EUF

Exame Unificado
das Pós-graduações em Física

Para o segundo semestre de 2017

04-05 de abril de 2017

FORMULÁRIO

Não escreva nada neste formulário. Devolva-o ao final da prova.

Constantes físicas

Velocidade da luz no vácuo	$c = 3{,}00\times10^{8}$ m/s
Constante de Planck	$h = 6{,}63\times10^{-34}$ J s $= 4{,}14\times10^{-15}$ eV s
	$\hbar = h/2\pi = 1{,}06\times10^{-34}$ J s $= 6{,}58\times10^{-16}$ eV s
	$hc \simeq 1240$ eV nm $= 1240$ MeV fm
	$\hbar c \simeq 200$ eV nm $= 200$ MeV fm
Constante de Wien	$W = 2{,}898\times10^{-3}$ m K
Permeabilidade magnética do vácuo	$\mu_0 = 4\pi\times10^{-7}$ N/A^2 $= 12{,}6\times10^{-7}$ N/A^2
Permissividade elétrica do vácuo	$\epsilon_0 = \dfrac{1}{\mu_0 c^2} = 8{,}85\times10^{-12}$ F/m
	$\dfrac{1}{4\pi\epsilon_0} = 8{,}99\times10^{9}$ Nm^2/C^2
Constante gravitacional	$G = 6{,}67\times10^{-11}$ N m^2/kg^2
Carga elementar	$e = 1{,}60\times10^{-19}$ C
Massa do elétron	$m_{\mathrm{e}} = 9{,}11\times10^{-31}$ kg $= 511$ keV/c^2
Comprimento de onda Compton	$\lambda_{\mathrm{C}} = 2{,}43\times10^{-12}$ m
Massa do próton	$m_{\mathrm{p}} = 1{,}673\times10^{-27}$ kg $= 938$ MeV/c^2
Massa do nêutron	$m_{\mathrm{n}} = 1{,}675\times10^{-27}$ kg $= 940$ MeV/c^2
Massa do dêuteron	$m_{\mathrm{d}} = 3{,}344\times10^{-27}$ kg $= 1{,}876$ MeV/c^2
Massa da partícula α	$m_\alpha = 6{,}645\times10^{-27}$ kg $= 3{,}727$ MeV/c^2
Constante de Rydberg	$R_H = 1{,}10\times10^{7}$ m^{-1} , $\quad R_H hc = 13{,}6$ eV
Raio de Bohr	$a_0 = 5{,}29\times10^{-11}$ m
Constante de Avogadro	$N_{\mathrm{A}} = 6{,}02\times10^{23}$ mol^{-1}
Constante de Boltzmann	$k_{\mathrm{B}} = 1{,}38\times10^{-23}$ J/K $= 8{,}62\times10^{-5}$ eV/K
Constante universal dos gases	$R = 8{,}31$ J mol^{-1} K^{-1}
Constante de Stefan-Boltzmann	$\sigma = 5{,}67\times10^{-8}$ W m^{-2} K^{-4}

Raio do Sol	=	$6{,}96\times10^{8}$ m	Massa do Sol	=	$1{,}99\times10^{30}$ kg
Raio da Terra	=	$6{,}37\times10^{6}$ m	Massa da Terra	=	$5{,}98\times10^{24}$ kg
Distância Sol-Terra	=	$1{,}50\times10^{11}$ m			

1 J $= 10^{7}$ erg   1 eV $= 1{,}60\times10^{-19}$ J   1 Å $= 10^{-10}$ m   1 fm $= 10^{-15}$ m

Constantes numéricas

$\pi \cong 3{,}142$	$\ln 2 \cong 0{,}693$	$\cos(30^\circ) = \sin(60^\circ) = \sqrt{3}/2 \cong 0{,}866$
$\mathrm{e} \cong 2{,}718$	$\ln 3 \cong 1{,}099$	$\sin(30^\circ) = \cos(60^\circ) = 1/2$
$1/\mathrm{e} \cong 0{,}368$	$\ln 5 \cong 1{,}609$	
$\log_{10}\mathrm{e} \cong 0{,}434$	$\ln 10 \cong 2{,}303$	

Regras de propagação de erros

Se o erro de X é σ_X (ou seja, medidas de X são dadas como $X \pm \sigma_X$), então

$$F = f(a,b) \Rightarrow \sigma_F = \sqrt{\left(\frac{\partial f}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial f}{\partial b}\right)^2 \sigma_b^2}$$

$$S = a + b,\ D = a - b \Rightarrow \sigma_S = \sigma_D = \sqrt{\sigma_a^2 + \sigma_b^2}$$

$$P = ab,\ Q = \frac{a}{b} \Rightarrow \frac{\sigma_P}{P} = \frac{\sigma_Q}{Q} = \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2}$$

Mecânica Clássica

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \qquad \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \mathbf{r} \times \mathbf{F} \qquad L_i = \sum_j I_{ij}\omega_j \qquad T_R = \sum_{ij} \frac{1}{2} I_{ij}\omega_i\omega_j \qquad I = \int r^2\, dm$$

$$\mathbf{r} = r\hat{\mathrm{e}}_r \qquad \mathbf{v} = \dot{r}\hat{\mathrm{e}}_r + r\dot{\theta}\hat{\mathrm{e}}_\theta \qquad \mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathrm{e}}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\mathrm{e}}_\theta$$

$$\mathbf{r} = \rho\hat{\mathrm{e}}_\rho + z\hat{\mathrm{e}}_z \qquad \mathbf{v} = \dot{\rho}\hat{\mathrm{e}}_\rho + \rho\dot{\varphi}\hat{\mathrm{e}}_\varphi + \dot{z}\hat{\mathrm{e}}_z \qquad \mathbf{a} = \left(\ddot{\rho} - \rho\dot{\varphi}^2\right)\hat{\mathrm{e}}_\rho + \left(\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi}\right)\hat{\mathrm{e}}_\varphi + \ddot{z}\hat{\mathrm{e}}_z$$

$$\mathbf{r} = r\hat{\mathrm{e}}_r \qquad \mathbf{v} = \begin{array}{l} \dot{r}\hat{\mathrm{e}}_r + r\dot{\theta}\hat{\mathrm{e}}_\theta \\ +r\dot{\varphi}\sin\theta\hat{\mathrm{e}}_\varphi \end{array} \qquad \mathbf{a} = \begin{array}{l} \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta\right)\hat{\mathrm{e}}_r \\ + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta\right)\hat{\mathrm{e}}_\theta \\ + \left(r\ddot{\varphi}\sin\theta + 2\dot{r}\dot{\varphi}\sin\theta + 2r\dot{\theta}\dot{\varphi}\cos\theta\right)\hat{\mathrm{e}}_\varphi \end{array}$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r) \qquad V(r) = -\int_{r_0}^{r} F(r')\mathrm{d}r' \qquad V_{\text{efetivo}} = \frac{L^2}{2mr^2} + V(r)$$

$$\int_{R_0}^{R} \frac{\mathrm{d}r}{\sqrt{E - V(r) - \frac{L}{2mr^2}}} = \sqrt{\frac{2}{m}}\,(t - t_0) \qquad \dot{\theta} = \frac{L}{mr^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0, \quad L = T - V \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = Q_k$$

$$Q_k = \sum_{i=1}^{N} F_{ix}\frac{\partial x_i}{\partial q_k} + F_{iy}\frac{\partial y_i}{\partial q_k} + F_{iz}\frac{\partial z_i}{\partial q_k} \qquad Q_k = -\frac{\partial V}{\partial q_k}$$

$$\left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\text{fixo}} = \left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\text{rotação}} + 2\boldsymbol{\omega} \times \left(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right)_{\text{rotação}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}$$

$$H = \sum_{k=1}^{f} p_k\dot{q}_k - L; \qquad \dot{q}_k = \frac{\partial H}{\partial p_k}; \qquad \dot{p}_k = -\frac{\partial H}{\partial q_k}; \qquad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

Eletromagnetismo

$$\oint \mathbf{E}\cdot d\mathbf{l} + \frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{S} = 0 \qquad \nabla\times\mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\oint \mathbf{B}\cdot d\mathbf{S} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{S} = Q/\epsilon_0 = 1/\epsilon_0 \int \rho dV \qquad \nabla\cdot\mathbf{E} = \rho/\epsilon_0$$

$$\oint \mathbf{B}\cdot d\mathbf{l} - \mu_0\epsilon_0 \frac{d}{dt}\int \mathbf{E}\cdot d\mathbf{S} = \mu_0 I = \mu_0 \int \mathbf{J}\cdot d\mathbf{S} \qquad \nabla\times\mathbf{B} - \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad d\mathbf{F} = I d\mathbf{l}\times\mathbf{B}$$

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{\mathrm{e}}_r}{r^2} \qquad \mathbf{E} = -\nabla V \qquad V = -\int \mathbf{E}\cdot d\mathbf{l} \qquad V = \frac{q}{4\pi\epsilon_0}\frac{\hat{\mathrm{e}}_r}{r}$$

$$\mathbf{F}_{2\to 1} = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{(\mathbf{r}_1 - \mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \qquad U_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{l}\times\hat{\mathrm{e}}_r}{r^2} \qquad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dV'$$

$$\mathbf{B} = \nabla\times\mathbf{A} \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')dV'}{|\mathbf{r}-\mathbf{r}'|}$$

$$\mathbf{J} = \sigma\mathbf{E} \qquad \nabla\cdot\mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

$$u = \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + \frac{1}{2\mu_0}\mathbf{B}\cdot\mathbf{B} \qquad \mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$$

$$\nabla\cdot\mathbf{P} = -\rho_P \qquad \mathbf{P}\cdot\hat{\mathbf{n}} = -\sigma_P \qquad \nabla\times\mathbf{M} = \mathbf{J}_M \qquad \mathbf{M}\times\hat{\mathbf{n}} = \mathbf{K}_M$$

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E} \qquad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu\mathbf{H}$$

Relatividade

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \qquad x' = \gamma\,(x - Vt) \qquad t' = \gamma\left(t - Vx/c^2\right)$$

$$v'_x = \frac{v_x - V}{1 - V v_x/c^2} \qquad v'_y = \frac{v_y}{\gamma\,(1 - V v_x/c^2)} \qquad v'_z = \frac{v_z}{\gamma\,(1 - V v_x/c^2)}$$

$$E = \gamma m_0 c^2 \qquad \mathbf{p} = \gamma m_0 \mathbf{V} \qquad T = T_0\sqrt{\frac{1 + V/c}{1 - V/c}}$$ (fonte e detector se afastando)

Mecânica Quântica

$$i\hbar\,\frac{\partial\Psi(x,t)}{\partial t}=H\Psi(x,t)$$

$$H=\frac{-\hbar^2}{2m}\,\frac{1}{r}\,\frac{\partial^2}{\partial r^2}\,r+\frac{\hat{L}^2}{2mr^2}+V(r)$$

$$p_x=\frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$[x,p_x]=i\hbar$$

$$\hat{a}=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}+i\frac{\hat{p}}{m\omega}\right)$$

$$\hat{a}|n\rangle=\sqrt{n}|n-1\rangle\ ,\qquad \hat{a}^\dagger|n\rangle=\sqrt{n+1}|n+1\rangle$$

$$L_\pm=L_x\pm iL_y$$

$$L_\pm Y_{\ell m}(\theta,\varphi)=\hbar\sqrt{l(l+1)-m(m\pm 1)}\ Y_{\ell m\pm 1}(\theta,\varphi)$$

$$L_z=x\,p_y-y\,p_x$$

$$L_z=\frac{\hbar}{i}\frac{\partial}{\partial\varphi}\ ,\qquad [L_x,L_y]=i\hbar L_z$$

$$E_n^{(1)}=\langle n|\delta H|n\rangle$$

$$E_n^{(2)}=\sum_{m\neq n}\frac{|\langle m|\delta H|n\rangle|^2}{E_n^{(0)}-E_m^{(0)}}\ ,\qquad \phi_n^{(1)}=\sum_{m\neq n}\frac{\langle m|\delta H|n\rangle}{E_n^{(0)}-E_m^{(0)}}\phi_m^{(0)}$$

$$\hat{\mathbf{S}}=\frac{\hbar}{2}\vec{\sigma}$$

$$\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix}\ ,\quad \sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\ ,\quad \sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\bar{\psi}(\vec{p})=\frac{1}{(2\pi\hbar)^{3/2}}\int d^3r\,e^{-i\vec{p}\cdot\vec{r}/\hbar}\,\psi(\vec{r})\qquad \psi(\vec{r})=\frac{1}{(2\pi\hbar)^{3/2}}\int d^3p\,e^{i\vec{p}\cdot\vec{r}/\hbar}\,\bar{\psi}(\vec{p})$$

$$e^{\hat{A}}\equiv\sum_{n=0}^{+\infty}\frac{\hat{A}^n}{n!}$$

Física Moderna

$$p=\frac{h}{\lambda}\qquad E=h\nu=\frac{hc}{\lambda}\qquad E_n=-Z^2\,\frac{hcR_H}{n^2}=-Z^2\,\frac{13{,}6}{n^2}\mathrm{eV}$$

$$R_T=\sigma T^4\qquad \lambda_{\max}T=W\qquad L=mvr=n\hbar$$

$$\lambda'-\lambda=\frac{h}{m_0c}(1-\cos\theta)\qquad n\lambda=2d\sin\theta\qquad \Delta x\,\Delta p\geq\hbar/2\qquad \Delta E\,\Delta t\geq\hbar/2$$

$\langle E\rangle=\frac{\sum E_nP(E_n)}{\sum P(E_n)}$, onde $P(E_n)$ é a função de distribuição.

Termodinâmica e Mecânica Estatística

$$dU = dQ - dW \qquad dU = TdS - pdV + \mu dN$$

$$dF = -SdT - pdV + \mu dN \qquad dH = TdS + Vdp + \mu dN$$

$$dG = -SdT + Vdp + \mu dN \qquad d\Phi = -SdT - pdV - Nd\mu$$

$$F = U - TS \qquad G = F + pV$$

$$H = U + pV \qquad \Phi = F - \mu N$$

$$\left(\frac{\partial T}{\partial V}\right)_{S,N} = -\left(\frac{\partial p}{\partial S}\right)_{V,N} \qquad \left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial p}{\partial T}\right)_{V,N}$$

$$\left(\frac{\partial T}{\partial p}\right)_{S,N} = \left(\frac{\partial V}{\partial S}\right)_{p,N} \qquad \left(\frac{\partial S}{\partial p}\right)_{T,N} = -\left(\frac{\partial V}{\partial T}\right)_{p,N}$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_{T,N} \qquad S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} = T\left(\frac{\partial S}{\partial T}\right)_{V,N} \qquad C_p = \left(\frac{\partial H}{\partial T}\right)_{p,N} = T\left(\frac{\partial S}{\partial T}\right)_{p,N}$$

Gás ideal: $pV = nRT$, $U = C_V T = nc_V T$,
Processo adiabático: $pV^\gamma = \text{const.}$, $\gamma = c_p/c_V = (c_V + R)/c_V$

$$S = k_B \ln W$$

$$Z = \sum_n e^{-\beta E_n} \qquad Z = \int d\gamma e^{-\beta E(\gamma)} \qquad \beta = 1/k_B T$$

$$F = -k_B T \ln Z \qquad U = -\frac{\partial}{\partial \beta} \ln Z \qquad S = \frac{\partial}{\partial T}\left(k_B T \ln Z\right)$$

$$\Xi = \sum_N Z_N e^{\beta \mu N} \qquad \Phi = -k_B T \ln \Xi$$

$$f_{\mathrm{FD}} = \frac{1}{e^{\beta(\epsilon - \mu)} + 1} \qquad f_{\mathrm{BE}} = \frac{1}{e^{\beta(\epsilon - \mu)} - 1}$$

Resultados matemáticos

$$\int_{-\infty}^{\infty} x^{2n} \mathrm{e}^{-ax^2} \mathrm{d}x = \frac{1.3.5...(2n+1)}{(2n+1)2^n a^n} \left(\frac{\pi}{a}\right)^{\frac{1}{2}} \quad (n = 0,1,2,\ldots)$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad (|x| < 1) \qquad\qquad \mathrm{e}^{i\theta} = \cos\theta + i\sin\theta$$

$$\int \frac{\mathrm{d}x}{(a^2+x^2)^{1/2}} = \ln\left(x + \sqrt{x^2+a^2}\right) \qquad\qquad \ln N! \cong N\ln N - N$$

$$\int \frac{\mathrm{d}x}{(a^2+x^2)^{3/2}} = \frac{x}{\left(a^2\sqrt{x^2+a^2}\right)} \qquad\qquad \int \frac{x^2\mathrm{d}x}{(a^2+x^2)^{3/2}} = \ln\left(x+\sqrt{x^2+a^2}\right) - \frac{x}{\sqrt{x^2+a^2}}$$

$$\int \frac{\mathrm{d}x}{1-x^2} = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad\qquad \int \frac{\mathrm{d}x}{x(x-1)} = \ln(1-1/x)$$

$$\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\ \arctan\frac{x}{a} \qquad\qquad \int \frac{x}{a^2+x^2}\, dx = \frac{1}{2}\ \ln(a^2+x^2)$$

$$\int_0^{\infty} \frac{z^{x-1}}{e^z+1}\, dz = (1-2^{1-x})\ \Gamma(x)\ \zeta(x) \qquad (x > 0)$$

$$\int_0^{\infty} \frac{z^{x-1}}{e^z-1}\, dz = \Gamma(x)\ \zeta(x) \qquad (x > 1)$$

$$\Gamma(2) = 1 \qquad \Gamma(3) = 2 \qquad \Gamma(4) = 6 \qquad \Gamma(5) = 24 \qquad \Gamma(n) = (n-1)!$$

$$\zeta(2) = \frac{\pi^2}{6} \cong 1{,}645 \qquad \zeta(3) \cong 1{,}202 \qquad \zeta(4) = \frac{\pi^4}{90} \cong 1{,}082 \qquad \zeta(5) \cong 1{,}037$$

$$\int_{-\pi}^{\pi} \sin(mx)\ \sin(nx)\ dx = \pi\delta_{m,n} \qquad\qquad \int_{-\pi}^{\pi} \cos(mx)\ \cos(nx)\ dx = \pi\delta_{m,n}$$

$$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z \qquad\qquad \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r^2\mathrm{d}r\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}} \qquad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\mathrm{e}^{\pm i\phi}$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right) \qquad Y_{2,\pm1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\mathrm{e}^{\pm i\phi} \qquad Y_{2,\pm2} = \mp\sqrt{\frac{15}{32\pi}}\sin^2\theta\mathrm{e}^{\pm 2i\phi}$$

$$P_0(x) = 1 \qquad P_1(x) = x \qquad P_2(x) = (3x^2-1)/2$$

Solução geral para a equação de Laplace em coordenadas esféricas, com simetria azimutal:

$$V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + \frac{B_l}{r^{l+1}})\ P_l(\cos\theta)$$

$$\nabla\cdot(\nabla\times\mathbf{V}) = 0 \qquad \nabla\times\nabla f = 0$$

$$\nabla\times(\nabla\times\mathbf{V}) = \nabla(\nabla\cdot\mathbf{V}) - \nabla^2\mathbf{V}$$

$$\oint \mathbf{A}\cdot\mathrm{d}\mathbf{S} = \int (\nabla\cdot\mathbf{A})\,\mathrm{d}V \qquad \oint \mathbf{A}\cdot\mathrm{d}\mathbf{l} = \int (\nabla\times\mathbf{A})\cdot\mathrm{d}\mathbf{S}$$

Coordenadas cartesianas

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathrm{e}}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathrm{e}}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathrm{e}}_x + \frac{\partial f}{\partial y}\hat{\mathrm{e}}_y + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Coordenadas cilíndricas

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right]\hat{\mathrm{e}}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\hat{\mathrm{e}}_\varphi + \left[\frac{1}{\rho}\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\varphi}\right]\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial\rho}\hat{\mathrm{e}}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial\varphi}\hat{\mathrm{e}}_\varphi + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Coordenadas esféricas

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(A_\varphi)}{\partial\varphi}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\varphi)}{\partial\theta} - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\varphi}\right]\hat{\mathrm{e}}_r$$

$$+ \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right]\hat{\mathrm{e}}_\theta + \left[\frac{1}{r}\frac{\partial(rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial\theta}\right]\hat{\mathrm{e}}_\varphi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathrm{e}}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\mathrm{e}}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\hat{\mathrm{e}}_\varphi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$