C23-A-AA-C-M-MET-MNG-MRAC-AEI-BM-CH-CHPP-CHPC-CHOT-CHST-EE-EEVT-TT-EC-ECII-CIOT-AIM-AI-CM-CAI-CCB-CCN–**102**

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BOARD DIPLOMA EXAMINATION, (C-23)

OCTOBER/NOVEMBER—2024

FIRST YEAR (COMMON) EXAMINATION

ENGINEERING MATHEMATICS—I

Time : 3 Hours] [*Total Marks* : 80

PART—A 3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

(3) Answer should be brief and straight to the point.

1. If $A = \left\{0, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f : A \to B$ is a function such that $f(x) = \cos x$, then find the range of *f*.

2. Resolve $\frac{x-1}{(x-2)(x+3)}$ into partial fractions.

3. If $A = \begin{bmatrix} 4 & 3 & -1 \\ 2 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & -3 \\ 3 & 5 & 2 \end{bmatrix}$, then find $3A - 4E$.

4. If ω is the cube root of unity, then prove that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$.

5. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then show that $A + B = \frac{\pi}{4}$.

6. Show that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$

7. Express the complex number $1+i\sqrt{3}$ in modulus amplitude form.

8. Find the equation of the straight line passing through the point (3,–4) and parallel to the line $x+7y+1=0$.

9. Evaluate $\lim_{x\to 0} \frac{x^2-7x+2}{3x^2-2x+4}$

10. Evaluate $\frac{d}{dx}(6x^2+12x-13)$

PART—B

10×5=50

Instructions : (1) Answer *any* **five** questions.

(2) Each question carries **ten** marks.

(3) Answer should be comprehensive and criteria for valuation is the content but not the length of the answer.

11. Solve the system of linear equations $2x-y+3z=9$, $x+y+z=6$ and $x-y+z=2$ using Cramer's rule.

12. *(a)* If $\cos x+\cos y=\frac{3}{5}$ and $\cos x-\cos y=\frac{2}{7}$, then show that

$$21\tan\left(\frac{x-y}{2}\right)+10\cot\left(\frac{x+y}{2}\right)=0$$

(b) Prove that $\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)=\sin^{-1}\left(\frac{56}{65}\right)$

13. *(a)* Solve $2\sin^2\theta-3\sin\theta+1=0$.

(b) Solve the ΔABC, if $a=1$, $b=2$, $c=\sqrt{3}$.

14. *(a)* Find the equation of the circle passing through the points (0,0), (2,0) and (0,3).

(b) Find the equation of the ellipse whose focus is (3,1), eccentricity is $\frac{1}{2}$ and directrix is $x - y + 6 = 0$.

15. *(a)* Find the derivative of $e^x \sin x$ w.r.t. x.

(b) Find $\frac{dy}{dx}$, if $y = \log(\sec x + \tan x)$.

16. *(a)* Find $\frac{dy}{dx}$, if $y = (\sin x)^x$.

(b) If $u(x, y, z) = \log(x + y + z)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 1$.

17. *(a)* Find the slopes of tangent and normal to the curve $y = x^2 - 2x - 3$ at (0,–3).

(b) A circular metal plate expands by heat so that its radius is increasing at the rate of 0·02 cm per second. At what rate its area is increasing when the radius is 20 cm?

18. *(a)* Find maximum or minimum value of $f(x) = 3 + 10x - 5x^2$.

(b) There is an error of 1% in measuring the radius of a spherical balloon. Find the approximate percentage error in its volume.

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