SHORT TYPE

① Evaluate $\int (x^3+3^x)\,dx$

Sol:- Let $\int (x^3+3^x)\,dx$

$\Rightarrow \int x^3dx+\int 3^x dx$

$\Rightarrow \frac{x^{3+1}}{3+1}+\frac{3^x}{\log 3}+c$

$\Rightarrow \frac{x^4}{4}+\frac{3^x}{\log 3}+c$

② Evaluate $\int (5^x+5x)\,dx$

Sol:- Let $\int (5^x+5x)\,dx$

$\Rightarrow \int 5^x dx+\int 5x\,dx$

$\Rightarrow \frac{5^x}{\log 5}+5\int x\,dx$

$\Rightarrow \frac{5^x}{\log 5}+5\frac{x^2}{2}+c$

③ Evaluate $\int (\sin 3x+\cos 2x)\,dx$

Sol:- Let $\int (\sin 3x+\cos 2x)\,dx$

$\Rightarrow \int \sin 3x\,dx+\int \cos 2x\,dx$

$$\Rightarrow -\frac{\cos 3x}{3} + \frac{\sin 2x}{2} + C \quad \left[\int \sin ax\,dx = -\frac{\cos ax}{a} + C\right]$$

(4) Evaluate $\int \left(\sin x + \frac{1}{1+x^2} + e^x\right) dx$

Sol:- Let $\int \left(\sin x + \frac{1}{1+x^2} + e^x\right) dx$

$$\Rightarrow \int \sin x\,dx + \int \frac{1}{1+x^2}\,dx + \int e^x\,dx$$

$$\Rightarrow -\cos x + \tan^{-1}x + e^x + C$$

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(5) Evaluate $\int \sqrt{1+\sin 2\theta}\,d\theta$

Sol:- Let $\int \sqrt{1+\sin 2\theta}\,d\theta$ $\quad [\sin 2A = 2\sin A\cos A]$

$$\Rightarrow \int \sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}\,d\theta$$

$$\Rightarrow \int \sqrt{(\sin\theta+\cos\theta)^2}\,d\theta$$

$$\Rightarrow \int (\sin\theta+\cos\theta)\,d\theta$$

$$\Rightarrow \int \sin\theta\,d\theta + \int \cos\theta\,d\theta$$

$$\Rightarrow -\cos\theta + \sin\theta + C$$

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(6) Evaluate $\int (\cos 5x + 4\sec^2 x + 8e^{4x} + \frac{2}{x})dx$

Sol:- Let $\int (\cos 5x + 4\sec^2 x + 8e^{4x} + \frac{2}{x})dx$

$\Rightarrow \int \cos 5x\,dx + \int 4\sec^2 x\,dx + \int 8e^{4x}dx + \int \frac{2}{x}dx$

$\Rightarrow \frac{\sin 5x}{5} + 4\int \sec^2 x\,dx + 8\int e^{4x}dx + 2\int \frac{1}{x}dx$

$\Rightarrow \frac{\sin 5x}{5} + 4\tan x + \cancel{8}\frac{e^{4x}}{\cancel{4}} + 2\log x + C$

$\Rightarrow \frac{\sin 5x}{5} + 4\tan x + 2e^{4x} + 2\log x + C$

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(7) Evaluate $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}dx$

Sol:- Let $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}dx$

put $\sin^{-1}x = t$

Diff w.r.t 'x'

$\frac{d}{dx}(\sin^{-1}x) = \frac{dt}{dx}$

$\frac{1}{\sqrt{1-x^2}}dx = dt$

$\therefore \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}dx = \int e^t dt = e^t + C$

$= e^{\sin^{-1}x} + C$

(8) Evaluate $\int \frac{e^{\tan^{-1}x}}{1+x^2}\, dx$

Sol:- Let $\int \frac{e^{\tan^{-1}x}}{1+x^2}\, dx$

Put $\tan^{-1}x = t$

Diff w.r.t 'x'

$$\frac{d}{dx}(\tan^{-1}x) = \frac{dt}{dx}$$

$$\frac{1}{1+x^2}\, dx = dt$$

$$\therefore \int \frac{e^{\tan^{-1}x}}{1+x^2}\, dx = \int e^t\, dt$$

$$= e^t + c$$

$$= e^{\tan^{-1}x} + c$$

(9) Evaluate $\int \sin 2x \cos 3x\, dx$

Sol:- Let $\int \sin 2x \cos 3x\, dx$

$$\Rightarrow \frac{1}{2}\int 2\sin 2x \cos 3x\, dx$$

$$\Rightarrow \frac{1}{2}\int \left[\sin(2x+3x) + \sin(2x-3x)\right] dx$$

$$\Rightarrow \frac{1}{2}\int \left[\sin 5x + \sin(-x)\right] dx$$

$$\Rightarrow \frac{1}{2}\int [\sin 5x - \sin x]\, dx$$

$$\Rightarrow \frac{1}{2}\left[\int \sin 5x\,dx - \int \sin x\, dx\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-\cos 5x}{5} - (-\cos x)\right] + C$$

$$\Rightarrow \frac{1}{2}\left[\frac{-\cos 5x}{5} + \frac{\cos x}{1}\right] + C$$

$$\Rightarrow \frac{1}{2}\left[\frac{-\cos 5x + 5\cos x}{5}\right] + C$$

$$\Rightarrow \frac{1}{10}\left[5\cos x - \cos 5x\right] + C$$

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(10) Evaluate $\int_0^1 (x+3)(x+1)\, dx$

Sol:- Let $\int_0^1 (x+3)(x+1)\, dx$

$$\Rightarrow \int_0^1 [x^2 + x + 3x + 3]\, dx$$

$$\Rightarrow \int_0^1 (x^2 + 4x + 3)\, dx$$

$$\Rightarrow \int_0^1 x^2 dx + 4\int_0^1 x\, dx + \int_0^1 3\, dx$$

$$\Rightarrow \left[\frac{x^3}{3} + 4\frac{x^2}{\cancel{2}} + 3x\right]_0^1$$

$$\Rightarrow \left(\frac{x^3}{3} + 2x^2 + 3x\right)_0^1$$

$$\Rightarrow \frac{(1)^3}{3} + 2(1)^2 + 3(1) - 0$$

$$\Rightarrow \frac{1}{3} + 2 + 3 = \frac{1}{3} + \frac{5}{1} = \frac{1+15}{3} = \frac{16}{3}$$

(11) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$

Sol:- Let $\int_0^1 \frac{1}{1+x^2} dx$

$$\Rightarrow \left[\tan^{-1} x\right]_0^1$$

$$\Rightarrow \left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$$\Rightarrow \left[\tan^{-1}\left(\tan\frac{\pi}{4}\right) - \tan^{-1}(\tan 0)\right]$$

$$\Rightarrow \frac{\pi}{4} - 0$$

$$\Rightarrow \frac{\pi}{4}$$

(12) Evaluate $\int_0^{\pi/2} \sin^2 x \, dx$

Sol:- Let $\int_0^{\pi/2} \sin^2 x \, dx$

$$\Rightarrow \int_0^{\pi/2} \left[\frac{1 - \cos 2x}{2}\right] dx$$

$$\Rightarrow \frac{1}{2}\int_0^{\pi/2}(1-\cos 2x)\,dx$$

$$\Rightarrow \frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$\Rightarrow \frac{1}{2}\left[\frac{\pi}{2}-\frac{\sin 2\left(\frac{\pi}{2}\right)}{2}-0+\frac{\sin 2(0)}{2}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{\pi}{2}-0-0+0\right]$$

$$\Rightarrow \frac{\pi}{4}$$

(13) Find The area bounded by $f(x)=x^2$ The x-axis Between The lines $x=1$ and $x=2$

Sol:- Given curve $f(x)=x^2$ and $x=1$ & $x=2$

The required Area $= \int_a^b f(x)\,dx$

$$= \int_1^2 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{2^3}{3}-\frac{1^3}{3}$$

$$= \frac{8}{3}-\frac{1}{3} = \frac{8-1}{3}$$

$$= \frac{7}{3} \text{ sq.units}$$

(14) Find The area bounded by The curve $y = x^2 - 5x + 6$, X-axis and The lines $x = 2$ and $x = 3$

Sol:- Given curve $y = x^2 - 5x + 6$

$$\text{Required Area} = \int_a^b y\, dx$$

$$= \int_2^3 (x^2 - 5x + 6)\, dx$$

$$= \left[\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right]_2^3$$

$$= \frac{3^3}{3} - \frac{5(3)^2}{2} + 6(3) - \frac{2^3}{3} + \frac{5(2)^2}{2} - 6(2)$$

$$= \frac{\cancel{27}}{\cancel{3}} - \frac{5(9)}{2} + 18 - \frac{8}{3} + \frac{5\cancel{(4)}}{\cancel{2}} - 12$$

$$= 9 - \frac{45}{2} + 18 - \frac{18}{3} + 10 - 12$$

$$= \left|\frac{-21}{6}\right|$$

$$= \frac{21}{6} \text{ sq. Units}$$

(15) Find the area of the region bounded by the curve $2y = x^2$ the x-axis and the ordinates at $x=1$, and $x=3$

Sol:- Given curve $2y = x^2$ Here $a=1$, $b=3$

$$\Rightarrow y = \frac{x^2}{2}$$

$$\text{Required Area} = \int_a^b y\,dx$$

$$= \int_1^3 \left(\frac{x^2}{2}\right) dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{x^3}{3}\right]_1^3$$

$$\Rightarrow \frac{1}{2}\left[\frac{3^3}{3} - \frac{1^3}{3}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{27}{3} - \frac{1}{3}\right] = \frac{1}{2}\left[\frac{27-1}{3}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{26}}{3}\right]$$

$$= \frac{13}{3} \text{ sq.units}$$

(16) Find Mean value of $f(x) = x^3 + x$ over the interval $[0,1]$

Sol:- Given $f(x) = x^3 + x$ Here $a=0$, $b=1$

$$\text{Mean value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{1-0} \int_0^1 (x^3+x)\,dx$$

$$= \frac{1}{1} \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1$$

$$= 1 \left[\frac{1}{4} + \frac{1}{2} - 0 - 0\right]$$

$$= 1 \left[\frac{1}{4} + \frac{1}{2}\right]$$

$$= \frac{1+2}{4} = \frac{3}{4}$$

(17) Find The mean value of The function $f(x) = \frac{1}{1+x^2}$ in The interval $[0,1]$

sol:- Given $f(x) = \frac{1}{1+x^2}$ Here $a=0$, $b=1$

$$\text{Mean value} = \frac{1}{b-a} \int_a^b f(x)\,dx$$

$$= \frac{1}{1-0} \int_0^1 \frac{1}{1+x^2}\,dx$$

$$= 1 \left[\tan^{-1} x\right]_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$

(18) Find The mean value of $Y = \sin x$ over $(0, 2\pi)$

Sol:- Given $Y = \sin x$

Here $a = 0$, $b = 2\pi$

$$\text{Mean value} = \frac{1}{b-a}\int_a^b y\,dx$$

$$= \frac{1}{2\pi - 0}\int_0^{2\pi} \sin x\,dx$$

$$= \frac{1}{2\pi}\left[-\cos x\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[-\cos 2\pi + \cos 0\right]$$

$$= \frac{1}{2\pi}\left[-1+1\right]$$

$$= \frac{1}{2\pi}(0)$$

$$= 0$$

(19) Find The order and degree of The Differential equation $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 5y = 0$

Sol:- Given Diff eq' is

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 5y = 0$$

order $= 3$

degree $= 1$

(20) Find The ~~Diffe~~ order and degree of The Differential Equation $2\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2+5y=0$

Sol:- Given Diff Eq is $2\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2+5y=0$

order is 3 and
degree is 1

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(21) Find The order and degree of The Differential Equation $\left(\frac{dy}{dx}\right)^3-\left(\frac{d^2y}{dx^2}\right)^{3/2}+5x=0$

Sol:- Given D.E is $\left(\frac{dy}{dx}\right)^3-\left(\frac{d^2y}{dx^2}\right)^{3/2}+5x=0$

The given D.E is not a polynomial Eq in The derivatives and it is not free from radicals for The derivatives of y

So The given D.E can be written as

$$\left(\frac{dy}{dx}\right)^3+5x=\left(\frac{d^2y}{dx^2}\right)^{3/2}$$

squaring o. B.S

$$\left[\left(\frac{dy}{dx}\right)^3+5x\right]^2=\left[\left(\frac{d^2y}{dx^2}\right)^{3/2}\right]^2$$

$$\left(\frac{dy}{dx}\right)^6 + 25x^2 + 2(5x)\left(\frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^3$$

$$\left(\frac{dy}{dx}\right)^6 + 10x\left(\frac{dy}{dx}\right)^3 + 25x^2 = \left(\frac{d^2y}{dx^2}\right)^3$$

This is The D.E which can be expressed as a polynomial eq in The derivatives.

order = 2
degree = 3

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(22) From The differential Equation from ~~for~~ $y = A\sin 3x + B\cos 3x$ where A, B are constants

sol:- Let $y = A\sin 3x + B\cos 3x$ —(1)

Diff w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(A\sin 3x + B\cos 3x)$$

$$= 3A\cos 3x - 3B\sin 3x$$

$$\frac{dy}{dx} = 3(A\cos 3x - B\sin 3x)$$

Again Diff w.r.t 'x'

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(3(A\cos 3x - B\sin 3x)\right)$$

$$\frac{d^2y}{dx^2} = 3[-3A\sin 3x - 3B\cos 3x]$$

$$= -9[A\sin 3x + B\cos 3x]$$

$$= -9y \quad [\text{from } (1)]$$

$$\boxed{\frac{d^2y}{dx^2} + 9y = 0}$$

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(23) From The Diff Eq for The family of Curve $y = ae^x + be^{-x}$ Eliminating The arbitary constants a and b

Sol:- Let $y = ae^x + be^{-x}$ — ①

Diff w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(ae^x + be^{-x})$$

$$\frac{dy}{dx} = ae^x - be^{-x}$$

Again Diff w.r.t 'x'

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(ae^x - be^{-x})$$

$$\frac{d^2y}{dx^2} = ae^x + be^{-x}$$

$$\frac{d^2y}{dx^2} = y \quad [\text{From } ①]$$

$$\boxed{\frac{d^2y}{dx^2} - y = 0}$$

(24) solve $(1+x)\,dy = (1+y)\,dx$

Sol:- Given $(1+x)\,dy = (1+y)\,dx$

Apply variable separable method

$$\frac{1}{1+y}\,dy = \frac{1}{1+x}\,dx$$

I.O.B.S

$$\int \frac{1}{1+y}\,dy = \int \frac{1}{1+x}\,dx$$

$$\Rightarrow \log(1+y) = \log(1+x) + \log c$$

$$\Rightarrow \log(1+y) - \log(1+x) = \log c$$

$$\Rightarrow \log\left(\frac{1+y}{1+x}\right) = \log c$$

$$\Rightarrow \frac{1+y}{1+x} = c$$

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25 Solve $\dfrac{dy}{dx} + \sqrt{\dfrac{1-y^2}{1-x^2}} = 0$

Sol:- Given $\dfrac{dy}{dx} = -\sqrt{\dfrac{1-y^2}{1-x^2}}$ $\left[\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\right]$

$$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-y^2}}\,dy = -\frac{1}{\sqrt{1-x^2}}\,dx$$

I.O.B.S

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = -\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\sin^{-1}y = -\sin^{-1}x + c$$

$$\boxed{\sin^{-1}x + \sin^{-1}y = c}$$

(26) Solve $\frac{dy}{dx} = \frac{y}{x}$

Sol:- Given $\frac{dy}{dx} = \frac{y}{x}$ [variable separable]

$$\frac{1}{y}\,dy = \frac{1}{x}\,dx$$

I.O.B.S

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

$$\log y = \log x + \log c$$

$$\log y = \log cx$$

$\boxed{y = cx}$ where 'c' is an arbitary constant

(27) Solve $(D^2 - 5D + 4)y = 0$

Sol:- Given $(D^2 - 5D + 4)y = 0$

$\therefore$ Auxiliary Equation of $f(D)y = 0$ is $f(m) = 0$ is

$$m^2 - 5m + 4 = 0$$

$$m^2 - 4m - m + 4 = 0$$

$$m(m-4) - 1(m-4) = 0$$

$$(m-4)(m-1) = 0$$

$m = 1, 4$

$$\boxed{C.F = c_1 e^{x} + c_2 e^{4x}}$$

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(28) Solve $(D^2 + 4D + 3)y = 0$

Sol:- Given $(D^2 + 4D + 3)y = 0$

A.E is

$m^2 + 4m + 3 = 0$

$m^2 + 3m + m + 3 = 0$

$m(m+3) + 1(m+3) = 0$

$(m+3)(m+1) = 0$

$m = -1, -3$

$$\boxed{C.F = c_1 e^{-x} + c_2 e^{-3x}}$$

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(29) Solve $(D^2 + 4)y = 0$

Sol:- A.E is $m^2 + 4 = 0$

$m^2 = -4$

$m = \sqrt{-4}$ $\quad [i^2 = -1]$

$m = \sqrt{(2i)^2}$

$m = \pm 2i$

$m = 0 \pm 2i$ $\quad$ Here $\alpha = 0, \beta = 2$

$$C.F = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$$

$$= e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$$

$$\boxed{C.F = c_1 \cos 2x + c_2 \sin 2x}$$

30 Solve $(D^2+4D+4)y=0$

Sol:- A.E is $m^2+4m+4=0$

$$m^2+2m+2m+4=0$$

$$m(m+2)+2(m+2)=0$$

$$(m+2)(m+2)=0$$

$$m=-2,-2$$

The roots are Equal

$\therefore$ The General solution is

$$C.F = (c_1+c_2 x)e^{mx}$$

$$\boxed{C.F = (c_1+c_2 x)e^{-2x}}$$

ESSAY TYPE

31) (a) Evaluate $\int \frac{1}{\sqrt{25x^2+9}}\,dx$

(b) Evaluate $\int \frac{1}{x^2+25}\,dx$

Sol:- (a) Let $\int \frac{1}{\sqrt{25x^2+9}}\,dx$

$$\Rightarrow \int \frac{1}{\sqrt{25\left(x^2+\frac{9}{25}\right)}}\,dx$$

$$\Rightarrow \int \frac{1}{\sqrt{25}\sqrt{x^2+\left(\frac{3}{5}\right)^2}}\,dx$$

$$\Rightarrow \frac{1}{5}\int \frac{1}{\sqrt{x^2+\left(\frac{3}{5}\right)^2}}\,dx$$

$$\boxed{\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right)+c}$$

$$\Rightarrow \frac{1}{5}\sinh^{-1}\left(\frac{x}{3/5}\right)+c$$

$$\Rightarrow \boxed{\frac{1}{5}\sinh^{-1}\left(\frac{5x}{3}\right)+c}$$

(b) Let $\int \frac{1}{x^2+25}\,dx$

$$\Rightarrow \int \frac{1}{x^2+5^2}\,dx$$

$$\boxed{\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c}$$

$$\Rightarrow \boxed{\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)+c}$$

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(32) (a) Evaluate $\int \frac{1}{\sqrt{25-x^2}}\,dx$ (b) Evaluate $\int \sqrt{9-x^2}\,dx$

Sol:- (a) Let $\int \frac{1}{\sqrt{25-x^2}}\,dx$

$\Rightarrow \int \frac{1}{\sqrt{5^2-x^2}}\,dx$ $\quad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+c$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right)+c$

(b) Let $\int \sqrt{9-x^2}\,dx$

$\Rightarrow \int \sqrt{3^2-x^2}\,dx$ $\quad \boxed{\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c}$

$\Rightarrow \frac{x}{2}\sqrt{3^2-x^2}+\frac{3^2}{2}\sin^{-1}\left(\frac{x}{3}\right)+c$

$\Rightarrow \boxed{\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)+c}$

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(33) (a) Evaluate $\int \frac{1}{x^2+8x+25}\,dx$

(b) Evaluate $\int \frac{1}{x^2+6x+25}\,dx$

Sol:- (a) Let $\int \frac{1}{x^2+8x+25}\,dx$

$\Rightarrow \int \frac{1}{x^2+2(4)x+16+9}\,dx$

$$\Rightarrow \int \frac{1}{x^2+2(4)x+4^2+9}\,dx$$

$$\boxed{\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c}$$

$$\Rightarrow \int \frac{1}{(x+4)^2+3^2}\,dx$$

$$\Rightarrow \frac{1}{3}\tan^{-1}\left(\frac{x+4}{3}\right)+c$$

(b) Evaluate $\int \frac{1}{x^2+6x+25}\,dx$

Sol:-

Let $\int \frac{1}{x^2+6x+25}\,dx$

$$\Rightarrow \int \frac{1}{x^2+2(3)x+9+16}\,dx$$

$$\Rightarrow \int \frac{1}{x^2+2(3)x+3^2+16}\,dx$$

$$\Rightarrow \int \frac{1}{(x+3)^2+4^2}\,dx$$

$$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{x+3}{4}\right)+c$$

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(34) Evaluate $\int \frac{x}{(x+2)(x+4)}\,dx$

Sol:-

Let $\frac{x}{(x+2)(x+4)} = \frac{A}{x+2}+\frac{B}{x+4} \rightarrow (1)$

$$\frac{x}{\cancel{(x+2)(x+4)}} = \frac{A(x+4)+B(x+2)}{\cancel{(x+2)(x+4)}}$$

$\Rightarrow \quad x = A(x+4) + B(x+2)$

put $x=-2$

$-2 = A(-2+4) + B(-2+2)$

$-2 = A(2) + 0$

$-2 = A(2)$

$A = \frac{-\cancel{2}}{\cancel{2}}$

$\boxed{A=-1}$

put $x=-4$

$-4 = A(-4+4) + B(-4+2)$

$-4 = 0 + B(-2)$

$-4 = B(-2)$

$B = \frac{\cancel{-}4}{\cancel{-}2}$

$\boxed{B=2}$

A, B values are sub in Eq (1) we get

$$\frac{x}{(x+2)(x+4)} = \frac{-1}{x+2} + \frac{2}{x+4}$$

I. O. B. S

$$\int \frac{x}{(x+2)(x+4)}\,dx = \int\left[\frac{-1}{x+2} + \frac{2}{x+4}\right]dx$$

$$= -\int \frac{1}{x+2}\,dx + 2\int \frac{1}{x+4}\,dx$$

$$= -\log(x+2) + 2\log(x+4) + C$$

(35) Evaluate $\int \frac{3x+1}{(x-1)(x-2)}\,dx$

Sol:- Let $\frac{3x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \rightarrow (1)$

$$\Rightarrow \frac{3x+1}{(x-1)\cancel{(x-2)}} = \frac{A(x-2)+B(x-1)}{(x-1)\cancel{(x-2)}}$$

$$\Rightarrow 3x+1 = A(x-2)+B(x-1)$$

put $x=1$

$3(1)+1 = A(1-2)+B(1-1)$

$4 = A(-1)+0$

$4 = A(-1)$

$A = \frac{4}{-1}$

$\boxed{A=-4}$

put $x=2$

$3(2)+1 = A(2-2)+B(2-1)$

$6+1 = 0+B(1)$

$7 = B(1)$

$B = \frac{7}{1}$

$\boxed{B=7}$

A, B values are sub in Eq (1) we get

$$\frac{3x+1}{(x-1)(x-2)} = \frac{-4}{x-1} + \frac{7}{x-2}$$

I.O.B.S

$$\int \frac{3x+1}{(x-1)(x-2)}\,dx = \int\left(\frac{-4}{x-1} + \frac{7}{x-2}\right)dx$$

$$= -4\int\frac{1}{x-1}\,dx + 7\int\frac{1}{x-2}\,dx$$

$$= -4\log(x-1) + 7\log(x-2) + C$$

(36) Evaluate (a) $\int e^x(\sin x + \cos x)\,dx$

(b) $\int e^x(\tan x + \sec^2 x)\,dx$

Sol:- (a) Let $\int e^x(\sin x + \cos x)\,dx$

Here $f(x) = \sin x$, $f'(x) = \cos x$

$$\boxed{\therefore \int e^x(f(x) + f'(x))\,dx = e^x f(x) + C}$$

$$\int e^x(\sin x + \cos x)\,dx = e^x \sin x + C$$

(b) Let $\int e^x(\tan x + \sec^2 x)\,dx$

Here $f(x) = \tan x$, $f'(x) = \sec^2 x$

$$\boxed{\int e^x(\tan x + \sec^2 x)\,dx = e^x \tan x + C}$$

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(37) Evaluate $\int x^2 e^x\,dx$

Sol:- Let $\int x^2 e^x\,dx$

we take $u = x^2$ and $dv = e^x\,dx$

$du = 2x\,dx$ $\quad v = e^x$.

Then. Integration by parts gives
$\int u\,dv = uv - \int v\,du$

$\int x^2 e^x dx = x^2 e^x - \int e^x (2x)\,dx$

$= x^2 e^x - 2\int x e^x\,dx$

Repeating The process of integration by parts we have

$= x^2 e^x - 2\left[x e^x - \int e^x dx\right] + C$

$= x^2 e^x - 2x e^x + 2e^x + C$

$= e^x\left[x^2 - 2x + 2\right] + C.$

38 Find $\int x \tan^{-1}x\,dx$

Sol:- Let $\int x \tan^{-1}x\,dx$

we take $u = \tan^{-1}x$ $\quad dv = x\,dx$

Diff w.r.t 'x' $\quad$ I.O.B.S

$\frac{du}{dx} = \frac{1}{1+x^2}$ $\quad \int dv = \int x\,dx$

$du = \frac{dx}{1+x^2}$ $\quad v = \frac{x^2}{2}$

Using The formula for Integrating by parts, we have

$\int u\,dv = uv - \int v\,du$

$\int x \tan^{-1}x\,dx = \tan^{-1}x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\frac{dx}{1+x^2}$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \left[\frac{1+x^2-1}{1+x^2}\right]dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \left[\frac{\cancel{1+x^2}}{\cancel{1+x^2}} - \frac{1}{1+x^2}\right]dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \left[1 - \frac{1}{1+x^2}\right]dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[\int 1\,dx - \int \frac{1}{1+x^2}\,dx\right]$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[x - \tan^{-1}x\right] + c$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{\tan^{-1}x}{2} + c$$

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39 Evaluate $\int x^3 e^{2x}\,dx$

Sol:- Let $\int x^3 e^{2x}\,dx$

As per ILATE rule

$u = x^3$ $\quad\quad v = e^{2x}$

$u' = 3x^2$ $\quad\quad v_1 = \frac{e^{2x}}{2}$

$u'' = 6x$

$u''' = 6$ $\quad\quad v_2 = \frac{e^{2x}}{4}$

$v_3 = \frac{e^{2x}}{8}$

$v_4 = \frac{e^{2x}}{16}$

By The Bernoulli's Rule

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$$

$$\int x^3 e^{2x}\,dx = x^3\left(\frac{e^{2x}}{2}\right) - 3x^2\left(\frac{e^{2x}}{4}\right) + 6x\left(\frac{e^{2x}}{8}\right) - 6\left(\frac{e^{2x}}{16}\right)$$

$$= x^3\frac{(e^{2x})}{2} - \frac{3x^2e^{2x}}{4} + \frac{3xe^{2x}}{4} - \frac{3e^{2x}}{8}$$

$$= \frac{e^{2x}}{8}\left[4x^3 - 6x^2 + 6x - 3\right] + C$$

(40) Evaluate $\int_0^{\pi/2} x\cos x\,dx$

Sol:- Let $\int_0^{\pi/2} x\cos x\,dx$

Here $u = x$ $\quad dv = \cos x\,dx$

$du = dx$ $\quad v = \sin x$

$$\int_0^{\pi/2} x\cos x\,dx = \left[x\sin x - \int \sin x\,dx\right]_0^{\pi/2}$$

$$= \left[x\sin x - (-\cos x)\right]_0^{\pi/2}$$

$$= \left[x\sin x + \cos x\right]_0^{\pi/2}$$

$$= \left[\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} - 0 - \cos 0\right]$$

$$= \frac{\pi}{2}(1) + 0 - 1]$$

$$= \frac{\pi}{2} - 1$$

(41) Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

sol:- Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ —— (1)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)}+\sqrt{\cos(\frac{\pi}{2}-x)}}\,dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \text{—— (2)}$$

Adding (1) & (2) we get

$$I+I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_0^{\pi/2} \left[\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right] dx$$

$$2I = \int_0^{\pi/2} \left[\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right] dx$$

$$2I = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

$$\boxed{\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx = \frac{\pi}{4}}$$

---o---

(42) Evaluate $\int_0^{\pi/2} \log \tan x \, dx$

Sol:-

Let $I = \int_0^{\pi/2} \log(\tan x)\, dx \rightarrow (1)$

$$I = \int_0^{\pi/2} \log \tan\left(\frac{\pi}{2} - x\right) dx$$

$$I = \int_0^{\pi/2} \log \cot x \, dx \rightarrow (2)$$

Adding (1) & (2) we get

$$I + I = \int_0^{\pi/2} \log \tan x \, dx + \int_0^{\pi/2} \log \cot x \, dx$$

$$2I = \int_0^{\pi/2} [\log \tan x + \log \cot x]\, dx$$

$$2I = \int_0^{\pi/2} [\log \tan x \cot x]\, dx$$

$$2I = \int_0^{\pi/2} \log(1)\, dx$$

$$2I = \int_0^{\pi/2} 0\, dx$$

$$2I = 0$$

$$I = 0$$

$$\boxed{\int_0^{\pi/2} \log(\tan x)\, dx = 0}$$

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(43) Find the R.M.S value of $\sqrt{8-4x^2}$ b/w $x=0$ and $x=2$

Sol:- Let $f(x) = \sqrt{8-4x^2}$. Here $a=0$, $b=2$

Mean square value of

$$M.S.V = \frac{1}{b-a}\int_a^b (f(x))^2\, dx$$

$$= \frac{1}{2-0}\int_0^2 \left(\sqrt{8-4x^2}\right)^2 dx$$

$$= \frac{1}{2}\int_0^2 (8-4x^2)\, dx$$

$$= \frac{1}{2}\left[\int_0^2 8\, dx - 4\int_0^2 x^2\, dx\right]$$

$$= \frac{1}{2}\left[8x - 4\frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{2}\left[8(2) - 4\frac{2^3}{3} - 8(0) + 4\frac{(0)^3}{3}\right]$$

$$= \frac{1}{2}\left[16 - \frac{4(8)}{3} - 0 + 0\right]$$

$$= \frac{1}{2}\left[\frac{16}{1} - \frac{32}{3}\right]$$

$$= \frac{1}{2}\left[\frac{48-32}{3}\right]$$

$$= \frac{1}{6}[16]$$

$$M.S.V = \frac{8}{3}$$

$$\therefore R.M.S \text{ value} = \sqrt{M.S.V}$$

$$= \sqrt{\frac{8}{3}}$$

———o———

(44) Find The R.M.S value of $\sqrt{\log x}$ over the range $x=1$ to $x=e$

Sol:- Let $f(x) = \sqrt{\log x}$

Here $a=1$, $b=e$

$$M.S.V = \frac{1}{b-a}\int_a^b [f(x)]^2\,dx$$

$$= \frac{1}{e-1}\int_1^e \left(\sqrt{\log x}\right)^2 dx$$

$$= \frac{1}{e-1} \int_1^e \log x \, dx$$

$$u = \log x \qquad dv = dx$$

$$du = \frac{1}{x} dx \qquad v = x$$

$$= \frac{1}{e-1} \left[uv - \int v \, du \right]_1^e$$

$$= \frac{1}{e-1} \left[\log x (x) - \int x \frac{1}{x} dx \right]_1^e$$

$$= \frac{1}{e-1} \left[x \log x - x \right]_1^e$$

$$= \frac{1}{e-1} \left[e \log e - e - 1 \log 1 + 1 \right]$$

$$= \frac{1}{e-1} \left[e(1) - e - 0 + 1 \right]$$

$$= \frac{1}{e-1} \left[\cancel{e} - \cancel{e} + 1 \right]$$

$$M.S.V = \frac{1}{e-1}$$

$$R.M.S \text{ value} = \sqrt{M.S.V}$$

$$= \sqrt{\frac{1}{e-1}}$$

$$= \frac{1}{\sqrt{e-1}}$$

———o———

(45) Solve $\frac{dy}{dx} + y\tan x = \sec x$

sol:- Let $\frac{dy}{dx} + y\tan x = \sec x$

Comparing The given Eq with $\frac{dy}{dx} + Py = Q$

we have $P = \tan x$, $Q = \sec x$

Now. $I.F = e^{\int P dx}$

$= e^{\int \tan x \, dx}$

$= e^{\log \sec x}$

$I.F = \sec x$

$\therefore$ The solution is

$y \times I.F = \int Q \times I.F \, dx + c$

$y \sec x = \int \sec x \sec x \, dx + c$

$y \sec x = \int \sec^2 x \, dx + c$

$y \sec x = \tan x + c$

$$\boxed{y \sec x - \tan x = c}$$