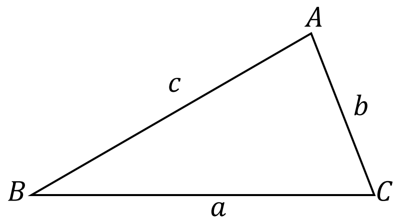
Pure Sector 2: Trigonometry 1

Aims:

- To be able to use the sine and cosine rule to find missing angles and lengths in triangles.
- To be able to find the area of triangles using trigonometry.
- To convert angles between degree and radian measure.
- To be able to find the length of an arc and the area of a sector.

To find missing angles or lengths in triangles that are **not** right angled we use the sine and cosine rules. The triangle ABC below has sides of length a, b and c. The angle A is opposite a, angle B is opposite b and angle C is opposite c. Make sure you label your diagram carefully!



Remember:

- Acute angles are between $0°$ and $90°$.
- Obtuse angles are between $90°$ and $180°$.
- Reflex angles are between $180°$ and $360°$.

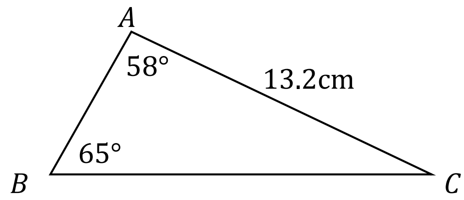
Sine Rule

We can use the sine rule to find a missing angle or length when one side and its opposite angle are known.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1

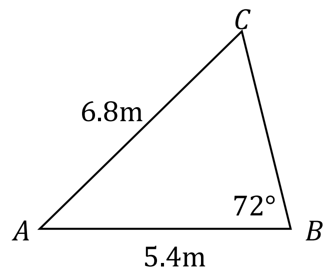
The diagram shows the triangle ABC. The length of AC is 13.2cm, and the sizes of angles ABC and BAC are $65°$ and $58°$ respectively. Show that the length of $BC = 12.4$cm, correct to the nearest 0.1cm.



When answering a show that question you must always show the full unrounded version before the required rounded version otherwise you will lose marks!

Example 2

The size of angle B is $72°$, and the lengths AB and AC are 5.4m and 6.8m respectively. Find the size of the angle ACB, give your answer to 3sf.



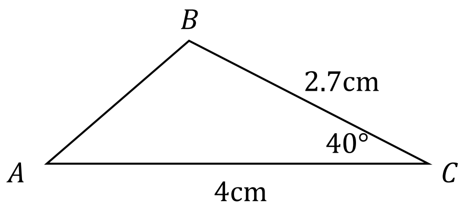
Cosine Rule

We can use the cosine rule to find a missing angle if we are given all three sides or a missing length if we are given two sides and the angle opposite the missing side. You need to remember this formula.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

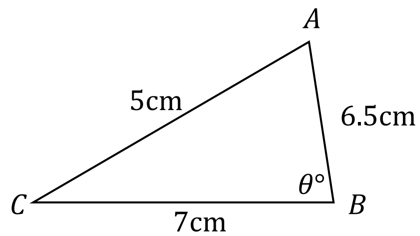
Example 3

The diagram show the triangle ABC. The lengths of AC and BC are 4cm and 2.7cm respectively. The size of angle BCA is $40°$. Calculate the length of AB, giving your answer to 2 significant figures.

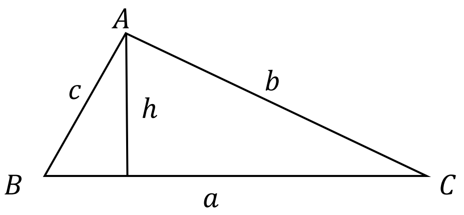


Example 4

The triangle ABC, shown in the diagram, is such that $AB = 6.5$cm, $AC = 5$cm, $BC = 7$cm and angle $ABC = \theta$. Show that $\theta = 43.3°$, correct to the nearest $0.1°$.



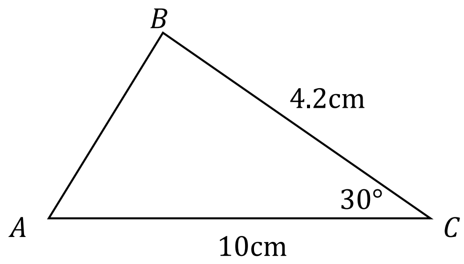
Area of a Triangle



$$\text{Area of triangle} = \frac{1}{2}ab\sin C$$

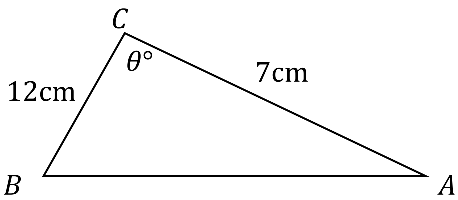
Example 5

The diagram show the triangle ABC. The lengths of AC and BC are 10cm and 4.2cm respectively. The size of angle BCA is $30°$. Calculate the area of the triangle ABC.



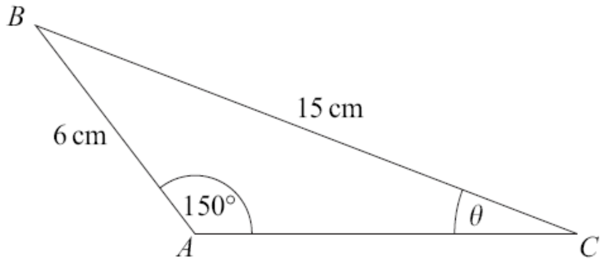
Example 6

The triangle ABC is such that $AC = 7\text{cm}$, $BC = 12\text{cm}$ and the acute angle $ACB = \theta°$. The area of the triangle is 32cm^2. Show that the value of $\theta = 49.6$ correct to three significant figures.



Exam Question

The triangle ABC, shown in the diagram, is such that $AB = 6\,\text{cm}$, $BC = 15\,\text{cm}$, angle $BAC = 150°$ and angle $ACB = \theta$.



(a) Show that $\theta = 11.5°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

Degrees and Radians
Angles can be measured in degrees or in radians where $360° = 2\pi$ rads. 1 radian can be written as 1 rads or 1^c.

Converting degrees to radians	Converting radians to degrees
$360° = 2\pi$ $180° = \pi$ $1° = \frac{\pi}{180}$	$2\pi = 360°$ $\pi = 180°$ $1^c = \frac{180}{\pi}$
$\theta \times \frac{\pi}{180}$	$\theta \times \frac{180}{\pi}$

Example 7

Convert the following angles from degrees into radians:

a) $20°$

b) $495°$

Example 8

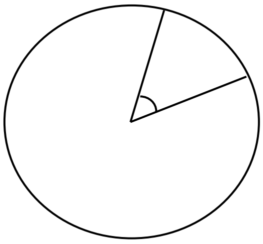
Convert the following angles from radians into degrees:

a) $\frac{3\pi}{5}$

b) $\frac{7\pi}{3}$

Check your calculator is in the correct mode!

Arc Length
A sector of a circle is the region bounded by two radii and an arc. The larger region is called the major sector and the smaller region is called the minor sector.



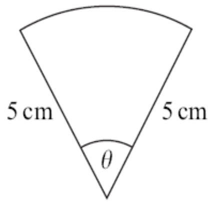
The length l of an arc of a circle is given by:

$$l = r\theta$$

where r is the radius and θ is the angle, in radians.

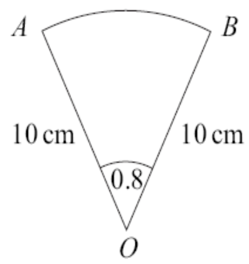
Example 9

The diagram shows the sector of a circle of radius 5cm and angle 0.6^c. Find the perimeter of the sector.



Example 10

The diagram shows a sector OAB of a circle with centre O and radius 10cm. The perimeter of the sector OAB is equal to the perimeter of a square. Find the area of the square.



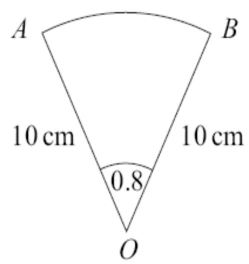
Area of a Sector
The area A of a sector of a circle is given by:

$$A = \frac{1}{2}r^2\theta$$

where r is the radius and θ is the angle, in radians.

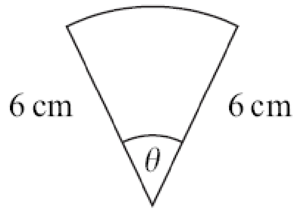
Example 11

The diagram shows a sector OAB of a circle with centre O and radius 10cm. Find the area of the sector.



Example 12

The diagram shows a sector of a circle of radius 6cm and angle θ radians. The area of the rectangle, length 6cm and width 3cm, is twice the area of the sector. Show that $\theta = 0.5$

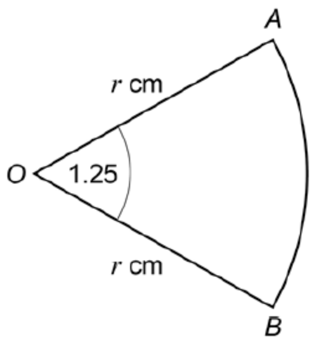


Example 13 – Exam Style Question

The diagram shows a sector of OAB of a circle with centre O and radius r cm.
The angle AOB is 1.25 radians. The perimeter of the sector is 39 cm.

a) Show that r = 12

b) Calculate the area of the sector OAB.



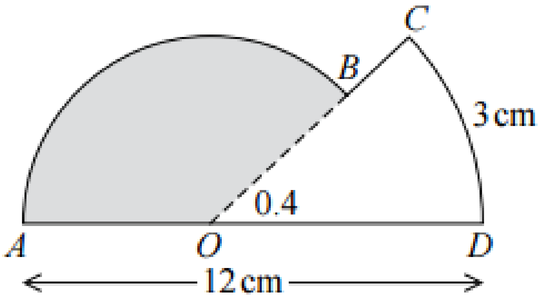


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

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