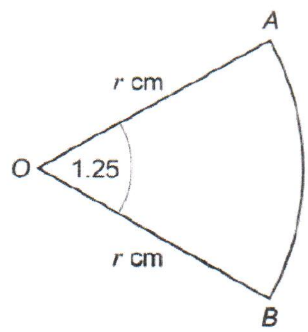
$$= 0.5^c$$

Example 13 – Exam Style Question

The diagram shows a sector of OAB of a circle with centre O and radius r cm. The angle AOB is 1.25 radians. The perimeter of the sector is 39 cm.

a) Show that r = 12

b) Calculate the area of the sector OAB.



a) $2r + 1.25r = 39$

$$r = \frac{39}{3.25}$$

$$= 12$$

b) $A = \frac{1}{2}(12)^2(1.25)$

$$= 90 \text{ cm}^2$$

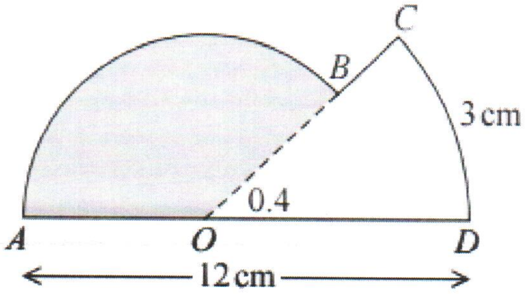


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length $CD = 3$ cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

a) let $OD = r$

$0.4r = 3$

$r = \frac{3}{0.4} = 7.5$ cm

b) $AO = 12 - 7.5$

$= 4.5$

$\theta = \pi - 0.4$

AREA $= \frac{1}{2}(4.5)^2(\pi - 0.4)$

$= 27.7586\ldots$

$= 27.8 \text{ cm}^2$ (3 s.f.)