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# Reconstruction of Weyl's history of geometry, following the guideline of "purification of the a priori"

JULIEN BERNARD

## 1. INTRODUCTION

In "Schlick, Weyl, Husserl: On Scientific philosophy" [1], I have studied the open polemic between Husserl and Schlick which arose around 1918. This concerned the question: What does it mean for philosophy to become "scientific"? I have shown that, beside their strong philosophical opposition, Schlick and Husserl had difficulty in understanding each other because they used totally different notions of "intuition" (*Anschauung*) and "lived experience" (*Erlebnis*).

In the last part of this article, I dealt with the highly controversial question about the pertinence of the use of the *synthetic a priori* at the foundations of science. As it is well known, Schlick strongly advocated for the elimination of any such appeal to the *synthetic a priori*. Concerning this last question, instead of turning to Husserl's own response, I have chosen to question Hermann Weyl who, at least at the period, was a defender of phenomenology, engaged in the controversy.

Independently of Schlick's specific attacks, since the beginning of the XIX$^{th}$ century, defenders of the *synthetic a priori* had to conciliate their position with the fact that most of the elements of knowledge that Kant had considered to be *synthetic a priori* eventually turned out to be contingent ele-

1. [Berté].

ments that had been abolished by the evolution of mathematics and physics. Above all, this concerns the axioms of Euclidean geometry. As it is now well known, neo-Kantians and the young Reichenbach (with his notion of "relativized *a priori*") had proposed their own solutions to this issue [2].

**Apriority of space vs historicity of geometry.** Let us restrict our attention to the domain of space and geometry. So the issue explained above takes a new form. We can say that the defenders of the transcendental-ideality of space had to resolve the tension between 1) the supposed *apriority of space* and 2) the recognized *historicity of geometry*. Here [3], by "history of geometry" I never mean an history of changes *within* a given geometrical framework, but always a change *of* geometrical framework. For example, I am not interested here in the history of Euclidean geometry (history of its theorems and concepts, history of their reception by mathematical communities, history of the different formulations of its concepts, etc.). I am rather interested in the historical shift that compelled us to abandon Euclidean geometry and adopt other geometrical frameworks in order to give foundations to the spatial properties of physical phenomena. Therefore, this issue concerns the relationships between geometry as a mathematical discipline *and physics*. The fundamental problem is: Among the different kinds of geometrical frameworks, which is (are) the "good" or "true" one(s) [4]? Here and in the following, I put the brackets around "good" and "true" in order to remember that this does not necessarily refer to a realistic conception of space, but it merely refers to the space(s) that is (are) the most pertinent for the needs of physics. This fundamental problem is part of what Weyl (and a lot of his contemporaries) called "the problem of space".

Along with neo-Kantians and phenomenologists, Weyl is highly involved in the debates emerging from the tension between apriority of space and historicity of geometry. Indeed, on one hand, Weyl defended an idealistic philosophy of space, mainly inspired by "Kantian" ideas in a broad sense and by Husserlian phenomenology, assuming that space is a *form of appearances* (*Form der Erscheinungen*) [5], and admitting the importance of the *synthetic a*

2. See [Berté, section 3] and the references there.

3. As in [Berté].

4. The plural should be used here in order to leave open the possibility of a (at least partly) conventional position about geometry.

5. Actually, this specific point is closer to Kantianism (but in a modified vocabulary) than to Husserlian phenomenology. As Husserl teaches us in *Ding und Raum*, this was an important mistake of Kant not having considered space as a "form of thinghood" (*Form der Dinglichkeit*) instead of a mere form of sensible phenomena. See [HL89, chapitre III,

*priori* in the foundations of geometry[6]. On the other hand, he was very conscious about the historicity of geometry, since he wrote several remarks about history of geometry –in the precise sense we are interested in here– and since he himself proposed us to enter in a new stage of this History (see below). As Weyl suggested himself[7]: Geometry will always be "alive", evolving together with physics. The [geometrical] Truth will never accept to be "buried" in a fixed framework, once and for all. In so far as the axioms of geometry evolve with the history of science, they cannot be considered as apodictic, in the Kantian sense. How is this compatible with the idealistic position on space defended by Weyl?

I have showed that Weyl's solution to get out of this dilemma is different from Reichenbach's famous idea of a *relativized a priori*[8]. Indeed, for Weyl, the *a priori* itself is not historical (this would be contradictory). What is evolving is only the more or less pure *expression*[9] of the *a priori*, within the axioms of science. More precisely, I have summarised Weyl's position like this:

> The true *a priori* has no history. It is constituted of apodictic elements of knowledge, which can be justified by a lucid epistemological analysis. But we have usually no direct access to them. Within the axioms of geometry, this true *a priori* is unconsciously mixed together with empirical impurities, or contingent elements inherited from the past. The history of geometry is not an history of the *a priori*. Rather, throughout the history of geometry, the *a priori* should become *expressed*[8] in a purer and purer way within the axioms.
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> *The guideline of history of geometry should be the purification of the a priori within the axioms.*

This interpretation of the historicity of geometry is not thematised at length by Weyl. That is why I have had to reconstruct it, mainly from the

§14]. Moreover, neo-Kantians of that period often questioned the role of sensibility in the foundations of the space used in physics.

6. See for example [Ber13] and [BL19a].
7. [WBA15, $7^{th}$ conference].
8. See the last part of [Berté].
9. By speaking about « expression » here, I don't mean that this would be a *linguistic* problem. The difficulty involved here does not consist principally in finding a correct *linguistic expression* of the *a priori*. This is rather a question of not unconsciously mixing it with impurities.

collection of four passages from Weyl, in which he speaks, in a very general way, about the relationships between theory of knowledge and the positive advance of sciences [10] (in particular of geometry).

Now there is a second type of textual source that we can use in order to check this reconstruction; namely: the texts where Weyl gives his own vision of the different steps of the history of geometry. The objective of the current article is to check that, according to Weyl, the driving force of the history of science should actually be the idea of *purification of the a priori*. Weyl's conception of the history of geometry will be mainly reconstructed by focusing on the eight conferences of *Mathematische Analyse des Raumproblems* [11]. I shall show that Weyl's detailed understanding of the different steps confirms his understanding of the history of geometry as a process of clarification of the *aprioristic* foundations of space.

We must understand that the idea of history of geometry involved here is a *normative* and *teleological* one. It is normative since this is a reflection about what *should* direct the history of geometry, in order for it to play correctly its role within the building of knowledge. That is why I used modal expressions above ("The guideline of history of geometry *should* be the ..."). And it is *teleological* in the sense that it is directed toward the goal of a purification of the *a priori*. But I don't mean necessarily that the aimed-at "pure geometry" could be reached in a finite time, and that the last step of this history should be that of Weyl geometry (in the sense of the geometry reached by the "fourth jump" below). We will even see below that Weyl seriously doubted about it.

Therefore, according to my interpretation, Weyl was not in those texts directly interested by the *concrete and factual* history of geometry, but by a *normative* one, guided by epistemological considerations of a transcendental-idealistic nature. Nevertheless, Weyl speaks about this idealised history of geometry through certain steps that are extracted from the concrete and factual history of geometry (Euclid, Helmholtz-Lie, Riemann...). Indeed, *some of* the concrete and actual steps of the history of geometry were adequate, according to Weyl's conception, since they were or could be motivated by correct epistemological analyses. Concerning those peculiar "moments" of the history of geometry, the factual and the normative coincided. That is why these steps, extracted from the concrete history of geometry, can be used as examples of the idea of purification of the *a priori* as a guideline.

10. The involved texts are listed in [Berté, footnote 51].
11. [WBA15].

However, obviously, the factual history of geometry is also full of wrong attempts to generalise or modify the spatial structures. By "wrong", I mean some proposed generalisations that should not be accepted, according to Weyl's conception, because one can show that they violate some legitimate *a priori* requirement. We will see two examples of such wrong generalisations below (Finsler's and Cartan's). Because of the divergence between the factual and the normative histories of geometry, we understand that Weyl's intention, within *Mathematische Analyse des Raumproblems*, was not to speak about all the concrete developments of the factual geometry.

## 2. RECONSTRUCTION OF WEYL'S HISTORY OF GEOMETRY

### A. First jump in the history of geometry: From Euclid to Riemann-Helmholtz (constant curvature geometries)

The first jump [12], in this sketchy history of geometry, is from *Euclidean geometry* to the more general *geometry of constant curvature* (XIX$^{th}$ century). At this step, the empirical impurity was revealed not by any empirical discovery but rather by reflections on purely mathematical results, above all the discovery of non-Euclidean geometry.

When Riemann [13] and Helmholtz-Lie [14] gave foundations to constant curvature geometry, what did they retain from the old framework (Euclidean geometry), and what did they remove? They kept the idea that the metrical structure of space must be *homogeneous.* Space must have the same properties everywhere. But they, in fact, gave a stronger meaning to the idea of homogeneity, by assuming the possibility to move freely a rigid body within space and give it any possible orientation. The axiom of free mobility expresses not

12. [WBA15, pp. 25-sq.].

13. In [Rie19], Riemann adopts several positions on the foundations of the "real" metric. By putting here Riemann together with Helmholtz and Lie, I insist in this section on the main passages of his memoir where he defends that the metric of real space is Euclidean, and where he expresses the hypothesis of free mobility. In another passage of the same text, which interested Weyl very much, Riemann considered the possibility that the real metric would be dynamical, depending on the present forces. For this reason, Weyl considered Riemann as a prophet of relativity. This second position of Riemann will be considered in the next paragraph. Cf. also [Ber18, section 2.2] for more details on the structure of Riemann's memoir.

14. I put here Lie together with Helmholtz, not referring to his general work on the so called "Lie groups", but for his work of correction/completion of Helmholtz's solution to the problem of space. See [Mer10] and [Ber18].