

ELECTROSTATICS -

Charge

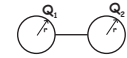
Quantization of charge

$$Q = \pm ne \quad Q = \text{Total charge}$$
$$n = 1, 2, 3, \dots$$
$$e = 1.6 \times 10^{-19} \text{C}$$

Additivity of charge

$$Q' = Q_1 + Q_2$$

Redistribution of charge



$$Q' = \frac{Q_1 + Q_2}{2}$$

Q' = Charge on each shell after redistribution

Charge Density

Linear Charge density, $\lambda = \frac{Q}{L}$ Unit = $\frac{C}{m}$

Surface Charge density, $\sigma = \frac{Q}{S}$ Unit = $\frac{C}{m^2}$

Volume Charge density, $\rho = \frac{Q}{V}$ Unit = $\frac{C}{m^3}$

Q = Total charge V = Volume
 L = Length S = Area



If a charge on the body is 1 nC, then how many electrons are present on the body?

a) 1.6×10^{19} b) 6.25×10^9

c) 6.25×10^{27} d) 6.25×10^{28}

Coloumb's Law

$$Q_1 \xrightarrow{r} Q_2 \quad F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

ϵ_0 = Permittivity of free space

$$\epsilon_0 = \frac{[Q_1][Q_2]}{[r^2][F]} = \frac{[AT][AT]}{[L^2][MLT^{-2}]} = M^{-1}L^{-3}T^4A^2$$

$$Q_1 \xrightarrow{r} Q_2 \quad F_{\text{med}} = \frac{F_{\text{air}}}{K}$$

Superposition

Direction:

a) Like- Towards the point at which force has to be evaluated (repulsion)

b) Unlike- Away from the point at which force has to be evaluated (attraction)

General rule

$$F_{\text{Net}} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta}$$

When $\theta = 60^\circ$

$$F_{\text{net}} = \sqrt{3}F$$

When $\theta = 90^\circ$

$$F_{\text{net}} = \sqrt{2}F$$

When $\theta = 120^\circ$

$$F_{\text{net}} = F$$

Equilibrium of Charges

Calculation of Charge

$$Q_1 \xrightarrow{r_1} q \xleftarrow{r_2} Q_2$$

$$\frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \quad q \text{ in equilibrium}$$

$$q = -\left(\frac{r_1}{r_1 + r_2}\right)^2 Q_2 \quad Q_1 \text{ in equilibrium}$$

$$q = -\left(\frac{r_2}{r_1 + r_2}\right)^2 Q_1 \quad Q_2 \text{ in equilibrium}$$



A charge is placed at the centre of the line joining two equal charges Q . The system of the three charges will be in equilibrium if q is equal to

a) $-Q/2$ c) $+Q/4$

b) $-Q/4$ d) $+Q/2$

Charge on pendulum

$$\tan \theta = \frac{qE}{mg}$$

$$\sin \theta = \frac{r}{2l}$$

$$r = 2l \sin \theta$$

if θ is very small

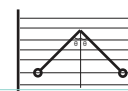
$$\tan \theta \approx \sin \theta$$

$$\frac{r}{2l} = \frac{qE}{mg}$$

$$\frac{r}{2l} = \frac{kq^2/r^2}{mg} \quad r^3 \propto q^2$$



it θ does not change on submerging in liquid Dielectric constant of liquid,



density of liquid = σ

Density of ball " ρ "

$$K = \frac{\rho}{\rho - \sigma}$$

Electric Field

$$E \propto \frac{1}{r^2}$$

Electric field at a point, $E = \frac{Kq}{r^2}$

$$K = \frac{1}{4\pi\epsilon_0}$$

Superposition

$$E_1 \rightarrow E_{\text{net}} \rightarrow E_2$$

General rule

$$E_{\text{net}} = \sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \cos \theta}$$

$E_1 \rightarrow E_{\text{net}} = E_1 + E_2$

$E_1 \rightarrow E_{\text{net}} = E_1 - E_2$

If $E_1 = E_2 = E$ Then, $E_{\text{net}} = \sqrt{3}E$

If $E_1 = E_2 = E$ Then, $E_{\text{net}} = \sqrt{2}E$

If $E_1 = E_2 = E$ Then, $E_{\text{net}} = E$

If $E_1 = E_2 = E$ Then, $E_{\text{net}} = E$

Direction

1) Positive charge:- Towards the point at which electric field has to be evaluated

2) Negative charge:- Away from the point at which electric field has to be evaluated

Neutral Point

Like Charges

$$x_1 = \frac{\sqrt{Q_1} r}{\sqrt{Q_1} + \sqrt{Q_2}}$$

$$x_2 = \frac{\sqrt{Q_2} r}{\sqrt{Q_1} + \sqrt{Q_2}}$$

Unlike Charges

Outside closer to smaller charge

$$|Q_2| < |Q_1|$$

$$x = \frac{\sqrt{Q_2} r}{\sqrt{Q_1} - \sqrt{Q_2}}$$

Distance from $Q_1 = x + r$



Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is:

a) $8L$

b) $4L$

c) $2L$

d) $L/4$

Charged particle in an electric field

1) Force, $F = qE$

2) Acceleration, $a = \frac{qE}{m}$

3) Velocity, $V = \frac{qE}{m} t$

4) Velocity, $V = \sqrt{\frac{2qEx}{m}}$

5) Kinetic energy, $K.E = \frac{q^2 E^2 t^2}{2m}$

accelerated in the direction of electric field

accelerated opposite to the direction of electric field

$$V = \sqrt{V_x^2 + V_y^2}$$
$$= \sqrt{U^2 + \left(\frac{qE}{m} t\right)^2}$$

accelerated in the direction perpendicular to the electric field

$$\frac{M_p}{M_e} = 1837, \quad \frac{e}{m} = 1.7 \times 10^{-11}$$

$$\frac{1}{2} at^2 = \text{Constant}$$
$$t^2 \propto m$$
$$\frac{t_1}{t_2} = \left[\frac{m_p}{m_e}\right]^{1/2}$$

Time period of Charged Pendulum in an electric field

$$T = 2\pi \sqrt{\frac{l}{g - \frac{QE}{m}}}$$

Time will increase

$$T = 2\pi \sqrt{\frac{l}{g + \frac{QE}{m}}}$$

Time will decrease

$$T = 2\pi \sqrt{\frac{l}{g^2 + \left(\frac{QE}{m}\right)^2}}$$

Electric field inside a dielectric medium

$$E_{\text{net}} = E - E_{\text{induced}}$$
$$E_{\text{induced}} = E - E_{\text{net}} = E\left(1 - \frac{1}{K}\right)$$

Properties of field lines

1) Start from positive charge and end on negative charge

2) Never intersect each other, if they intersect there will be 2 directions for electric field which is not possible

3) Always perpendicular to Conducting surface

4) $E \propto$ Electric field line density

5) Never form closed loops (Conservative force)

6) $q \propto$ no. of field lines

$$|q_1| > |q_2|$$

Electric lines of force about negative point charge are:

a) circular, anticlockwise

b) circular, clockwise

c) radial, inward

d) radial, outward

Electric flux

Total no. of field lines passing through an area

$$\Phi = \int E \cdot ds \cos \theta$$

Gauss Law:- $\Phi = \frac{q}{\epsilon_0} = \oint E \cdot ds \cos \theta$

Zero flux:- $\Phi = \frac{q_{\text{net}}}{\epsilon_0} = 0$, where $q_{\text{net}} = 0$

Electric flux for Cube

1) No charge inside the cube

$$\Phi = \frac{q}{\epsilon_0} = 0$$

2) Charge placed at the center

$$\Phi_{\text{total}} = \frac{q}{\epsilon_0}$$

$$\Phi_{\text{one side}} = \frac{q}{6\epsilon_0}$$

3) Charge placed at the face

$$\Phi_{\text{total}} = \frac{q}{2\epsilon_0}$$

4) Charge placed at the corner

$$\Phi_{\text{total}} = \frac{Q}{8\epsilon_0}$$
$$\Phi_{\text{one face}} = \frac{Q}{8\epsilon_0} \times \frac{1}{3} = \frac{Q}{24\epsilon_0}$$

5) Charge placed at the edge

$$\Phi_{\text{total}} = \frac{Q}{4\epsilon_0}$$
$$\Phi_{\text{face}} = \frac{Q}{\epsilon_0} \times \frac{1}{4} = \frac{Q}{16\epsilon_0}$$

6) Flux through curved surface

$$\Phi_{\text{effective}} = \Phi_{\text{curve}} + 2\Phi_{\text{cross section}}$$

Application of Gauss Theorem

1) Point charge $E = \frac{Kq}{r^2}$

2) Metal sphere

$$E_{\text{surface}} = \frac{KQ}{R^2}$$

$$E_{\text{outside}} = \frac{KQ}{r^2}$$

$$E_{\text{inside}} = 0$$

3) Non-Conducting sphere

$$E_{\text{inside}} = \frac{KQr}{R^3}$$

$$E_{\text{surface}} = \frac{KQ}{R^2}$$

$$E_{\text{outside}} = \frac{KQ}{r^2}$$

4) Conducting sheet

$$E = \frac{\sigma}{\epsilon_0}$$

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$$Q_{\text{outer}} \quad Q_{\text{inner}} \quad Q_{\text{plate}} \quad Q_{\text{inner}} = Q_{\text{plate}} - Q_{\text{outer}}$$

7) Electric field due to a finite linear charge distribution

$$E = \frac{2k\lambda}{r} \sin(\theta/2)$$

8) Electric field due to a infinite linear charge distribution

$$E = \frac{2k\lambda}{r}$$

9) Electric field due to circular arc

$$E = \frac{2k\lambda}{r} \sin(\theta/2)$$

eg: For a semicircle $\theta = 180^\circ$

$$E = \frac{2k\lambda}{r} \sin(90^\circ)$$
$$= \frac{2k\lambda}{r}$$

10) Electric field at the center of a circular ring

$$E_{\text{total}} = 0$$

11) Electric field due to a circular ring of charge

$$E = \frac{1}{4\pi\epsilon_0} \frac{qx}{(x^2 + a^2)^{3/2}}$$

(For large distance)

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$$

- Density (ρ) = $\frac{\text{mass (m)}}{\text{volume (v)}}$



- unit = kg/m^3
- density of water = $1000 \text{ kg / m}^3 = 1 \text{g/cc}$
- for same mass
 $\rho_1 V_1 = \rho_2 V_2$
 if $\rho_1 > \rho_2$
 $V_1 < V_2$

Mixing of liquid

Calculation of resultant/final density

1) Volumes are equal

$$d = \frac{d_1 + d_2}{2}$$

2) Masses are equal

$$\text{For 2- liquid} \Rightarrow d = \frac{2d_1 d_2}{d_1 + d_2}$$

$$\text{For 3- liquid} \Rightarrow d = \frac{3d_1 d_2 d_3}{d_1 d_2 + d_2 d_3 + d_1 d_3}$$

3) Masses and volumes are different

$$d = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{V_1 + V_2}$$

$$m_1 = \rho_1 V_1 \text{ \& } m_2 = \rho_2 V_2$$

$$V_1 = \frac{m_1}{\rho_1} \text{ \& } V_2 = \frac{m_2}{\rho_2}$$

Relative density (R.D)/Specific gravity

1) Relative density of a body

$$(R.D)_s = \frac{d_s}{d_w} = \frac{w_a}{w_a - w_w}$$

2) Relative density of liquid

$$(R.D)_L = \frac{d_L}{d_w} = \frac{w_a - w_L}{w_a - w_w}$$

3) Relative density of a solid to that of liquid

$$\frac{(R.D)_s}{(R.D)_L} = \frac{w_a}{w_a - w_L}$$

Pressure

Normal force or thrust exerted by liquid at rest per unit area

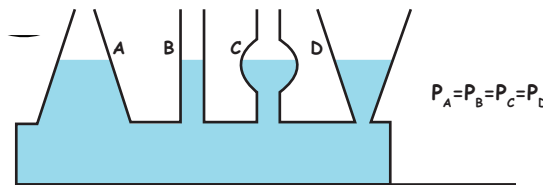
$$P = \frac{F}{A}$$

Pressure depth relation

$$P = h \rho g$$

Hydrostatic paradox

Whatever the shape or width the pressure at any particular depth is same



- Gauge pressure = $P - P_{atm} = h \rho g$

- $P_{atm} = 1.01325 \times 10^5 \text{ Pa}$

Inclined barometer

if θ = angle with horizontal

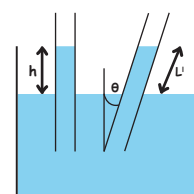
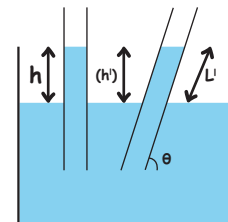
$$\sin \theta = \frac{h}{L'}$$

$$L' = \frac{h}{\sin \theta}$$

if θ = angle with vertical

$$\cos \theta = \frac{h}{L'}$$

$$L' = \frac{h}{\cos \theta}$$

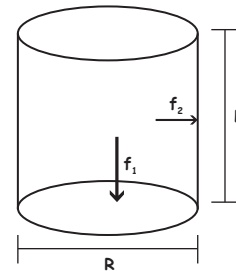


Bubble rising up at constant temperature

radius 'r' becomes 'R' when bubble rises in liquid from bottom to the surface

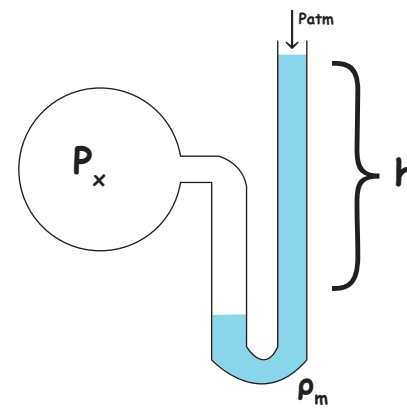
$$\rho g h = P_{atm} [n^3 - 1] \quad R = n r$$

Conditions for equal forces on wall and bottom in a cylinder



If $L = R$
then, $F_1 = F_2$

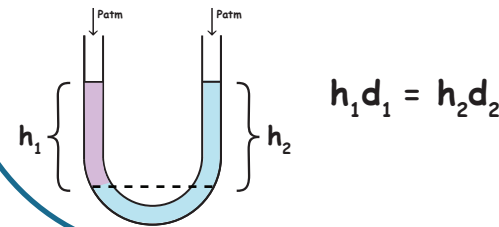
Manometer



$$P_x = P_o + h \rho_m g$$

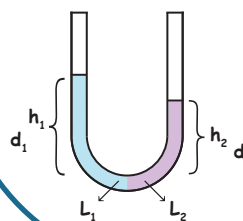
$$h \rho_m g = P_x - P_o$$

1) U-Tube manometer



$$h_1 d_1 = h_2 d_2$$

2) U-Tube type

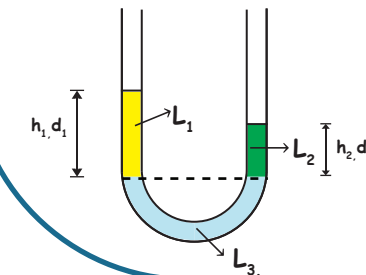


$$h_1 d_1 = h_2 d_2$$

$$\frac{d_1}{d_2} = \frac{h_2}{h_1}$$

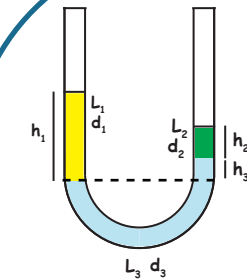
Note:
If one of the liquid H_2O ,
sp. gravity = $(R.D) = \frac{d_L}{d_w} = \frac{h_w}{h_L}$
 $(R.D) = \frac{h_w}{h_L}$

3) The third liquid is in level with other



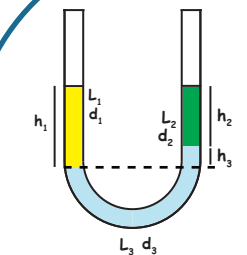
$$h_1 d_1 = h_2 d_2$$

4)



$$h_1 d_1 = h_2 d_2 + h_3 d_3$$

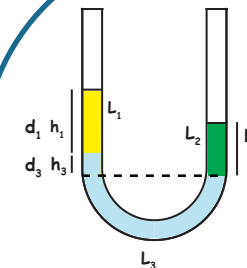
5)



$$h_1 = h_2 + h_3$$

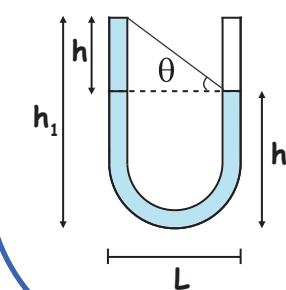
$$h_1 d_1 = h_2 d_2 + h_3 d_3$$

6)



$$h_1 d_1 + h_3 d_3 = h_2 d_2$$

U - tube accelerating

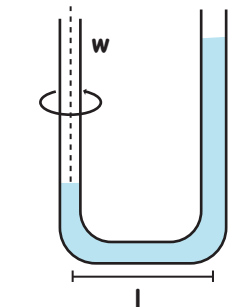


$$h = h_1 - h_2$$

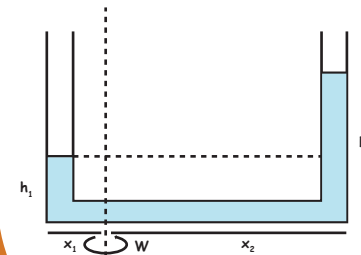
$$\frac{h}{L} = \frac{a}{g}$$

$$\tan \theta = \frac{h}{L} = \frac{a}{g}$$

Special case: U - tube rotating



$$h = \frac{w^2 l^2}{2g}$$



$$h_1 = \frac{w^2 x_1^2}{2g}$$

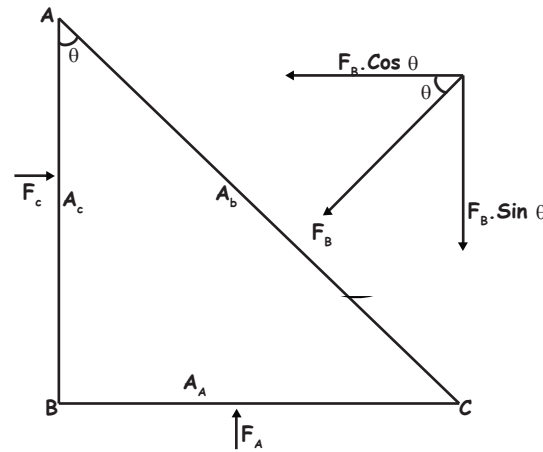
$$h_2 = \frac{w^2 x_2^2}{2g}$$

$$h = h_2 - h_1 = \frac{w^2 (x_2^2 - x_1^2)}{2g}$$

FLUID MECHANICS 01

Pascals Law

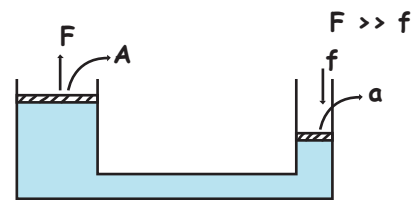
If gravity effect is neglected, the pressure at every point of liquid in static equilibrium is same



Application

Hydraulic Lift

As $A \gg a$ therefore



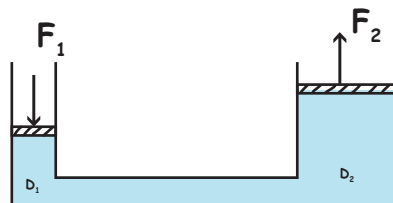
$$\frac{F}{A} = \frac{f}{a}$$

If the cylinders are connected

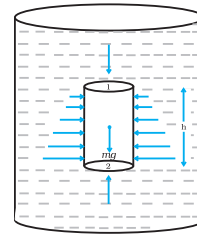
$$\frac{F_1}{\pi R_1^2} = \frac{F_2}{\pi R_2^2}$$

$$\frac{F_1}{R_1^2} = \frac{F_2}{R_2^2}$$

$$\frac{F_1}{D_1^2} = \frac{F_2}{D_2^2}$$



Archimedes principle



upthrust=weight of the liquid displaced= $V\rho g$

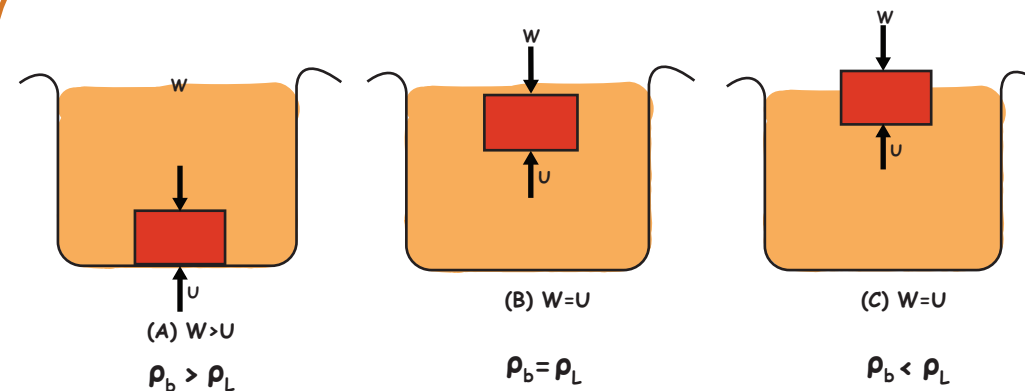
Apparent weight=Actual weight-upthrust

$$W_{app} = W_{air} - U$$

$$= W_{air} \left[1 - \frac{\sigma}{\rho} \right]$$

net force acting upward= $V \times \rho_L \times g$

Law of floatation



$W \Rightarrow$ Weight
 $U \Rightarrow$ Upthrust

Unit of Coefficient of viscosity

1) The CGS Unit of η is dyne s cm⁻² and is called poise.

2) The SI unit of η is Nsm⁻² or decapoise or poiseuille

1 poiseuille = 10 poise

Poiseuille's formula

$$Q = \frac{\pi P r^4}{8 \eta l}$$

Stoke's law

$$F = 6 \eta \pi r v$$

$F_{net} =$ Apparent weight-viscous force

Terminal velocity

$$V_t = \frac{2 r^2}{9 \eta} (\rho - \sigma) g$$

- 1) If $\rho > \sigma$, the body will attain terminal velocity in the downward direction.
- 2) If $\rho < \sigma$ the terminal velocity will be negative and the body will move in the upward direction.
- 3) $\rho = \sigma$, the body remain suspended in the fluid.

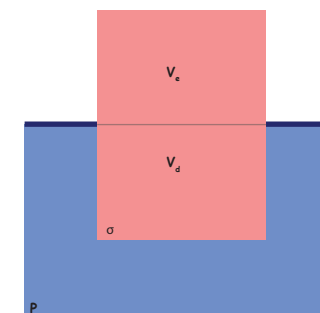
Fractional submerged volume

$$\frac{\text{Displaced volume}(V_d)}{\text{Total volume}(V)} = \frac{\sigma}{\rho} \text{ (submerged fraction)}$$

$$\frac{\text{Exposed volume}(V_e)}{\text{Total volume}(V)} = 1 - \frac{\sigma}{\rho} \text{ (Exposed fraction)}$$

$$\text{Relative density of a solid} = \frac{\text{weight of solid in air}}{\text{Loss of weight in water}} = \frac{W_a - W_w}{W_a - W_w} = \frac{\rho_b}{\rho_w}$$

$$\text{Relative density of a liquid} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{W_a - W_L}{W_a - W_w} = \frac{\rho_L}{\rho_w}$$



Critical velocity

Reynold number

$$R_e = \frac{\rho v D}{\eta}$$

Significances of Reynold number.

- If R_e lies between 0 and 2000 the flow is stream lined or laminar.
- If $R_e > 3000$, the liquid is turbulent.
- If R_e lies between 2000 & 3000 the flow of liquid is unstable. It may change from laminar to turbulent and vice versa.

Velocity gradient

$$\text{Velocity gradient} = \frac{dv}{dx}$$

$$F \propto A \frac{dv}{dx} \Rightarrow F = -\eta A \frac{dv}{dx}$$

$$\eta = \frac{F}{A \frac{dv}{dx}} \Rightarrow \text{coefficient of viscosity} \Rightarrow \eta = \frac{F/A}{\frac{dv}{dx}} = \frac{F/A}{\frac{d}{dt}(x/l)}$$

$$\Rightarrow \eta = \frac{\text{shearing stress}}{\text{strain rate}}$$

Equation of continuity

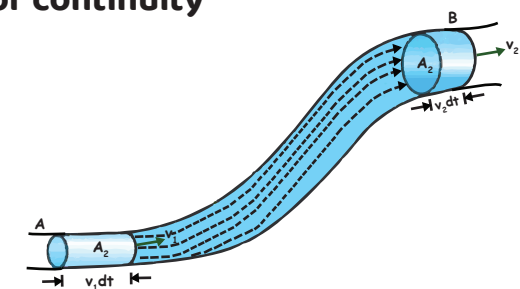
$$V_1 A_1 \Delta t \rho_1 = V_2 A_2 \Delta t \rho_2$$

since the liquid is incompressible $\rho_1 = \rho_2$

$$V_1 A_1 = V_2 A_2$$

$Av =$ constant.

$$Av = \frac{dv}{dt} = Q \Rightarrow \text{Volume rate of flow}$$



FLUID MECHANICS 02

Energy of fluid in a study flow

$$\text{kinetic Energy} = \frac{1}{2} mv^2$$

$$\text{kinetic energy per unit mass} = \frac{1}{2} v^2$$

$$\text{kinetic energy per unit volume} = \frac{1}{2} \rho v^2$$

$$\text{Potential Energy} = mgh$$

$$\text{Potential energy per unit mass} = gh$$

$$\text{Potential energy per unit volume} = \rho gh$$

$$\text{Pressure energy} = PV$$

$$\text{Pressure energy per unit mass} = \frac{P}{\rho}$$

$$\text{Pressure energy per unit volume} = P$$

BERNOULLI'S PRINCIPLE

$$P_1 V_1 - P_2 V_2 = \frac{1}{2} m (v_2^2 - v_1^2) + mg (h_2 - h_1)$$

$$(P_1 - P_2) V = \frac{1}{2} m (v_2^2 - v_1^2) + mg (h_2 - h_1)$$

$$(P_1 - P_2) V = \frac{1}{2} \frac{m}{V} (v_2^2 - v_1^2) + \frac{mg}{V} (h_2 - h_1)$$

$$\Rightarrow P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$$

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

$$\frac{P}{\rho g} = \text{pressure head}$$

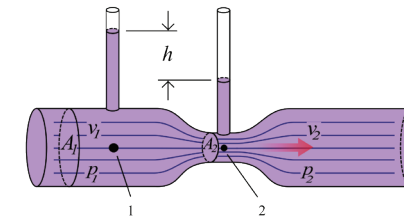
$$\frac{v^2}{2g} = \text{velocity head}$$

$$h = \text{Gravitational head}$$

VENTURIMETER

Device to measure the flow of speed of incompressible fluid

$$v_1 = \sqrt{\frac{2hg}{(A_1^2/A_2^2) - 1}}$$

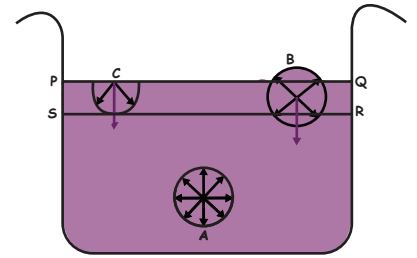


SURFACE TENSION

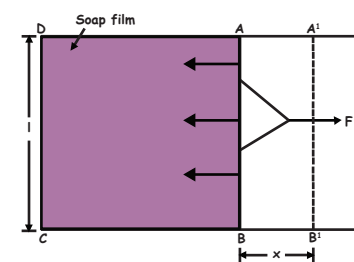
$$\text{Surface tension } T = \frac{\text{Force}}{\text{Length}} = \frac{F}{l}$$

$$\text{Unit in SI system} = \frac{\text{N}}{\text{m}}$$

$$\text{Unit in CGS system} = \text{dyne} / \text{cm}$$



SURFACE ENERGY



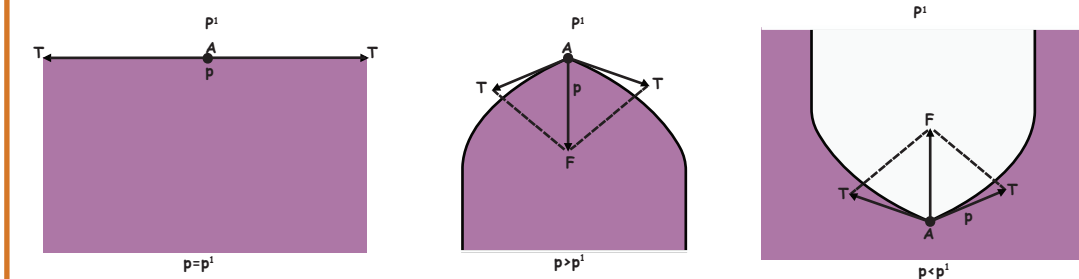
$$\text{Work done } W = F \times x$$

$$\text{But } F = 2TL$$

$$\Rightarrow W = 2TL \times x$$

$$\text{Energy of the additional surface} = W = 2TLx \\ = T(2Lx) = T\Delta A$$

PRESSURE DIFFERENCE ACROSS A CURVED LIQUID SURFACE

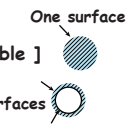


Pressure on concave side > pressure on convex side

$$P_{\text{concave}} - P_{\text{convex}} = \frac{2T}{R}$$

$$P_{\text{inside}} - P_{\text{outside}} = \frac{2T}{R} \quad [\text{Liquid drop or air bubble}]$$

$$P_{\text{inside}} - P_{\text{outside}} = \frac{4T}{R} \quad [\text{Soap bubble}]$$



APPLICATIONS OF BERNOULLI'S PRINCIPLE

Torricelli's Law of Efflux

$$v = \sqrt{\frac{2(P - P_o)}{\rho} + 2gh}$$

If tank is open, $P = P_o$

$$\text{Then } v = \sqrt{2gh}$$

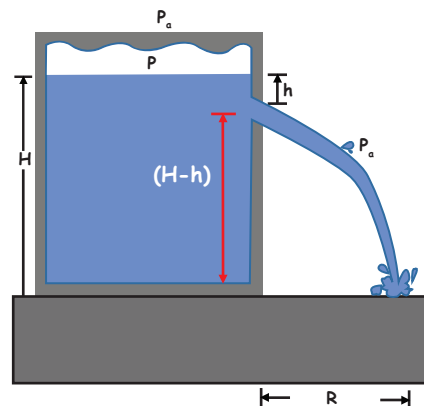
$$\text{Time of fall, } t = \sqrt{\frac{2(H-h)}{g}}$$

$$\text{Range } R = v_x \times t \\ = \sqrt{2gh} \times \sqrt{2(H-h)/g}$$

$$\Rightarrow R = 2\sqrt{h(H-h)}$$

v_x = Horizontal component of velocity

R is max. when $h = \frac{H}{2}$



Excess pressure inside a liquid drop

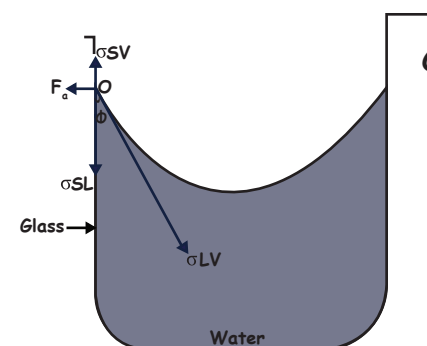
$$P_i - P_o = \frac{2T}{R}$$

Excess pressure inside a soap bubble

$$P_i - P_o = \frac{4T}{R}$$

FLUID MECHANICS 03

Shape of liquid meniscus



Consider the equilibrium of at line of contact

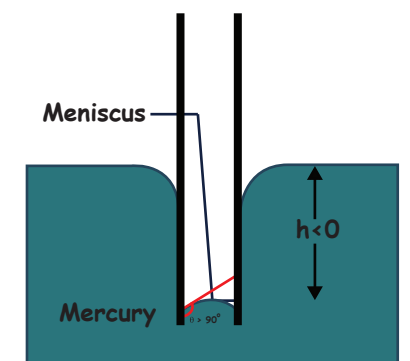
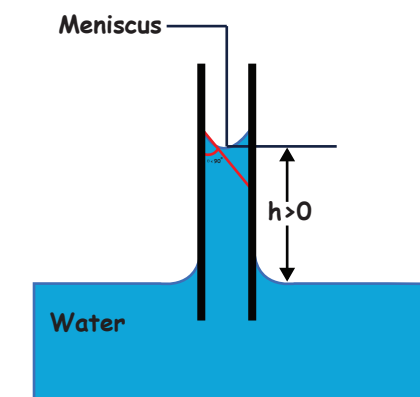
$$F_a = \sigma_{LV} \sin \theta$$

$$\sigma_{SV} = \sigma_{SL} + \sigma_{LV} \cos \theta$$

$$\cos \theta = \frac{\sigma_{SV} - \sigma_{SL}}{\sigma_{LV}}$$

θ = Angle of contact.

Capillarity

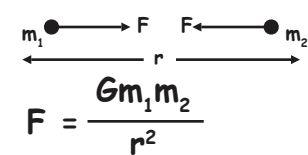


Ascent formula:

$$h = \frac{2T}{R\rho g} \quad h > 0 (\theta < 90^\circ)$$

$$h = \frac{2T \cos \theta}{r\rho g} \quad h < 0 (\theta > 90^\circ)$$

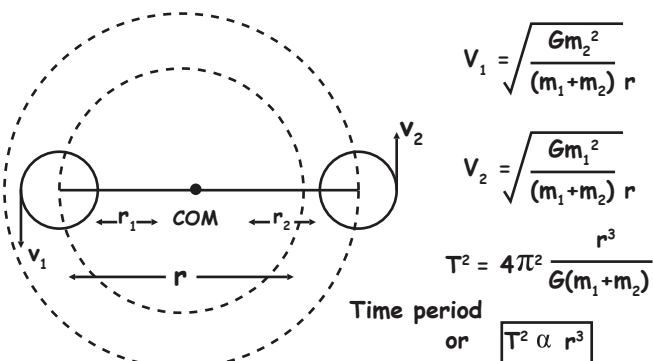
NEWTON'S LAW OF GRAVITATION



$$F = \frac{Gm_1m_2}{r^2}$$

G - Universal gravitational constant
 Value of G
 $6.67 \times 10^{-11} \text{ Nm}^2\text{Kg}^{-2}$ (SI or MKS)
 $6.67 \times 10^{-8} \text{ dyne cm}^2\text{g}^{-2}$ (CGS)
 Dimensional formula $[G]$
 $M^{-1}L^3T^{-2}$

ROTATION OF 2 MASSES UNDER MUTUAL GRAVITATIONAL FORCE OF ATTRACTION



$$v_1 = \sqrt{\frac{Gm_2^2}{(m_1+m_2)r}}$$

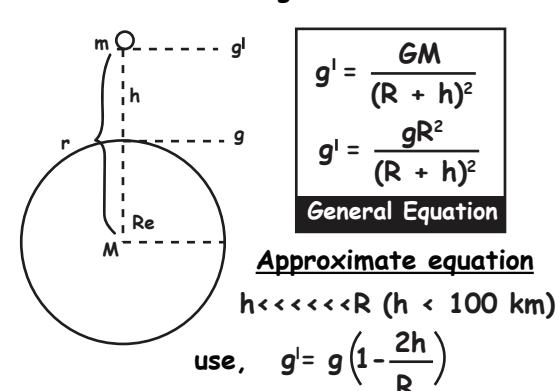
$$v_2 = \sqrt{\frac{Gm_1^2}{(m_1+m_2)r}}$$

$$T^2 = 4\pi^2 \frac{r^3}{G(m_1+m_2)}$$

Time period or $T^2 \propto r^3$

VARIATION IN THE VALUE OF ACCELERATION DUE TO GRAVITY

• Variation due to height 'h'



$$g' = \frac{GM}{(R+h)^2}$$

$$g' = \frac{gR^2}{(R+h)^2}$$

General Equation

Approximate equation
 $h \ll R$ ($h < 100 \text{ km}$)
 use, $g' = g \left(1 - \frac{2h}{R}\right)$

Note the point

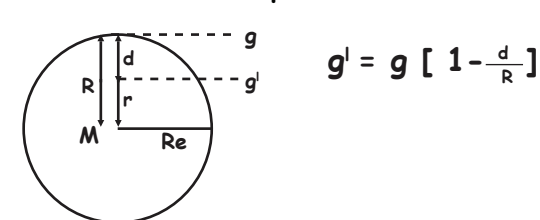
If $h \ll R$, then decrease in the value of g with height

Absolute decrease $= \Delta g = g - g' = \frac{2hg}{R}$

Fractional decrease $= \frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$

Percentage decrease $= \frac{\Delta g}{g} \times 100 = \frac{2h}{R} \times 100$

• Variation due to depth 'd'



$$g' = g \left[1 - \frac{d}{R}\right]$$

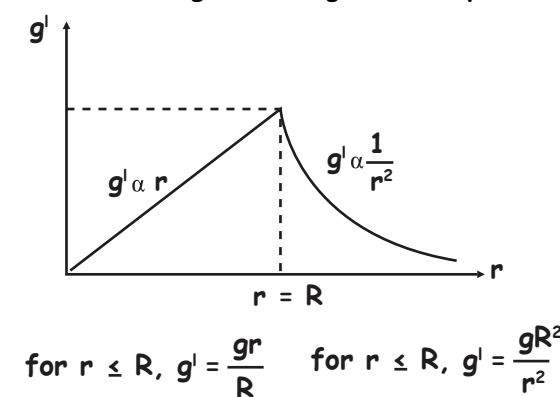
Absolute decrease $= \frac{\Delta g}{g} = g - g' = \frac{dg}{R}$

Fractional decrease $= \frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R}$

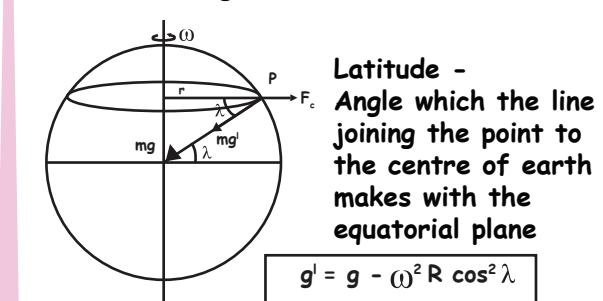
Percentage decrease $= \frac{\Delta g}{g} \times 100 = \frac{d}{R} \times 100$

Very imp graph

The graphical representation of change in the value of g' with height and depth



• Variation of g due to rotation of earth



Latitude - Angle which the line joining the point to the centre of earth makes with the equatorial plane

$$g' = g - \omega^2 R \cos^2 \lambda$$

Note $\Rightarrow$ value of $\omega^2 R = 0.034$

For poles $\lambda = 90^\circ$ $g' = g$

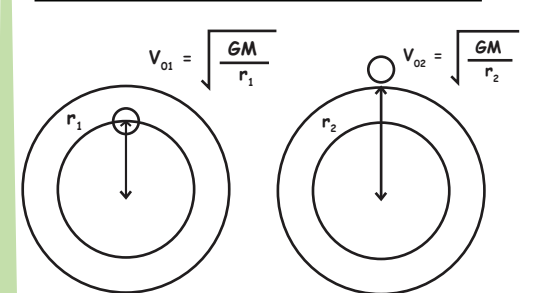
There is no effect of rotational motion of the earth on the value of g at poles.

For equator $\lambda = 0^\circ$ $g' = g - \omega^2 R$

The effect of rotational motion of the earth on the value of g at the equator is maximum.

When a body of mass m is moved from equator to the poles, weight increases by an amount $m(g_p - g_e) = m\omega^2 R$

WORK DONE IN MOVING OBJECT FROM ONE ORBIT TO ANOTHER



$$v_{01} = \sqrt{\frac{GM}{r_1}}$$

$$v_{02} = \sqrt{\frac{GM}{r_2}}$$

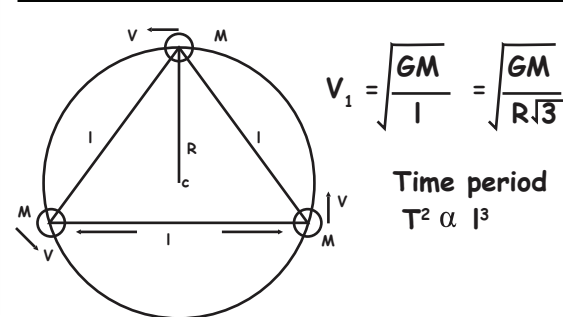
CONCEPT - WORK DONE BY EXTERNAL AGENT = CHANGE IN MECHANICAL ENERGY

$$W = E_2 - E_1 = \frac{GMm}{2} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

IMPORTANT POINTS ABOUT GRAVITATIONAL FORCE

1. Gravitational force
 - * Always attractive in nature
 - * Independent of the nature of medium between masses
 - * Independent of presence or absence of other bodies
2. Are central forces, acts along the centre of gravity of two bodies.
3. Conservative force
4. Force between any two masses - Gravitational force
 Force between earth and any other body - Force of gravity

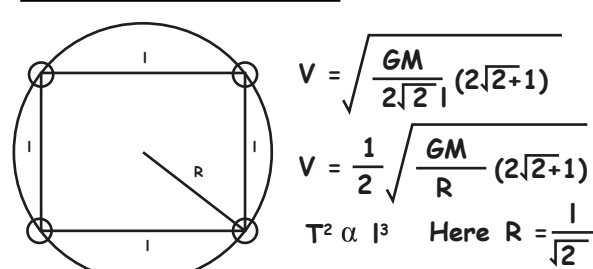
THREE MASSES(EQUAL) REVOLVING UNDER MUTUAL GRAVITATIONAL FORCE



$$v_1 = \sqrt{\frac{GM}{l}} = \sqrt{\frac{GM}{R\sqrt{3}}}$$

Time period $T^2 \propto l^3$

FOUR EQUAL MASSES UNDER MUTUAL GRAVITATIONAL FORCE



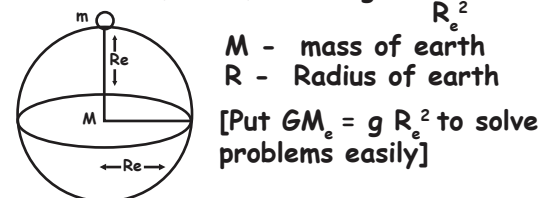
$$v = \sqrt{\frac{GM}{2\sqrt{2}l}} (2\sqrt{2}+1)$$

$$v = \frac{1}{2} \sqrt{\frac{GM}{R}} (2\sqrt{2}+1)$$

$T^2 \propto l^3$ Here $R = \frac{l}{\sqrt{2}}$

GRAVITY

Acceleration due to gravity
 On the surface of earth $g = \frac{GM_e}{R_e^2}$



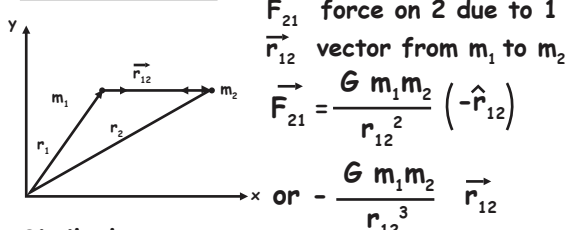
M - mass of earth
 R - Radius of earth
 [Put $GM_e = gR_e^2$ to solve problems easily]

g IN TERMS OF DENSITY OF EARTH

$$g = 4\pi G \rho R_e \quad g \propto \rho R_e$$

"If density is mentioned use the above equation"

VECTOR FORM



$\vec{F}_{21}$ force on 2 due to 1
 $\vec{r}_{12}$ vector from m_1 to m_2

$$\vec{F}_{21} = \frac{Gm_1m_2}{r_{12}^2} (-\hat{r}_{12})$$

or $-\frac{Gm_1m_2}{r_{12}^3} \vec{r}_{12}$

Similarly

$\vec{F}_{12}$ force on 1 due to 2

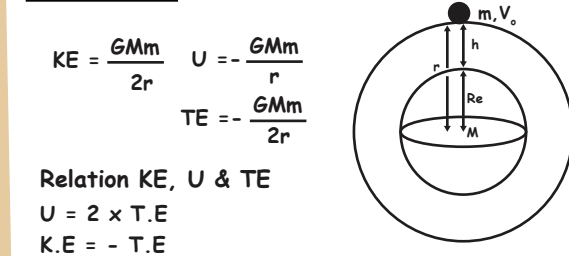
$$\vec{F}_{21} = \frac{Gm_1m_2}{r_{12}^2} (\hat{r}_{12}) \text{ or } \frac{Gm_1m_2}{r_{12}^3} \vec{r}_{12}$$

Clearly

Newtons third law $\vec{F}_{21} = -\vec{F}_{12}$

Gravitational force is a two body interaction. Force between two particles does not depend on the presence or absence of other particles. The principle of superposition is valid here. "Force on a particle due to a no. of particles is the resultant of forces due to individual particles."

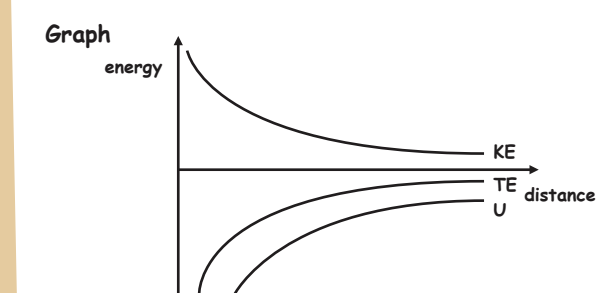
KE, PE OR TE FOR AN ORBITING SATELLITE



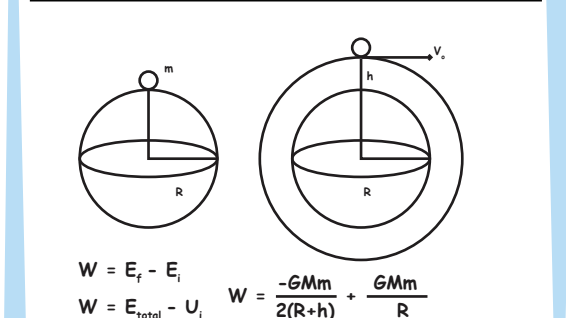
$$KE = \frac{GMm}{2r} \quad U = -\frac{GMm}{r}$$

$$TE = -\frac{GMm}{2r}$$

Relation KE, U & TE
 $U = 2 \times T.E$
 $K.E = -T.E$



WORK DONE IN MOVING OBJECT FROM SURFACE TO CIRCULAR ORBIT



$$W = E_f - E_i$$

$$W = E_{\text{total}} - U_i = \frac{-GMm}{2(R+h)} + \frac{GMm}{R}$$

Work done to move object to a height $h = R$ $W = \frac{mgR}{2}$

Work done to move object to a height $h = R/2$ $W = \frac{mgR}{3}$

GRAVITATION

Kinetic Theory of Gases

01

DEGREES OF FREEDOM

- For monoatomic gas, $f = 3$
- For diatomic gas,
 - (a) at room temperature, $f = 5$
 - (b) at high temperature, $f = 7$
- For triatomic gas,
 - (a) Linear $f = 5$
 - (b) Non-linear $f = 6$
- 1 Vibrational mode $f = 2$

Q1

Ideal gas is composed of polyatomic molecule that has 4 vibrational modes. Total degree of freedom is
a) 12 b) 14 c) 8 d) 6

02

SPECIFIC HEAT CAPACITY

- a) $C_p - C_v = R$ (MSH)
- b) $C_p - C_v = \frac{R}{M}$ (GSH) Mono- $\gamma = \frac{5}{3}$
- c) $C_v = \frac{R}{\gamma - 1} = \frac{f}{2} R$ Dia $\gamma = \frac{7}{5}$
- d) $C_p = \frac{\gamma R}{\gamma - 1} = 1 + \frac{f}{2}$ Tri $\gamma = \frac{4}{3}$
- e) $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

Q2

If C_p and C_v denoted the specific heats of unit mass of nitrogen at constant pressure and volume respectively, then

- a) $C_p - C_v = \frac{R}{28}$ b) $C_p - C_v = \frac{R}{14}$ c) $C_p - C_v = \frac{R}{7}$ d) $C_p - C_v = R$

03

MIXING OF GASES

$$C_{Vmix} = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}$$

$$C_{Pmix} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 + n_2}$$

$$\gamma_{mix} = \frac{C_{Pmix}}{C_{Vmix}}$$

Q3

Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. It's C_p/C_v value will be:

- a) 19/13 b) 67/45 c) 40/27 d) 23/15

04

LAW OF EQUIPARTITION OF ENERGY

- 1 Molecule per $f = \frac{1}{2} K T$
- Total for a molecule = $\frac{f}{2} K T$
- Monoatomic Molecule = $\frac{3}{2} K T$
- Total for a mole = $\frac{f}{2} R T$
- Monoatomic = $\frac{3}{2} R T$ (mole)
- Diatomic = $\frac{5}{2} R T$ (mole)
- Translatory Kinetic energy = $\frac{3}{2} R T$ (mole $f = 3$)

Q4

A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T . Neglecting all vibrational modes, the total internal energy of the system is
a) $4RT$ b) $15RT$ c) $9RT$ d) $11RT$

06

VELOCITY OF GAS

$$V_{mps} : V_{avg} : V_{rms} = 1 : 1.14 : 1.228$$

07

MEAN FREE PATH

Average distance travelled by molecules between two successive collision
 $\lambda_{mean} = \frac{1}{\sqrt{2} \pi d^2 n}$
 d = diameter of molecules.
 n = no. of molecules per unit volume

$$\lambda \propto \frac{1}{d^2}$$

$$\lambda \propto \frac{1}{r^2}$$

$$\lambda \propto \frac{T}{P}$$

05

FIRST LAW OF THERMODYNAMICS

$$\Delta Q_p = \Delta U + \Delta W$$

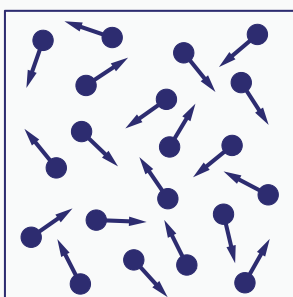
$$\Delta U = n C_v \Delta T$$

$$\Delta W = \int P dv$$

$$\frac{\Delta U}{\Delta Q_p} = \frac{1}{\gamma}$$

$$\frac{\Delta W}{\Delta Q_p} = 1 - \frac{1}{\gamma}$$

KINETIC THEORY OF GAS



Root Mean square speed:

Square root of mean of square of speed of different molecules,

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + \dots + V_n^2}{n}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{d}} = \sqrt{\frac{3k_B T}{m}}$$

Average Speed:

Arithmetic mean of speed of molecules of gas at given temperature.

$$V_{avg} = \frac{|V_1| + |V_2| + \dots + |V_n|}{n}$$

$$V_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8P}{\pi d}}$$

Most probable speed:

Speed possessed by maximum number of molecules of gas.

$$V_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{d}}$$

Q5

Consider a gas of triatomic molecules. The molecules are assumed to be triangular made up of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is:

- a) $\frac{5}{2} RT$ b) $\frac{3}{2} RT$ c) $\frac{9}{2} RT$ d) $3RT$

Q6

The rms speeds of the molecules of Hydrogen, Oxygen & Carbon dioxide at the same temperature are V_H , V_O and V_C respectively then:

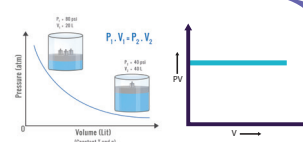
- a) $V_H > V_O > V_C$ b) $V_C > V_O > V_H$
- c) $V_H = V_O > V_C$ d) $V_H = V_O = V_C$

Q7

The mean free path of molecules of gas, (radius r) is inversely proportional to

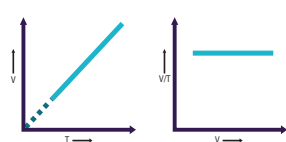
- a) r^3 b) r^2
- c) r d) $\sqrt{r}$

BOYLE'S LAW



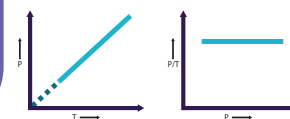
- $PV = \text{constant}$, if $T = \text{constant}$
- $P_1 V_1 = P_2 V_2$, When gas changes its state under constant temperature.

CHARLE'S LAW



- $V \propto T$; $\frac{V}{T} = \text{constant}$; $P = \text{constant}$.
- $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, When gas change its state under constant pressure.

GAY LUSSAC'S LAW



- $P \propto T$; $\frac{P}{T} = \text{constant}$; $V = \text{constant}$.
- $\frac{P_1}{T_1} = \frac{P_2}{T_2}$, When gas change its state under constant Volume.

PRESSURE OF GAS

$$PV = \frac{1}{3} mn V_{rms}^2 = \frac{1}{3} mn \bar{V}^2$$

Relation between pressure and Kinetic Energy.

$$E = \frac{3}{2} PV$$

IDEAL GAS LAW

$$PV = nRT$$

$$R = 8.314 \text{ JK}^{-1} \text{mol}^{-1}$$

$$d = \frac{PM}{RT}$$

Specific heat of Solid = $3R$
WATER = $9R$

INERTIA
A body cannot change its state of rest or uniform motion along a straight line. This property is called inertia. Inertia has no unit and no dimension.

INERTIA

- Inertia of Rest
- Inertia of Motion
- Inertia of Direction

- Inertia of Rest
Inability to change state of rest by itself.

- Inertia of Motion
Inability of a body to change its state of uniform motion by itself.

- Inertia of Direction
Inability of a body to change direction of motion by itself.

Newton's Second Law
 $F_{\text{net}} = \text{Rate of change of linear momentum.}$

Instantaneous
 $\vec{F} = \frac{d\vec{p}}{dt}$

Average
 $\vec{F}_{\text{av}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$

MOMENTUM
 $\vec{p} = m\vec{v}$
-It is a vector quantity having direction same as that of velocity
-Unit is kg m/s.

LIQUID JETS

Area = A, u , wall, $v=0$

$F_{\text{jet}} = \frac{P_f - P_i}{\Delta t} = \frac{m \cdot 0 - mu}{\Delta t} = \frac{-mu}{\Delta t} = -\rho A u^2$

$F_{\text{wall}} = \rho A u^2$ ($m = \rho A \Delta x$, $\frac{\Delta x}{\Delta t} = u$)

When liquid jet bounce back

$F_{\text{jet}} = \frac{P_f - P_i}{\Delta t} = \frac{-2mu}{\Delta t}$

$F_{\text{jet}} = -2\rho A u^2$

$F_{\text{wall}} = 2\rho A u^2$

When liquid jet strikes obliquely

$F_{\text{jet}} = -2\rho A u^2 \cos\theta$

$F_{\text{wall}} = 2\rho A u^2 \cos\theta$

Change in momentum = $-2mu \cos\theta$

NEWTON'S THIRD LAW

- To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.
- Forces in nature always occurs in pairs.
- A single isolated force is not possible.
- Counter force experienced by a body- reaction
- Action and reaction never act on the same body

* Force exerted on body A by body B (action)
* force exerted on body B by body A (reaction)

$\vec{F}_{\text{Action}} = -\vec{F}_{\text{Reaction}}$

Normal Reaction
- Occurs when two surfaces are in contact with each other
Always perpendicular to the surface and is self-adjusting.

Tension

Restoring force developed when a longitudinal force is applied on a body

Ideal Rope

- *Massless
- *Tension is same everywhere
- *Can support only elongation
- *Cannot support compressive force
- *On compression it becomes slack.
- *Tension always acts away from the object.

Rod

Elongation

Compressive

can support both elongation and compression

SINGLE BLOCK

Horizontal Force
Acceleration is along x-axis only
Along y-axis $a=0$,
 $N - Mg = Ma$ ($Ma=0$)
 $N = Mg$

Along x-axis
 $F - 0 = Ma$ $a = \frac{F}{M}$

Inclined Forces

$F \sin\theta$, N , Mg , $F \cos\theta$, a

- If, $F \sin\theta < Mg$ block remains in contact with ground
- If, $F \sin\theta = Mg$ block just leaves contact with ground
- If, $F \sin\theta > Mg$ the block leaves contact with ground and it begins to accelerate.

MOTION OF CONNECTED BODIES

Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$ $f = m_2 a$	$a = \frac{F}{m_1 + m_2}$ $f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$ $F - f = m_2 a$	$a = \frac{F}{m_1 + m_2}$ $f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$ $f_1 - f_2 = m_2 a$ $f_2 = m_3 a$	$a = \frac{F}{m_1 + m_2 + m_3}$ $f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$ $f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

MOTION OF BLOCKS CONNECTED BY MASSLESS STRING

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$ $F - T = m_2 a$	$a = \frac{F}{m_1 + m_2}$ $T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$ $T = m_2 a$	$a = \frac{F}{m_1 + m_2}$ $T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$ $T_2 - T_1 = m_2 a$ $F - T_2 = m_3 a$	$a = \frac{F}{m_1 + m_2 + m_3}$ $T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$ $T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

PULLEY

Ideal
 $M_1 > M_2$
 $a = \frac{F}{M} = \frac{M_1 - M_2}{M_1 + M_2} g$
 $T = \frac{2M_1 M_2}{M_1 + M_2} g$

INCLINED PLANE + PULLEY

		$m_1 a = T - m_1 g \sin\theta$	$a = \frac{(m_2 - m_1 \sin\theta)}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$
		$T - m_1 g \sin\theta = m_1 a$	$a = \frac{(m_2 \sin\theta - m_1 \sin\theta)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin\theta - T$	$T = \frac{m_1 m_2 (\sin\theta + \sin\theta)}{m_1 + m_2} g$
		$m_1 g \sin\theta - T = m_1 a$	$a = \frac{m_1 g \sin\theta}{m_1 + m_2}$

THICK ROPE

Tension will be different at different points.

Mass per unit length = $\frac{M}{L}$
Mass of x length of rope = $\frac{M}{L} x$

Note :
Mass of given length = $\frac{\text{total mass}}{\text{total length}} \times \text{given length}$

$\frac{\text{mass}}{\text{length}} = \text{constant}$

$T = \frac{m_2 F}{M}$

$\frac{M}{L} (L-x) = m_2$, $\frac{M}{L} x = m_1$

LIFT PROBLEMS

Apparent weight of body in a lift
Reading of weighing machine = reaction force exerted by weighing machine (N)

Apparent weight, $(W_{\text{apparent}}) = \text{Reaction force (R)}$

Case 1: Lift is at rest
 $R = mg$, $W_{\text{apparent}} = W_{\text{actual}} = mg$

Case 2: Lift moving up or down with constant velocity
 $R = mg$, $W_{\text{apparent}} = W_{\text{actual}} = mg$

Case 3: Accelerated upward at a rate of 'a'
 $R = m(g+a) = W_{\text{apparent}}$

$W_{\text{apparent}} > W_{\text{actual}} \rightarrow \text{Feels over weight}$

Accelerated upward at a rate of 'g'
 $R - mg = mg$, $R = 2mg$, $W_{\text{app}} = 2 \times W_{\text{act}}$

Case 4: Accelerated downward at a rate of 'a'
 $mg - R = ma$, $R = m(g-a) = W_{\text{app}}$, $W_{\text{app}} < W_{\text{act}}$

Accelerated downward at a rate of 'g' [Freefall]
 $mg - R = mg$, $R = mg - mg = 0$, $W_{\text{app}} = 0$

If $a > g$: body loses contact with the weighing machine and R becomes zero

Lift moves from ground floor to first floor

C $v=0$, $a=0$ $N=mg$ (true weight)

first floor

$N=m(g-a)$

v decrease $\uparrow$ a (retardation)

B

$N=m(g+a)$

Ground floor

$N=mg$ (true weight)

Graph

$N_1 = m(g+a_1)$, $N_2 = m(g-a_2)$

Acceleration, $a_1 = \tan\theta_1$
Retardation, $a_2 = \tan\theta_2$

FRAME OF REFERENCE & PSEUDO FORCE

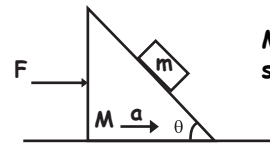
Frame of Reference

A frame in which and observer is situated and makes his observation

Inertial frame of reference	Non-Inertial frame of reference
At rest or moving with uniform velocity along straight line. i.e unaccelerated	Accelerated frame of reference.
Holds Newton's law of motion	Newton's law of motion not applicable.

LAWS OF MOTION

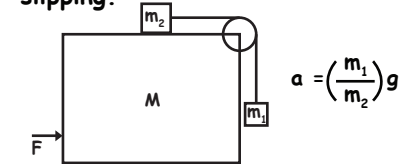
RELATIVE SLIPPING



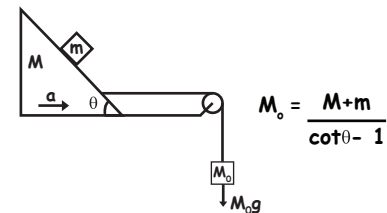
Minimum force required to push the inclined plane such that "m" does not slip with respect to "M"

$$F = (m+M) g \tan \theta, \quad a = g \tan \theta$$

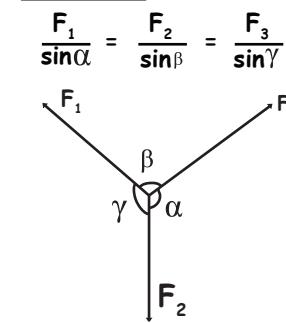
Minimum acceleration "M" must be pushed such that there is no relative slipping.



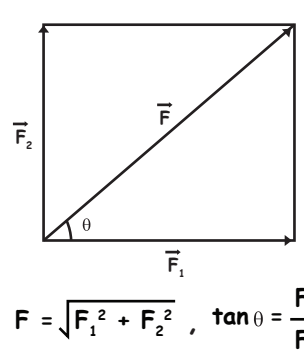
Minimum mass such that there is no relative slipping



EQUILIBRIUM & LAMI'S THEOREM

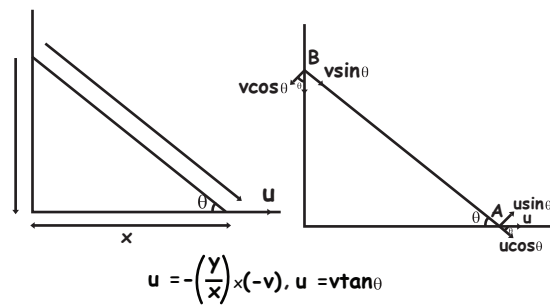


PARALLELOGRAM LAW



$$\vec{F} = \vec{F}_1 + \vec{F}_2, \quad F = \sqrt{F_1^2 + F_2^2}, \quad \tan \theta = \frac{F_2}{F_1}$$

ROD SLIDING ON A WALL



velocity of B towards A = $v \sin \theta$
velocity of A away from B = $u \cos \theta$

ANGLE OF FRICTION

Angle made by resultant of normal (N) & frictional force (f_s) with normal

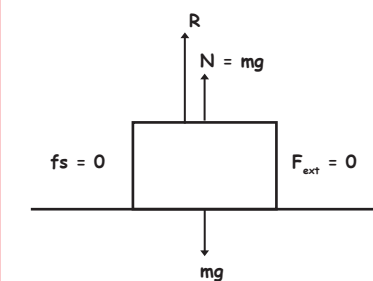
$$R = \sqrt{N^2 + f_s^2} \quad R \text{ is resultant}$$

$$\tan \theta = \frac{f_s}{N}$$

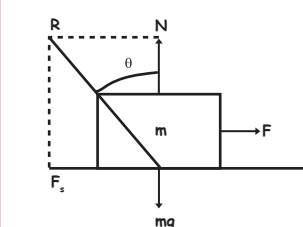
$$\text{When } f_s = f_{s \max} \quad R = N \sqrt{1 + \mu_s^2}$$

$$\tan \theta = \mu_s \quad \mu_s \text{ is Coefficient of friction}$$

$$\text{i) } F_{\text{ext}} = 0$$



$$\text{ii) } 0 < F_{\text{ext}} < f_{s \max}$$

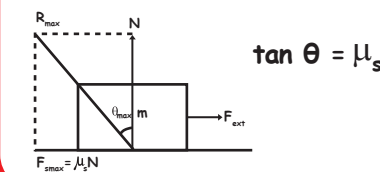


$$0 \leq \tan \theta \leq \mu_s$$

$$N < R < N \sqrt{1 + \mu_s^2}$$

$$\text{iii) } f_{\text{ext}} = f_{s \max}$$

$$R_{\max} = N \sqrt{1 + \mu_s^2}$$

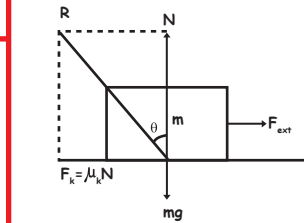


$$\tan \theta = \mu_s$$

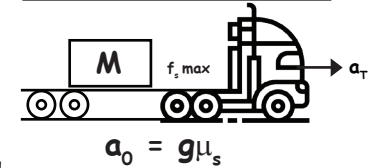
$$\text{iv) } f_{\text{ext}} > f_{s \max}$$

$$R = N \sqrt{1 + \mu_k^2}$$

θ = Angle of kinetic friction



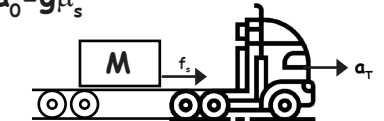
HORIZONTAL TRUCK BOX



$$a_0 = g \mu_s$$

Case-1

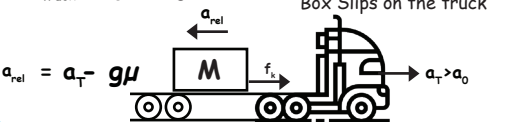
$$a_{\text{truck}} < a_0 = g \mu_s$$



$$f_s = M a_t \quad \text{Box does not slip on the truck}$$

Case-2

$$a_{\text{truck}} > a_0 = g \mu_s$$



$$a_{\text{rel}} = a_t - g \mu$$

ANGLE OF REPOSE

depends only on μ_s and is independent of mass.

$$\tan \theta_0 = \mu_s$$

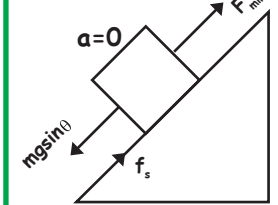
Variation of R

As angle of inclined plane increases, R remains constant and when sliding starts R starts decreasing.

Variation of angle of friction.

As angle of inclined plane increases, angle of friction will also increases and as sliding starts its value becomes constant and $\tan \theta = \mu_k$

Minimum & Maximum force (applied parallel to inclined plane)



$$F_{\min} = mg (\sin \theta - \mu_s \cos \theta)$$

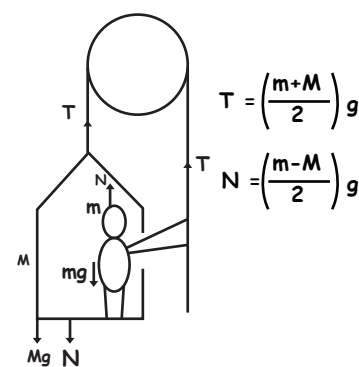
$F_{\min}$ is the minimum force required to make the block stationary if the angle of inclined plane is greater than angle of repose

$F_{\max}$ is the force required to move the block along the inclined plane if the angle of inclined plane is greater than angle of repose

$$F_{\max} = mg (\sin \theta + \mu_s \cos \theta)$$

$$mg (\sin \theta - \mu_s \cos \theta) \leq F \leq mg (\sin \theta + \mu_s \cos \theta)$$

MAN-CAGE PROBLEM



IMPULSE

Large force acting for short period of time, As a result of impulse there will be a sudden change in momentum

$$F_{\text{ip}} = \frac{dp}{dt}$$

$$P_f - P_i = \int_0^t F_{\text{ip}} dt = \text{area under } F-t \text{ graph}$$

$$I = \text{Impulse} = P_f - P_i = \int_0^t F_{\text{ip}} dt = \text{area of } F-t \text{ graph}$$

Case-1

Impulse

$$I = P_f - P_i$$

$$= m V_2 - (-m V_1)$$

$$I = m (V_1 + V_2)$$

Average impulsive force

$$F_{\text{avg}} = \frac{\text{impulse}}{t_0} = \frac{\Delta P}{t_0}$$

Case-2

Impulse

$$m(v+v) = J$$

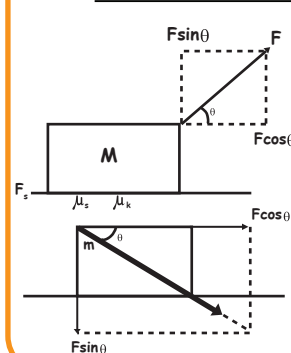
$$= m(2v)$$

$$J = 2mv$$

Average impulsive force

$$F_{\text{avg}} = \frac{\Delta P}{t_0} = \frac{2mv}{\Delta t}$$

PULLING FORCE & PUSHING FORCE



$$F \cos \theta \geq \mu_s (mg - F \sin \theta)$$

$$F \geq \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta}$$

$$F_{\text{pulling}} = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta}$$

$$F \cos \theta \geq \mu_s (mg + F \sin \theta)$$

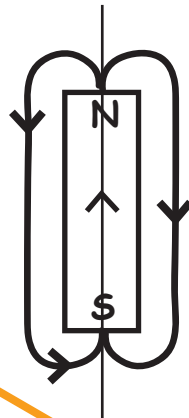
$$F \geq \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta}$$

$$F_{\text{pushing}} = \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta}$$

LAWS OF MOTION

MAGNETISM AND MATTER

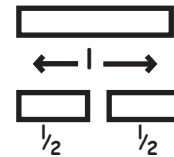
THE MAGNETIC FIELD LINES



1. The magnetic field lines of a magnet form continuous closed loops
2. The tangent to the field lines at a given point represents the direction of the net magnetic field B at that point
3. The larger no. of field lines $\rightarrow$ stronger $\vec{B}$
4. Do not intersect

CUTTING OF BAR MAGNET

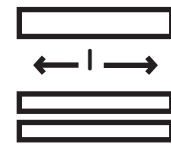
LENGTHWISE / TRANSVERSE



Pole strength $\rightarrow$ same
length $\rightarrow$ reduce to half

$$M_{\text{new}} = \frac{ml}{2}$$

HORIZONTAL



Pole strength $\rightarrow$ reduce to half
length $\rightarrow$ same

$$M_{\text{new}} = \frac{ml}{2}$$

FACTS

- A) Magnetic monopoles does not exist
B) A solenoid and bar magnet produce similar magnetic fields

POTENTIAL AT ANY GENERAL POINT

$$V = \frac{1}{4\pi\epsilon_0} \frac{P \cos\theta}{r^2}$$

$$V_m = \frac{\mu_0}{4\pi} \frac{M \cos\theta}{r^2}$$

TORQUE

$$1) F_{\text{net}} = 0$$

$$2) \vec{\tau} = \vec{p} \times \vec{E} = pE \sin\theta$$

$$1) F_{\text{net}} = 0$$

$$2) \vec{\tau} = \vec{M} \times \vec{B} = MB \sin\theta$$

WORK DONE IN ROTATING A DIPOLE

$$1. W = PE (\cos\theta_1 - \cos\theta_2) \quad 1. W_B = MB (\cos\theta_1 - \cos\theta_2)$$

Maximum work done is from

$$\theta_1 = 0^\circ \text{ to } \theta_2 = 180^\circ$$

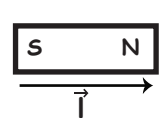
POTENTIAL ENERGY

$$U = -\vec{p} \cdot \vec{E}$$

$$U_B = -\vec{M} \cdot \vec{B}$$

$\theta_1 = 0^\circ$ Stable position; $\theta = 180^\circ$ Unstable position

MAGNETIC DIPOLE MOMENT ($\vec{M}$)



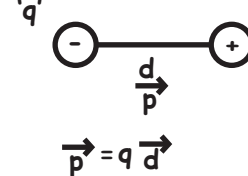
$\vec{M} = m\vec{l}$
 $m \rightarrow$ pole strength
 $l \rightarrow$ distance b/w poles

Direction from southpole to N pole

Unit of $M \rightarrow \text{Am}^2$
Unit of $m \rightarrow \text{Am}$

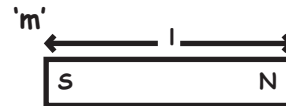
THE ELECTROSTATIC ANALOG (Help from electrostatics to magnetism)

SHORT ELECTRIC DIPOLE



$$\vec{p} = q\vec{d}$$

SHORT MAGNETIC DIPOLE



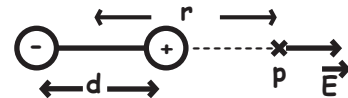
$$\vec{M} = m\vec{l}$$

COULOMB'S LAW

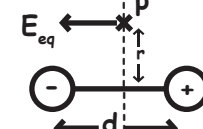
$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\vec{F}_m = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$$

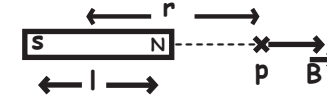
AXIAL & EQUATORIAL LINE OF DIPOLE



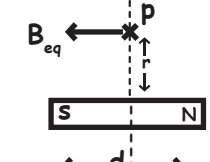
$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}$$



$$E_{\text{eq}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$



$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$$



$$B_{\text{eq}} = \frac{\mu_0}{4\pi} \frac{M}{r^3}$$

$$\vec{B}_{\text{axial}} = -2\vec{B}_{\text{eq}}$$

MAGNETISM AND GAUSS'S LAW

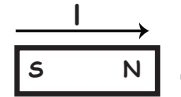
"The net magnetic flux through any closed surface is zero"

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

NOTE:

"The simplest magnetic element is a magnetic dipole or a current loop." Magnetic monopoles do not exist.

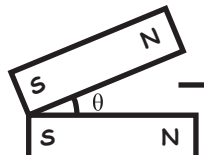
BAR MAGNET TO DIFFERENT SHAPES



$$M_{\text{new}} = \frac{M \sin(\frac{\theta}{2})}{(\frac{\theta}{2})}$$

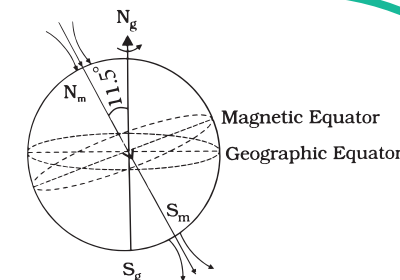
$$l' = \frac{l \sin(\frac{\theta}{2})}{(\frac{\theta}{2})}$$

RESULTANT DIPOLE MOMENT



$$M_{\text{net}} = \sqrt{M_1^2 + M_2^2 + 2M_1 M_2 \cos\theta}$$

THE EARTH'S MAGNETISM

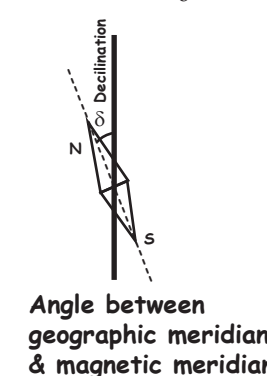


MAGNETIC ELEMENTS OF EARTH

Magnetic declination $-\delta$

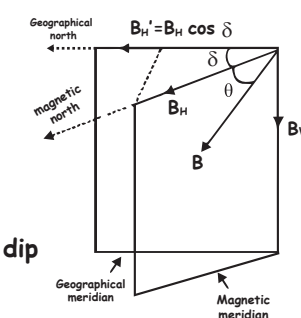
Inclination or Dip $-\theta$

Horizontal component $-B_H$



Angle between geographic meridian & magnetic meridian

$$\tan\theta = \frac{B_V}{B_H}$$



COMPASS NEEDLE AND DIP NEEDLE

1. A compass needle at the North pole can point along any direction.
2. A dip needle at the north pole points down and at South pole points straight up.

TIME PERIOD

of a magnetic dipole in uniform magnetic field

$$T = 2\pi \sqrt{\frac{I}{MB}}$$

I - Moment of Inertia
 M - Dipole moment magnetic
 B - Magnetic field

$$\text{Frequency } \nu = \frac{1}{2\pi} \sqrt{\frac{MB}{I}} \quad \text{To find } B, \quad B = \frac{4\pi^2 I}{MT^2}$$

APPARENT ANGLE OF DIP

Inclination of magnetic needle in plane other than magnetic meridian

$$\tan\delta' = \frac{\tan\delta}{\cos\theta}$$

δ' Apparent angle of dip
 δ true angle of dip

θ Angle between MM and the plane other than MM

RELATION BETWEEN TWO FALSE ANGLE OF DIP IN MUTUALLY PERPENDICULAR PLANE AND TRUE ANGLE OF DIP

$$\cot^2 \delta_1 + \cot^2 \delta_2 = \cot^2 \delta$$

FACTS

1. Declination is greater at poles and smaller near equator
2. Angle of dip is maximum at poles and minimum at equator

MAGNETIC PROPERTIES

1) Magnetic Permeability

Absolute Permeability of air or free space $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tesla metre}}{\text{Ampere}} \left[\frac{\text{Tm}}{\text{A}} \right]$

Relative Permeability of medium $\mu_r = \frac{\mu_{\text{medium}}}{\mu_0}$

2) Intensity of magnetizing field (H)

$\vec{H} = \frac{\vec{B}_{\text{ext}}}{\mu_0}$ vector quantity

SI unit $\rightarrow \frac{\text{A}}{\text{m}}$ CGS unit $\rightarrow$ Oersted

3) Magnetisation (M)

$\vec{M} = \frac{\vec{M}_{\text{net}}}{V} \rightarrow \left[\frac{\text{Induced dipole moment}}{\text{volume}} \right]$ also, $\vec{M} = \frac{\vec{B}_{\text{ind}}}{\mu_0}$

vector quantity

SI unit $\rightarrow \frac{\text{A}}{\text{m}}$ $[M] \rightarrow \text{L}^{-1}\text{A}$

4) Magnetic Susceptibility (χ_m)

$\chi_m = \frac{M}{H}$ Also $\chi = \frac{B_{\text{ind}}}{B_{\text{ext}}}$ scalar quantity with no unit no dimension

5) Relation between relative permeability and susceptibility

$\mu_r = (1 + \chi_m)$ Also $\mu = \mu_0 \mu_r = \mu_0 (1 + \chi_m)$

6) Relation between B , M and H

$B = \mu H$

MAGNETIC MATERIALS

1. Diamagnetic

- a. Weakly repelled by a magnet
- b. Eg: Cu, Ag, Au, NaCl, H_2O etc.
- c. Superconductors - Perfect conductivity perfect diamagnetism $\chi = -1, \mu_r = 0$

d. Perfect diamagnetism in superconductors
MEISSNER EFFECT

e. Important $-1 \leq \chi < 0$
 $0 \leq \mu_r < 1$
 $\mu < \mu_0$

- f. Individual atoms do not possess permanent magnet dipole moment
- g. No effect of temperature on magnetisation

2. Paramagnetic substances

- a. Weakly attracted by a magnet
- b. Eg : Al, Mn, Pt, Na, CuCl_2 , O_2 , Crown glass
- c. Individual atom possesses permanent dipole moment
- d. Curie's law
Magnetisation of a paramagnetic material is inversely proportional to the absolute temperature

$$\left. \begin{aligned} M &= C \frac{B_0}{T} \\ \chi &= C \frac{\mu_0}{T} \end{aligned} \right\} \text{Curie's law}$$

e. Important

$$\left. \begin{aligned} 0 < \chi < \epsilon \\ 1 < \mu_r < 1 + \epsilon \quad (\epsilon \rightarrow \text{Small positive number}) \\ \mu > \mu_0 \end{aligned} \right\}$$

f. Graph



3. Ferromagnetic substances

- a. Strongly attracted by a magnet
- b. Eg : Fe, Co, Ni, Cd, Fe_3O_4
- c. Individual atoms possess permanent magnetic moment and magnetic moments of neighbouring atoms tend to align due to a force called exchange coupling
- d. Due to exchange coupling, atoms form domains inside in which magnetic moment are aligned in the same direction

e. Important $\chi \gg 1$
 $\mu_r \gg 1$
 $\mu \gg \mu_0$

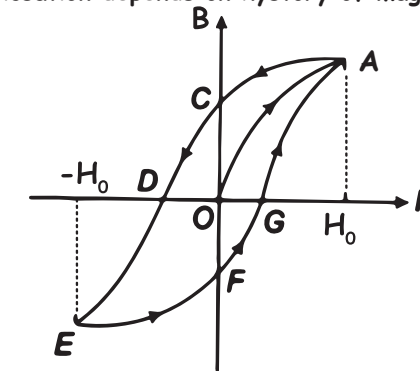
f. At high temperature, a ferromagnet becomes paramagnetic

Curie's temperature

$$\chi = \frac{C}{T - T_c} \quad (T > T_c)$$

HYSTERESIS CURVE / B-H CURVE

Magnetisation depends on history of magnetisation



Important terms

Retentivity - OC - Residual magnetism

Coercivity - Demagnetising process

- 1. High coercivity - Hard substance - Steel
- 2. Low coercivity - Soft substance - Soft iron

Important result

B-H curve signifies the energy loss/heat loss and is proportional to the area of the loop.

Area of hysteresis loop $\rightarrow$ Smaller for soft iron
 $\rightarrow$ Higher for steel

Permanent magnets

should have

- 1. High retentivity
- 2. High coercivity
- 3. High permeability

Steel is used for making permanent magnets

Steel $\longleftrightarrow$ soft iron
Smaller retentivity Higher retentivity than steel
High coercivity Smaller coercivity than steel

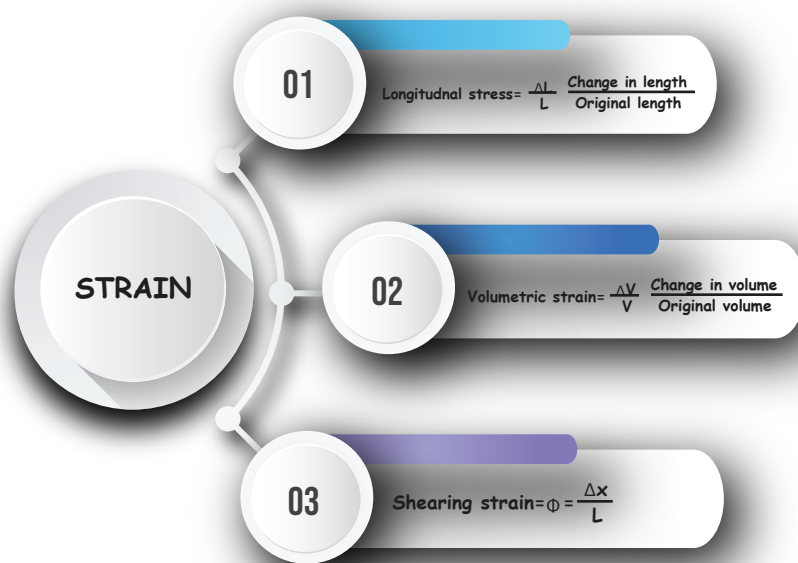
ELECTROMAGNETS

Materials should have

high permeability Soft iron is used
low retentivity

Used in electric bells, Loudspeakers, telephone diaphragms, heavy cranes to lift machinery

MECHANICS OF SOLIDS



LONGITUDINAL STRESS

Tensile stress causes increase in length

Tensile stress = $\frac{F}{A}$ unit N/m^2

$T.S = \frac{Fm}{(M+m)/A}$

Tensile stress $T.S = \frac{m(g+a)}{A}$

$T.S_1 = \frac{T_1}{A} = \frac{6mg}{A}$

$T.S_2 = \frac{T_2}{A} = \frac{5mg}{A}$

$T.S_3 = \frac{T_3}{A} = \frac{3mg}{A}$

VOLUME STRESS

- Same as pressure
- Causes change in volume

volume stress = $\frac{F}{A} = \text{pressure}$

$F = \text{normal force/thrust}$

SHEARING STRESS

- Causes change in shape

shearing stress = $\frac{F_t}{A}$

$F_t = \text{tangential force}$

HOKE'S LAW

$E = \frac{\text{Stress}}{\text{Strain}}$

(E = modulus of elasticity)

$\tan \theta = \text{modulus of elasticity}$

- For rigid body $E = \text{infinity}$
- Steel is more elastic than rubber
- Depends on :-
(a) Nature of metal
(b) Temperature
- Independent of dimensions

Young's modulus (E)

Bulk Modulus (K)

Modulus of rigidity (G)

STRESS STRAIN CURVE

OA - Hooke's law obeyed

A - Proportional limit

AB - Not proportional but body regains its original shape and size when load is removed

B - Yield point

B to D - Body doesn't regain its original dimension. Beyond B is plastic region.

D - Ultimate stress point beyond D - added strain is produced even for a small applied force.

E - Fracture occur

Plastic region is large for ductile materials and smaller for brittle materials

Plastic region is large for ductile materials and smaller for brittle materials

BREAKING STRESS

$B.F = \text{breaking stress} \times \text{area}, B.F \propto A$

$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad (B.S)_1 = (B.S)_2$

BREAKING OF WIRE UNDER IT'S OWN WEIGHT

$B.S \times A \times T = mg$

$B.S \times A = V \times \rho \times g = A \times L_{\text{max}} \times \rho \times g$

$L_{\text{max}} = \frac{B.S}{\rho g}$

RATIO OF DENSITY OF BODY TO THAT OF LIQUID IN WHICH BODY IS IMMERSSED

$W = K l_a$

$W(1 - \frac{\rho}{\sigma}) = K l_b$

$(1 - \frac{\rho}{\sigma}) = \frac{l_b}{l_a}$

$\frac{\rho}{\sigma} = \frac{l_a - l_b}{l_a}$

$\frac{\sigma}{\rho} = \frac{l_a}{l_a - l_b}$

YOUNG'S MODULUS

$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{FL}{A\Delta L}$

Comparing with a spring of force constant K

$Y = \frac{FL}{Ax} \quad F = \frac{AYx}{L} = kx \quad k = \frac{AY}{L}$

ELASTIC POTENTIAL ENERGY

$E.P.E = \frac{1}{2} kx^2 = \frac{F^2}{2k} = \frac{1}{2} Fx$

$= \frac{1}{2} \left(\frac{YA}{L} \right) x^2$

$= \frac{1}{2} \times \frac{\text{Stress}}{\text{Strain}} \times \text{volume} \times \text{strain}^2$

$= \frac{1}{2} \times \text{Stress} \times \text{volume} \times \text{strain}$

INCREASE IN LENGTH DUE TO IT'S OWN WEIGHT

$F = kx$

$Mg = \frac{YA}{L} x$

$x = \frac{mgL}{2YA}$

$P.E = \frac{F^2}{2k}$

$= \frac{M^2 g^2}{2 \frac{YA}{L}} = \frac{1}{4} \frac{M^2 g^2 L}{YA}$

RATIO OF EXTENSION

$\frac{x_1}{x_2} = \frac{\frac{F_1}{k_1}}{\frac{F_2}{k_2}} = \frac{\frac{3mg \times L_1}{Y_1 \times A_1}}{\frac{2mg \times L_2}{Y_2 \times A_2}} = \frac{3l}{2yd^2}$

where $l = \frac{L_1}{L_2} \quad y = \frac{Y_1}{Y_2} \quad d = \frac{D_1}{D_2}$

BULK MODULUS

Bulk modulus, $B = \frac{\Delta P}{-\frac{\Delta V}{V}}$

$K = \frac{1}{B} = \text{compressibility}$

$B_{\text{isothermal}} = P \quad B_{\text{adiabatic}} = \gamma P$

MODULUS OF RIGIDITY

$\eta = \frac{F}{A\phi} = \frac{Fl}{Ax}$

POISSON'S RATIO

$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

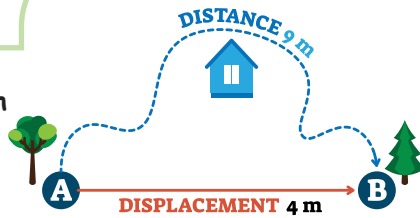
$\sigma = \frac{(-dr/r)}{(\frac{\Delta L}{L})} = \frac{-\frac{\Delta r}{r}}{\frac{\Delta L}{L}} = \frac{-\frac{\Delta D}{D}}{\frac{\Delta L}{L}}$

Theoretical value :- $-1 < \sigma < 0.5$

Practical value :- $0 < \sigma < 0.5$

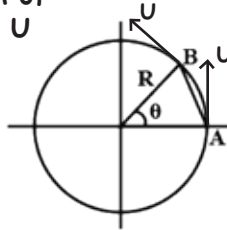
MOTION ALONG A STRAIGHT LINE

- Distance = Length of actual path
- Displacement = Length of shortest path
- Distance $\geq$ |displacement|



A particle moves from A to B in a circular path of radius R covering an angle θ with uniform speed U

- Distance = $R\theta$
- Displacement = $2R\sin\left(\frac{\theta}{2}\right)$
- Ratio of Displacement to Distance = $\frac{2\sin\left(\frac{\theta}{2}\right)}{\theta}$
- Time $t = \frac{R\theta}{U}$
- Average Velocity = $\frac{2U\sin\left(\frac{\theta}{2}\right)}{\theta}$
- Average Acceleration = $\frac{2U^2\sin\left(\frac{\theta}{2}\right)}{R\theta}$



For uniform motion

Displacement = velocity $\times$ time
Average speed = |average velocity| = |instantaneous velocity|

Time average speed

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

If $t_1 = t_2 = t_3 = \dots = t_n$
then $v_{av} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n} = \frac{v_1 + v_2}{2}$ (Arithmetic mean of speeds)

Distance average speed

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots + \frac{s_n}{v_n}}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots + \frac{s_n}{v_n}}$$

If $s_1 = s_2 = s_3 = \dots = s_n$
then $v_{av} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \dots + \frac{1}{v_n}} = \frac{2v_1 v_2}{v_1 + v_2}$ (Harmonic mean of speeds)

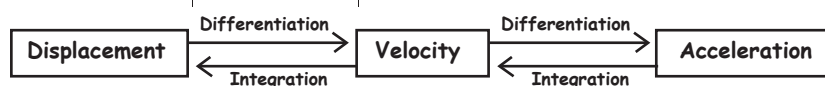
Instantaneous Velocity $v = \frac{dx}{dt}$ $\Delta x = \int v dt$

Instantaneous Acceleration $a = \frac{dv}{dt}$ $\Delta v = \int a dt$

Case 1: V or $x = f(t)$
then $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Case 2: $V = f(x)$
then $a = V \frac{dV}{dx}$

Case 3: $t = f(x)$
then $a = -(\text{double diff. of } t \text{ w.r. to } x) \times V^3$



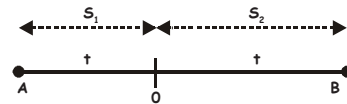
Motion with constant acceleration: Equations of motion

(i) $v = u + at$
(ii) $s = ut + \frac{1}{2} at^2$

- A Person travels from A to B covers unequal distances in equal interval of time with constant acceleration a then

initial velocity $U = \frac{3s_1 - s_2}{2t}$

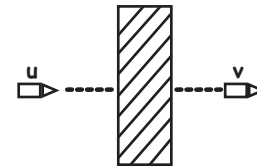
Acceleration $a = \frac{s_2 - s_1}{t^2}$



(iii) $v^2 = u^2 + 2as$

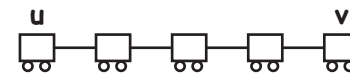
- The number of planks required to stop the bullet

$$N = \frac{u^2}{u^2 - v^2}$$

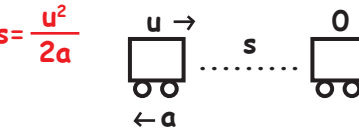


- The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. The velocity with which the middle point of the train passes the same point is

$$v_{\text{Mid}} = \sqrt{\frac{u^2 + v^2}{2}}$$



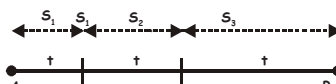
- Calculation of stopping distance $s = \frac{u^2}{2a}$



(iv) $s_n = u + \frac{a}{2}(2n-1)$

- Ratio of distance travelled in equal interval of time in a uniformly accelerated motion from rest

$$s_1 : s_2 : s_3 = 1 : 3 : 5$$



- for uniform accelerated motion $v_{avg} = \frac{u+v}{2}$

$$v_{avg} = \frac{u+v}{2}$$

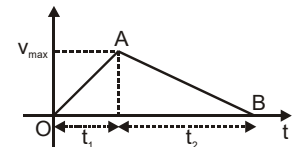
Important point about graphical analysis of motion

- Instantaneous velocity is the slope of position time curve $\left[v = \frac{dx}{dt} \right]$
- $v-t$ curve area gives displacement $\left[\Delta x = \int v dt \right]$
- Slope of velocity-time curve = instantaneous acceleration $\left[a = \frac{dv}{dt} \right]$
- $a-t$ curve area gives change in velocity $\left[\Delta v = \int a dt \right]$

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β , to come to rest. If the total time elapsed is t , the maximum velocity attained

$$V_{\text{max}} = \frac{\alpha\beta}{\alpha+\beta} t$$

$$\text{Total Distance} = \frac{1}{2} \left(\frac{\alpha\beta}{\alpha+\beta} \right) t^2$$



MOTION UNDER GRAVITY

Sign Convection

(i) initial velocity

- +ve = upward motion
- ve = downward motion

(ii) Acceleration

Always -ve

(iii) Displacement

- +ve = final position is above initial position
- ve = final position is below initial position
- Zero = final position & initial position are at same level

- Object is dropped from top of a tower

(i) Ratio of displacement in equal interval of time $s_1 : s_2 : s_3 : \dots = 1 : 3 : 5 : \dots$

(ii) Ratio of time of covering equal distance

$$t_1 : (t_2 - t_1) : (t_3 - t_2) : \dots : (t_n - t_{n-1}) = 1 : (\sqrt{2} - \sqrt{1}) : (\sqrt{3} - \sqrt{2}) : \dots : (\sqrt{n} - \sqrt{n-1})$$

(iii) Ratio of distance covered at the end of time $t : 2t : 3t : \dots = 1^2 : 2^2 : 3^2 : \dots$

- If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then

(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent $\frac{u}{g}$

(iii) Total time of flight = $\frac{2u}{g}$

(iv) Velocity of fall at the point of projection = u (downwards)

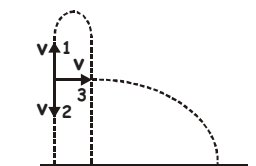
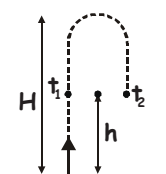
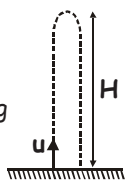
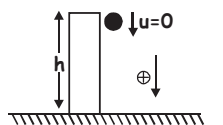
- At any point on its path the body will have same speed for upward journey and downward journey. If a body thrown upwards crosses a point in time t_1 & t_2 respectively then

$$\text{height of point } h = \frac{1}{2} g t_1 t_2 \quad \text{Maximum height } H = \frac{1}{8} g (t_1 + t_2)^2$$

$$t_1 + t_2 = \frac{2u}{g}$$

- A body is thrown upward, downward & horizontally with same speed takes time t_1 , t_2 & t_3 respectively to reach the ground then

$$t_3 = \sqrt{t_1 t_2} \quad \text{height from where the particle was throw is } h = \frac{1}{2} g t_1 t_2$$

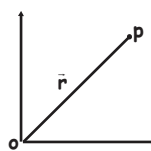


MOTION IN A PLANE

Magnitude Of Vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

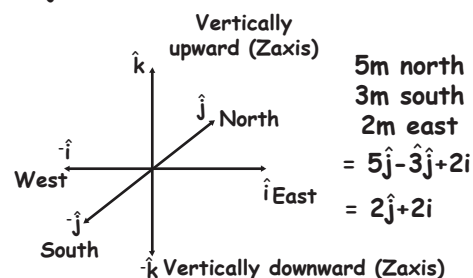
$$|\vec{r}| \text{ or } r = \sqrt{x^2 + y^2 + z^2}$$

$$\text{unit vector} = \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$

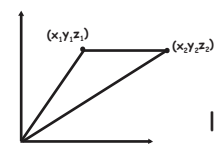
$$\vec{r} = \hat{i} - \hat{j} + \hat{k}$$

$$|\vec{r}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$



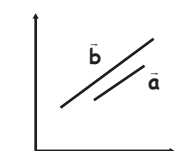
Displacement Vector



$$\Delta \vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\Delta \vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Parallel Vectors



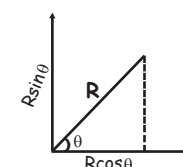
$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

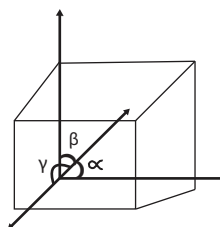
$$\vec{a} = m\vec{b}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = m$$

Components of Vector



$$\vec{R} = R \cos \theta \hat{i} + R \sin \theta \hat{j}$$



$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

Addition Of Vectors

$$\vec{R} = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\vec{R}_{\max} = \vec{A} + \vec{B}$$

$$\vec{R}_{\min} = \vec{A} - \vec{B}$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

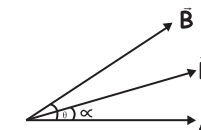
Vector product

$$\vec{C} = \vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}[A_y B_z - B_y A_z] - \hat{j}[A_x B_z - A_z B_x] + \hat{k}[A_x B_y - A_y B_x]$$

Dot product

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$



PROJECTILE MOTION

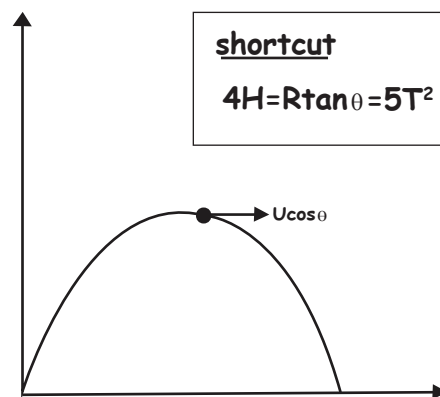
Projectile motion

$$\text{Horizontal component} = U \cos \theta$$

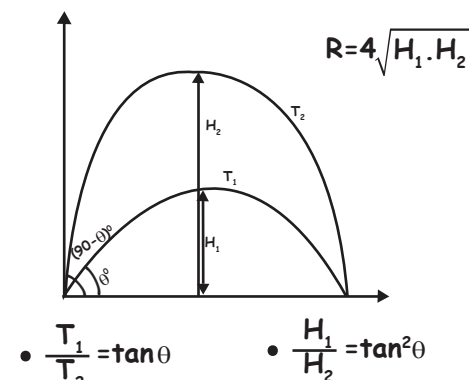
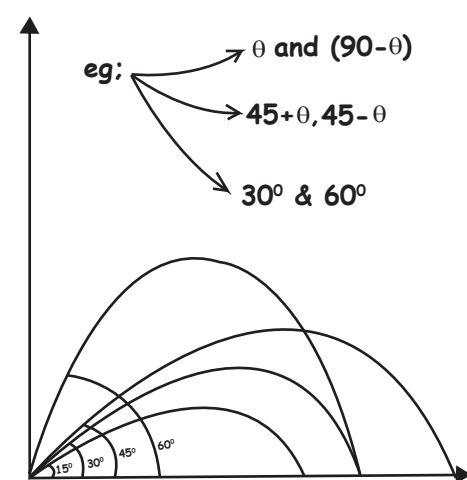
$$\text{Vertical component} = U \sin \theta$$

$$H = \frac{U^2 \sin^2 \theta}{2g} = \frac{(U \sin \theta)^2}{2g} = \frac{U_y^2}{2g}$$

$$R = \frac{U^2 \sin 2\theta}{g} = \frac{2U \sin \theta U \cos \theta}{g} = \frac{2U_x U_y}{g}$$



Same range for θ and $(90 - \theta)$



$$\frac{T_1}{T_2} = \tan \theta$$

$$\frac{H_1}{H_2} = \tan^2 \theta$$

$$T_1 \times T_2 = \frac{2R}{g}$$

$$H_1 \times H_2 = \frac{R^2}{16}$$

$$T_1^2 + T_2^2 = \frac{4R_{\max}}{g}$$

$$H_1 + H_2 = \frac{R_{\max}}{2}$$

Maximum range

$$\text{For } \theta = 45^\circ$$

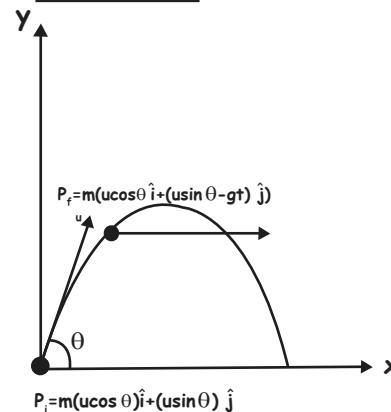
$$R_{\max} = \frac{U^2}{g}$$

$$\text{From the relation, } 4H = R \tan \theta = 5T^2$$

$$4H = R \tan 45^\circ \quad H = \frac{R_{\max}}{4} = \frac{U^2}{4g} \quad R_{\max} = \frac{U^2}{g}$$

$$\text{KE at maximum height} = K \cos^2 \theta$$

Momentum

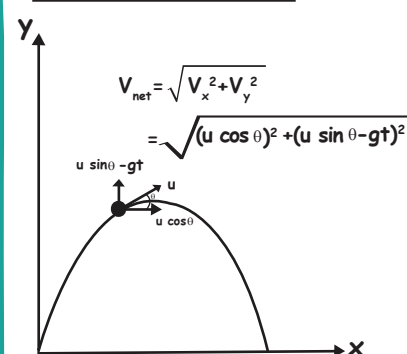


$$\Delta p = -mg \hat{j}$$

$$= -mg \times \frac{u \sin \theta}{g} \hat{j}$$

$$= -mu \sin \theta \hat{j}$$

Equation of Velocity



Net displacement

$$x = U \cos \theta t$$

$$y = U \sin \theta t - \frac{1}{2} g t^2$$

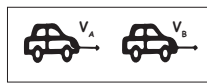
$$r = \sqrt{x^2 + y^2}$$

$$\beta = \tan^{-1}(y/x)$$

RELATIVE MOTION

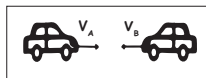
Relative Motion

$$1) \text{ Velocity of A with respect to B } V_{AB} = V_A - V_B$$



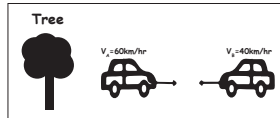
$$2) V_{A/B} = V_A - V_B$$

$$= V_A - (-V_B) = V_A + V_B$$



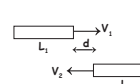
$$3) V_{A/\text{Tree}} = V_A - V_{\text{Tree}} = 60 - 0 = 60$$

$$V_{B/\text{Tree}} = V_B - V_{\text{Tree}} = -40$$

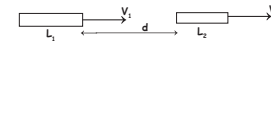


Relative Motion in one dimension overtaking & chasing

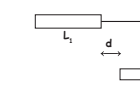
$$1) t = \frac{d + L_1 + L_2}{V_1 + V_2}$$



$$3) d + L_1 + L_2 = (u_1 - u_2)t + \frac{1}{2} (a_1 - a_2)t^2$$



$$2) t = \frac{d + L_1 + L_2}{V_1 - V_2}$$



Stopping distance

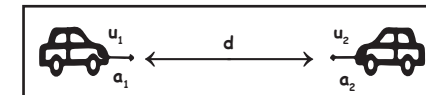
$$u_{\text{rel}} = u_1 - u_2$$

$$0 = u^2 - 2as$$

$$0 = u_{\text{rel}}^2 - 2a_{\text{rel}}s$$

$$a_{\text{rel}} = a_1 - a_2$$

$$S = \frac{u_{\text{rel}}^2}{2a_{\text{rel}}}$$



RELATIVE MOTION

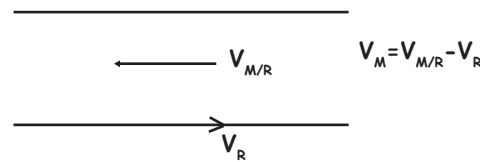
Man-river problem

1) $\vec{V}_{MR}$ or $\vec{V}_{M/Still\ water}$ = velocity due to effort of man [V]/ velocity in still water.

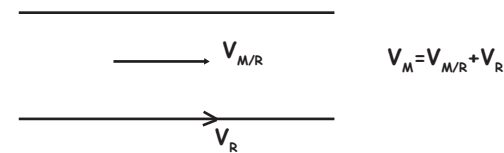
2) V_R = velocity of River

3) $\vec{V}_m$ = Resultant velocity of man with respect to ground

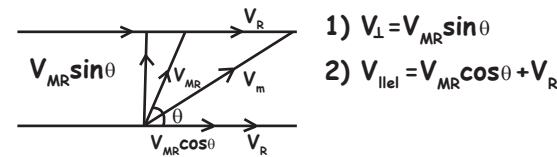
1) Upstream



2) Down stream



Swimming across the river



$$t_{cross} = \frac{d}{V_{MR} \cos \theta} = \frac{d}{V_{\perp}}$$

$$X_{drift} = (V_{MR} \sin \theta + V_R) \times t$$

$$= V_{MR} \cos \theta \times t + V_R t$$

Due to effort of man Additional

Shortest time	Shortest Path
• $t = \frac{d}{V_{MR} \sin \theta}$ • $t_{min} = \frac{d}{V_{MR}}$ • $X_{drift} = V_R \times t$ • $V_m = \sqrt{(V_{MR})^2 + (V_R)^2}$	Condition for no drifting $\Rightarrow \sin \theta = \frac{V_R}{V_{MR}}$ $\Rightarrow t_{cross} = \frac{d}{V_m}$ $\Rightarrow V_m = \sqrt{V_{MR}^2 - V_R^2}$ $\Rightarrow \text{Drift} = 0$

Escalator

$$t_3 = \frac{d}{V_E + V_{M/E}} = \frac{d}{\frac{d}{t_1} + \frac{d}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$$

t_1 = Time taken by a man to move distance d on a stationary escalator

t_2 = Time taken by a man to move distance d along a moving escalator

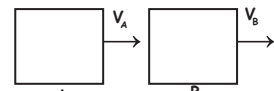
t_3 = Time taken by a man to move distance d while walking along a moving escalator

V_E = Velocity of escalator

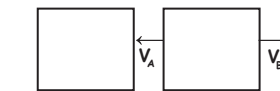
$V_{M/E}$ = Velocity of man w.r.t escalator

MAN RAIN PROBLEM

Man-rain problem

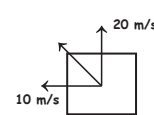
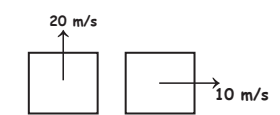


Inorder to find the relative velocity of B with respect to A we have to reverse the direction of vector A and add it with vector B



$$V_{BA} = V_B - V_A, \quad V_B \text{ w.r.t } A$$

$$V_{A/B} = V_B - V_A, \quad V_A \text{ w.r.t } B$$



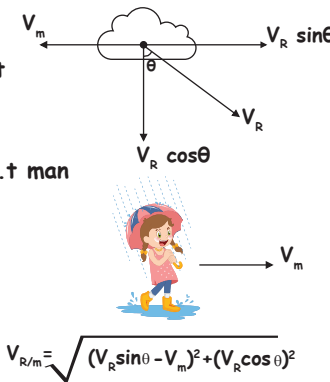
Terms

$V_R \Rightarrow$ Velocity of rain w.r.t stationary man

$V_m \Rightarrow$ Velocity of man

$V_{R/m} \Rightarrow$ Velocity of Rain w.r.t man

Method



Case 1

$$V_R \sin \theta > V_m$$

$$\tan \alpha = \frac{V_R \sin \theta - V_m}{V_R \cos \theta}$$

$$\alpha = \tan^{-1} \left(\frac{V_R \sin \theta - V_m}{V_R \cos \theta} \right)$$

Case 2

$$V_m > V_R \sin \theta$$

$$V_{R/m} = \sqrt{(V_R \cos \theta)^2 + (V_m - V_R \sin \theta)^2}$$

Case 3

$$V_m = V_R \sin \theta$$

$$V_{R/m} = V_R \cos \theta$$

Case 4

$$V_{R/m} = \sqrt{V_R^2 + V_m^2}$$

$$\tan \alpha = \frac{V_m}{V_R}$$

Shortest path

$$\text{shortest path, } x = \frac{d \times V}{\sqrt{V^2 + U^2}}$$

$$\text{Time taken to reach shortest path} = \frac{d \times U}{V^2 + U^2}$$

CIRCULAR MOTION

$$1^\circ = \frac{\pi}{180}$$

Angular velocity

$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{T}$$

$$\vec{V} = \vec{\omega} \times \vec{r}$$

$$\omega_{sec} = \frac{2\pi}{60} = \frac{2\pi}{T}$$

$$\omega_{min} = \frac{2\pi}{60 \times 60} \quad \omega_{hour} = \frac{2\pi}{12 \times 60 \times 60} \quad \frac{\omega_{min}}{\omega_{hour}} = \frac{12}{1}$$

Angular acceleration

$$\alpha = \frac{d\omega}{dt} \quad a_t = r\alpha$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

Equation of angular motion

1) Constant angular velocity :- ω = constant

2) Constant angular acceleration

$$\Rightarrow \omega = \omega_0 + \alpha t$$

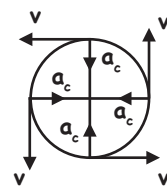
$$\Rightarrow \Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \omega^2 = \omega_0^2 + 2\alpha(\Delta \theta)$$

Centripetal acceleration

Directed towards centre

Not a constant vector



- change the direction of velocity

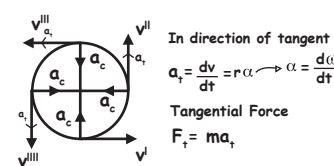
$$a_c = \frac{v^2}{R} = a_c = r\omega^2$$

$$\vec{a}_c \perp \vec{v} \rightarrow \text{Power} = \vec{F} \cdot \vec{v} = 0$$

$$\vec{F}_c \perp \vec{s} \rightarrow \text{Work} = \vec{F} \cdot \vec{s} = 0$$

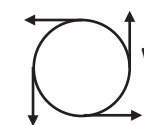
$$\Delta K = 0$$

Tangential acceleration



Resultant acceleration

$$a_r = \sqrt{a_c^2 + a_t^2}$$



Velocity is tangent to the circle

Circular Motion

Uniform Circular Motion

- Speed Constant
- Direction of velocity changes

- ω = Constant

$$a_c = \frac{v^2}{r}, r\omega^2$$

$$a_t = 0$$

Non-uniform Circular Motion

- Speed not Constant

- Velocity changes in direction and magnitude

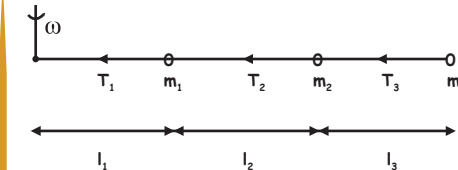
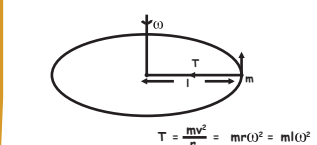
- a_c = Centripetal acceleration

- a_t = tangential acceleration

$$= \frac{dv}{dt} \quad \alpha = \frac{d\omega}{dt}$$

- ω = Changes $\rightarrow \alpha$ angular acceleration

Horizontal circular motion

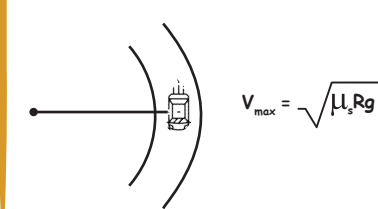


$$T_1 = m_1 l_1 \omega^2 + m_2 (l_1 + l_2) \omega^2 + m_3 (l_1 + l_2 + l_3) \omega^2$$

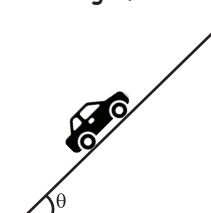
$$T_2 = m_2 (l_1 + l_2) \omega^2 + m_3 (l_1 + l_2 + l_3) \omega^2$$

$$T_3 = m_3 (l_1 + l_2 + l_3) \omega^2$$

Flat circular track



Banking of Road



$$v_{max} = \sqrt{\frac{rg(\tan \theta + \mu)}{1 - \mu \tan \theta}}$$

$$v_{opt} = \sqrt{rg \tan \theta}$$

$$v_{min} = \sqrt{\frac{rg(\tan \theta - \mu)}{1 + \mu \tan \theta}}$$

At Bottom

$$a) T_{max} = \frac{mv^2}{r} + mg$$

b) min velocity at bottom of the circle = $\sqrt{5gR}$

At Top

$$a) T_{min} = \frac{mv^2}{r} - mg$$

b) min velocity at top to complete the circle = $\sqrt{gR}$

MOVING CHARGES AND MAGNETISM

1

Biot-Savart Law

$$dB \propto \frac{Idl \sin \theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

μ_0 → magnetic permeability of free space or vacuum

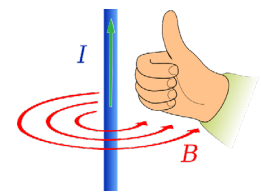
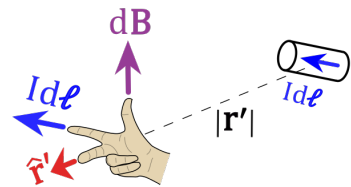
$$\mu_0 = 4\pi \times 10^{-7}$$

In vector form,

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3}$$

$d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\vec{r}$. By using right hand screw rule we can find direction of magnetic field

Here, B is into the plane



Magnetic field circulates around the current carrying wire

Formula of Field for straight wire

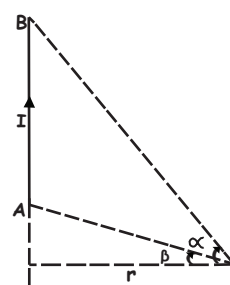
1) At point P

B is into the plane

$$B = \frac{\mu_0 I}{4\pi r} (\sin \alpha + \sin \beta), \otimes$$

2)

Extend AB downwards and draw a perpendicular from P



$$B = \frac{\mu_0 I}{4\pi r} (\sin \alpha - \sin \beta), \otimes$$

3) Wire of infinite length

$$\alpha = 90^\circ \text{ and } \beta = 90^\circ$$

$$B = \frac{\mu_0 I}{4\pi r} (\sin 90^\circ + \sin 90^\circ)$$

$$= \frac{\mu_0 I}{2\pi r}, \otimes$$

4) Wire of semi infinite length

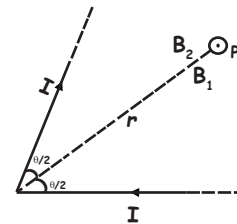
$$B = \frac{\mu_0 I}{4\pi r} (\sin 90^\circ + \sin 0^\circ)$$

$$= \frac{\mu_0 I}{4\pi r}$$

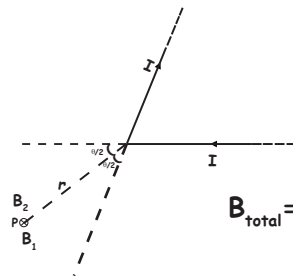


$$B(P_1) = B(P_2) = 0$$

Field Due to bent wire

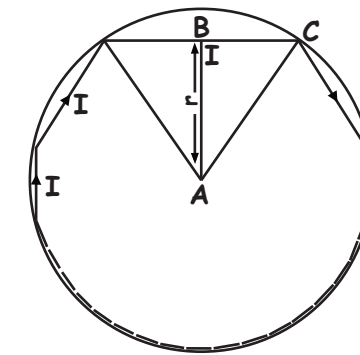


$$B_{\text{total}} = 2 \times \frac{\mu_0 I}{4\pi r} \frac{[1 + \cos \theta/2]}{\sin \theta/2}, \odot$$



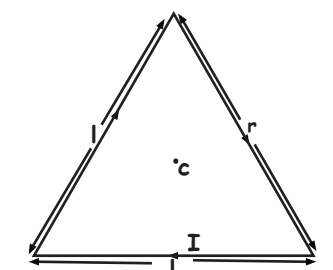
$$B_{\text{total}} = 2 \times \frac{\mu_0 I}{4\pi r} \frac{[1 - \cos \theta/2]}{\sin \theta/2}, \otimes$$

Field due to polygon



$$B_{\text{Total}} = \frac{\mu_0 n I}{\pi l} \tan \frac{\pi}{n} \sin \frac{\pi}{n}$$

eg: equilateral triangle

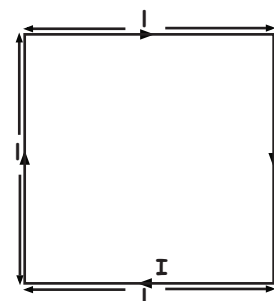


$$B_r = \frac{9}{2} \frac{\mu_0 I}{\pi l}$$

If a total length l is bent as an equilateral triangle each side has length l/3 then,

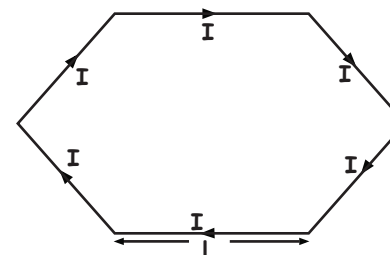
$$B_r = \frac{27}{2} \frac{\mu_0 I}{\pi l}$$

eg: Square



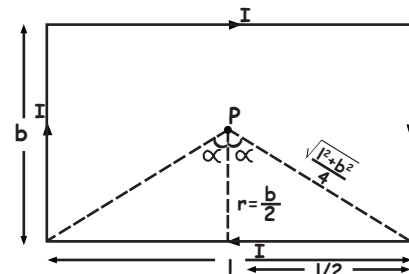
$$B = \frac{2\sqrt{2} \mu_0 I}{\pi l}$$

eg: Hexagon



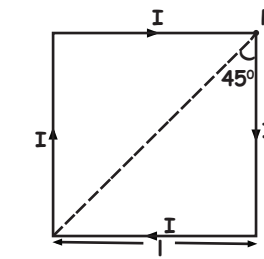
$$B = \frac{\sqrt{3} \mu_0 I}{\pi l}$$

Rectangle



$$B_r = \frac{8 \mu_0 I}{4\pi} \left(\frac{\text{diagonal}}{\text{area}} \right)$$

Note:- To find field at P



Here P is a point at vertex

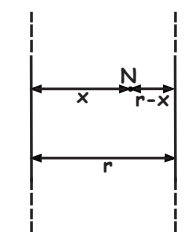
$$B_r = \frac{\mu_0 I}{2\sqrt{2} \pi l}$$

Neutral Points

Points at which magnetic field becomes zero are called neutral points.

Case 1: Parallel wires carrying current in same direction ($I_1 > I_2$)

$$\therefore x = \frac{I_1 r}{I_1 + I_2}$$

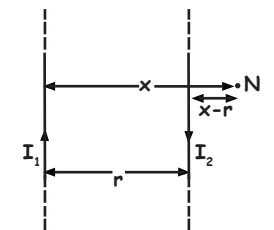


If y is the distance from 2nd conductor then

$$y = \frac{I_2 r}{I_1 + I_2}$$

Case 2: Parallel wires carrying currents in opposite direction ($I_1 > I_2$)

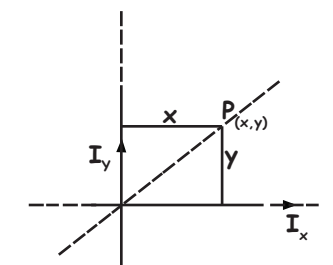
$$x = \frac{I_1 r}{I_1 - I_2}$$



Case 3: Wires perpendicular to each other

Comparing with $y = mx + C$ we get.

$$\text{slope } m = \frac{I_x}{I_y}$$



Wire perpendicular to plane

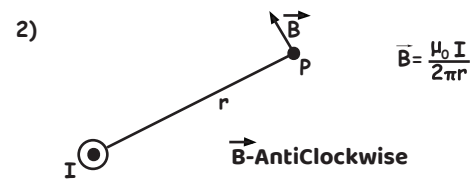
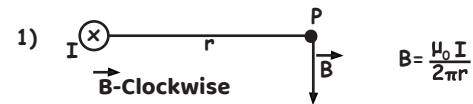
1) Current directed out of plane



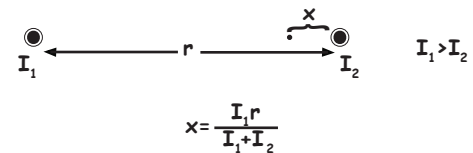
2) Current directed into the plane



Direction of field

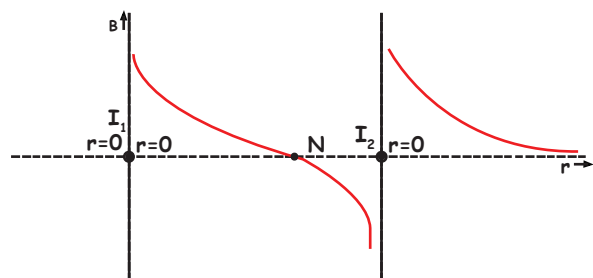


Neutral Point

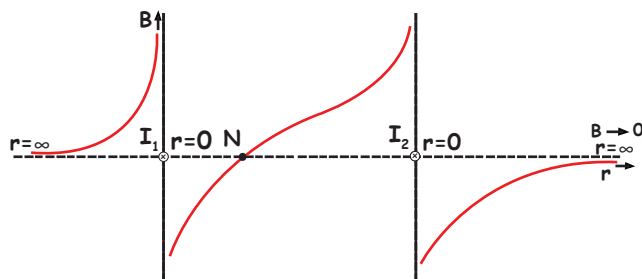


Graphs

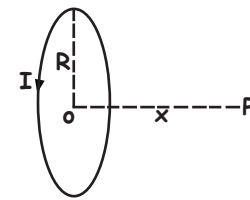
1) $I_1 > I_2$



2) I into the plane [$I_1 < I_2$]



Field due to a circular ring & Field at the axis of a ring



$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

Every current carrying loop act as a magnetic dipole

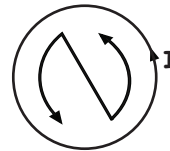
$$\vec{M} = I\vec{A}$$

Direction of A is determined using right hand thumb rule

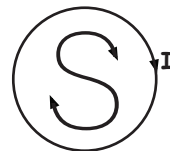
Here, $\vec{M} = I\vec{A} = I \times \pi R^2$

$$B = \frac{\mu_0}{4\pi} \frac{2M}{2(x^2 + R^2)^{3/2}}$$

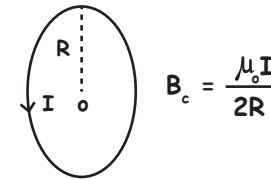
If current is flowing in anticlockwise direction



If current is flowing in clockwise direction



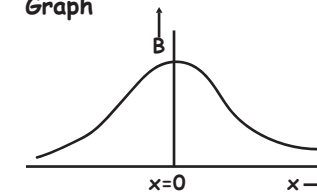
Field at the centre of ring



Field at the axis in terms of field at center

$$B_c = \frac{B}{\left(1 + \frac{x^2}{R^2}\right)^{3/2}}$$

Graph



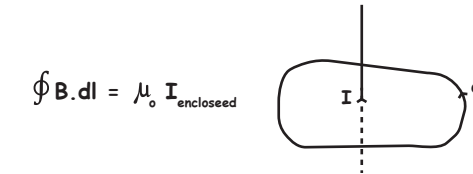
Percentage change in field at points near axis

$$\% \text{ change} = 1 - \left(1 + \frac{x^2}{R^2}\right)^{-3/2} \times 100\%$$

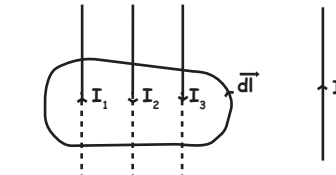
$$\text{For change} < 10\% = \left(\frac{3}{2} \times \frac{x^2}{R^2}\right) \times 100\%$$

Ampere's Circuital Law

The line integral of magnetic field over a closed loop is equal to μ_0 times the total current enclosed by the loop

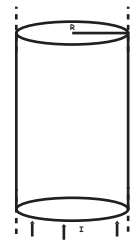


Example



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 [I_1 - I_2 + I_3]$$

Solid Cylinder



Outside : $r > R$

$$B_{\text{out}} = \frac{\mu_0 I}{2\pi r}$$

$$B_{\text{out}} \propto \frac{1}{r}$$

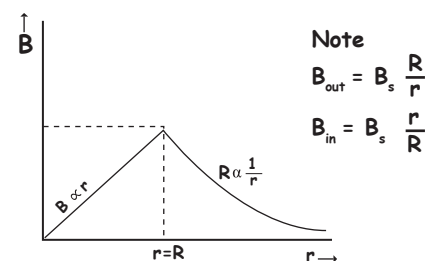
On the surface $r = R$

$$B_s = \frac{\mu_0 I}{2\pi R}$$

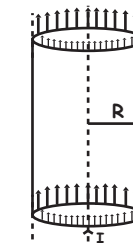
Inside ($r < R$)

$$B_{\text{in}} = \left(\frac{\mu_0 I}{2\pi R^2}\right) r \rightarrow B_{\text{in}} \propto r$$

Graph



Hollow cylinder (pipe)



Outside : $r > R$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B_{\text{out}} \propto \frac{1}{r}$$

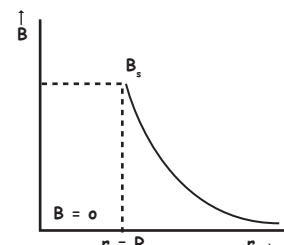
On the surface : $r = R$

$$B_s = \frac{\mu_0 I}{2\pi R}$$

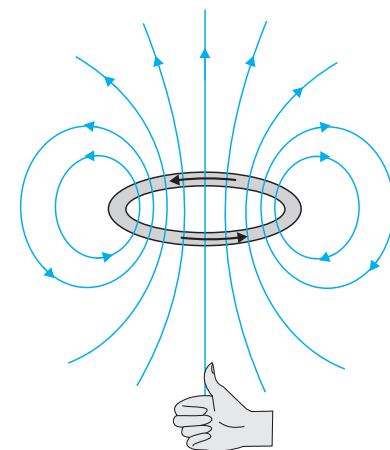
Inside : $r < R$

$$B_{\text{in}} = 0$$

Graph



Magnetic field lines for a current loop



$$\text{If there are } N \text{ loops, } B = \frac{\mu_0 N I R^2}{2(x^2 + R^2)^{3/2}}$$

Field due to circular arc at the Centre

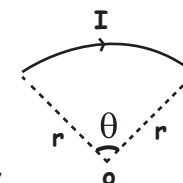
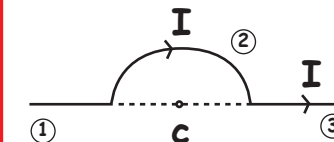
Field at centre of Circle,

$$B = \frac{\mu_0 I}{2R}$$

Then field at centre of arc,

where, θ is in radian

Example



$$B = \frac{\mu_0 I}{4\pi} \frac{\theta}{r}$$

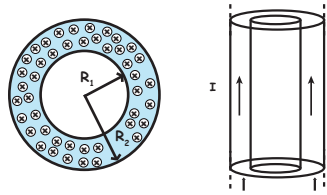
$$B_{\text{Total}} = B_1 + B_2 + B_3$$

$$= \frac{\mu_0 I}{4\pi} \frac{\pi}{r} = \frac{\mu_0 I}{4r}$$

MOVING CHARGES AND MAGNETISM

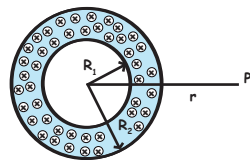
2

Annular pipe



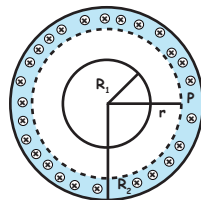
Outside ($r > R_2$)

$$B_{out} = \frac{\mu_0 I}{2\pi r}$$



Outside ($r > R_2$)

In between $R_1 < r < R_2$



$$I_{enc} = \frac{I}{\pi(R_2^2 - R_1^2)} \times \pi(r^2 - R_1^2)$$

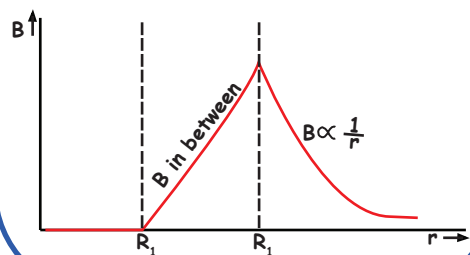
$$B_{btw} = \frac{\mu_0}{2\pi r} \times I \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$$

Inside

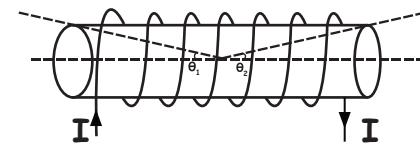
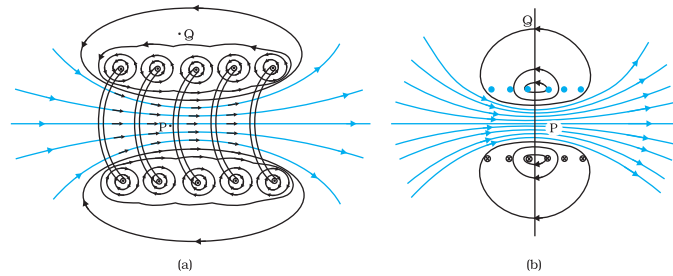
$$r < R_1$$

$$B_{in} = 0$$

Graph



Solenoid



$$B = \frac{1}{2} \mu_0 n I (\cos\theta_1 + \cos\theta_2)$$

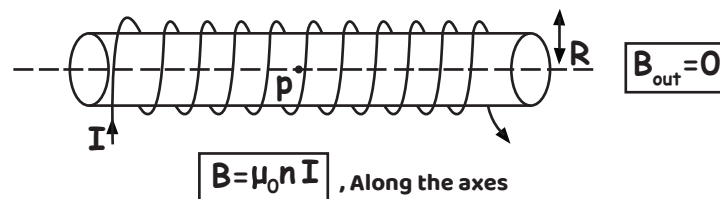
where $n \rightarrow$ no. of turns per unit length

Tightly packed long solenoid

Long solenoid: Radius is small compared to the length of the solenoid

For a long solenoid the middle region will have a uniform magnetic field

1) At centre



$$B = \mu_0 n I, \text{ Along the axes}$$

2) End point

$$\theta_1 = 0 \quad \theta_2 = 90^\circ$$

$$B_{end} = \frac{1}{2} \mu_0 n I$$

Charged particle in magnetic field ($\vec{v} \perp \vec{B}$)
Radius of circular path

$$\begin{aligned} \text{i) } F_m &= \frac{mv^2}{r} \\ qvB &= \frac{mv^2}{r} \\ r &= \frac{mv}{qB} \end{aligned}$$

ii) Momentum

$$\begin{aligned} mv &= p = \sqrt{2mk} \\ r &= \frac{\sqrt{2mk}}{qB} \end{aligned}$$

If charged particle is accelerated by particle of 'V' volt then,

$$K.E = qV$$

$$r = \frac{\sqrt{2mqV}}{qB}$$

Condition:

$$\text{Initial K.E.}, K_i = 0$$

$$W = \Delta K$$

$$qV = K_f - K_i$$

$$\text{If } K_i \neq 0, \text{ then } K_f = qV + K_i$$

$$r = \frac{\sqrt{2mk}}{qB} = \frac{\sqrt{2m(qV + K_i)}}{qB}$$

Calculation of ratio of radii

1) All are projected with same speed:

$$V = \text{Constant}$$

$$r = \frac{mv}{qB} \Rightarrow r \propto \frac{m}{q}$$

$$r_p : r_d : r_\alpha = \frac{1}{e} : \frac{2}{e} : \frac{4}{2e} = 1:2:2$$

2) All are projected with same momentum

$$r = \frac{mv}{qB} \Rightarrow r \propto \frac{1}{q}$$

$$r_p : r_d : r_\alpha = \frac{1}{e} : \frac{1}{e} : \frac{1}{2e} = 2:2:1$$

As radius $\uparrow$ Curvature $\downarrow$

3) All are projected with same kinetic energy

$$r = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$$

$$r \propto \frac{\sqrt{m}}{q}$$

$$r_p : r_d : r_\alpha = \frac{\sqrt{1}}{e} : \frac{\sqrt{2}}{e} : \frac{\sqrt{4}}{2e} = 1 : \sqrt{2} : 1$$

4) All are projected by same accelerating potential

$$r = \frac{\sqrt{2mqV}}{qB} \quad r \propto \frac{\sqrt{m}}{\sqrt{q}}$$

$$r_p : r_d : r_\alpha = \frac{\sqrt{1}}{e} : \frac{\sqrt{2}}{e} : \frac{\sqrt{4}}{2e} = 1 : \sqrt{2} : 1$$

Helical path &

$\vec{v}$ makes angle θ with $\vec{B}$ ($\theta \neq 0, \pi, \pi/2$)

Motion of charge on Helical path

$V_{||} \rightarrow ||$ to $\vec{B}$

$V_{\perp} \rightarrow \perp$ to $\vec{B}$

1) Radius

$$R = \frac{mv_{\perp}}{qB}$$

$$R = \frac{mv \sin \theta}{qB} = \frac{\sqrt{2mqV}}{qB} \sin \theta$$

2) Time period

$$T = \frac{2\pi m}{qB}$$

3) Pitch $= V_{||} \times T$

$$= V \cos \theta \times \frac{2\pi m}{qB}$$

$$= 2\pi \left(\frac{mv}{qB} \right) \cos \theta$$

$$= 2\pi \frac{\sqrt{2mK}}{qB} \cos \theta$$

$$= 2\pi \frac{\sqrt{2mqV}}{qB} \cos \theta$$

Time Period

$$T = \frac{2\pi m}{qB}$$

$$\Rightarrow T \propto r^0$$

$$T \propto v^0$$

$$\text{We have } \frac{q}{m} = \text{specific charge}$$

$$\Rightarrow T \propto \frac{1}{\text{specific charge}}$$

Toroid

r is the mean radius

$$r = \frac{r_1 + r_2}{2}$$

$$B_{out} = 0$$

$$B_{inside} = 0$$

$$B_{in \text{ between}} = \mu_0 n I$$

unit and dimension of B

$$F = qvB \sin \theta$$

$$B = \frac{F}{qv \sin \theta}$$

$$[B] = \frac{MLT^{-2}}{AT \times LT^{-1}} = MA^{-1}T^{-2}$$

$$\text{Unit} \rightarrow kgA^{-1}s^{-2} = \text{Tesla (T)}$$

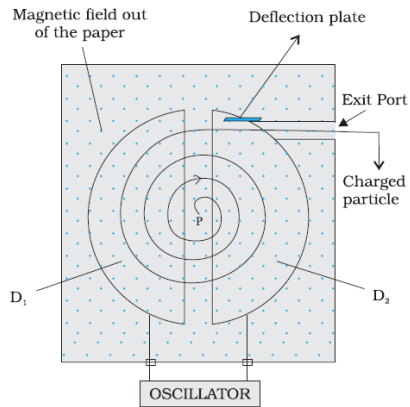
MOVING CHARGES AND MAGNETISM

3

MOVING CHARGES AND MAGNETISM

4

Cyclotron



1) Maximum kinetic energy

$$K_{\max} = \frac{q^2 B^2 R_{\text{dee}}^2}{2m}$$

2) Number of oscillation

$$\text{Work done} = \Delta K$$

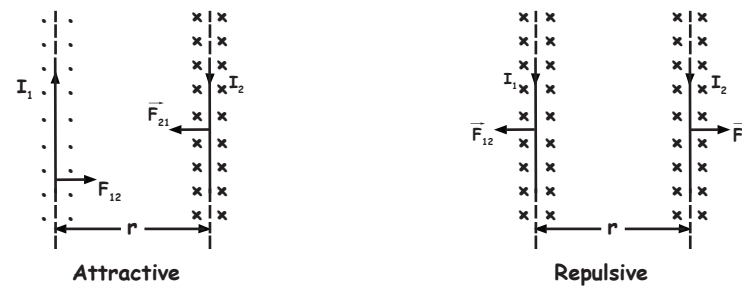
$$N2qEd = \frac{q^2 B^2 R^2}{2m}$$

3) Time period of revolution = $\frac{2\pi m}{qB}$

4) Cyclotron frequency $\nu_c = \frac{qB}{2\pi m}$

Force between parallel conductors

1) Force between two long parallel current conductors



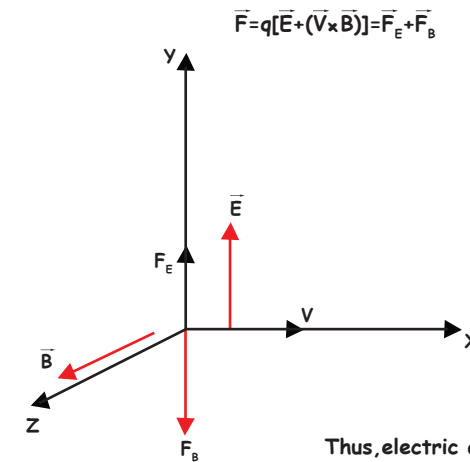
Parallel currents attract each other. Anti parallel currents repel each other
Force per unit length of conductor

$$F_{12} = F_{21} = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r}$$

$$= \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r}$$

$$\text{Net force on 'l' length of conductor} = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{r} \times l$$

Motion of charged particle in Crossed Electric and Magnetic field ($\vec{E} \perp \vec{B}$) [velocity selector]



$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] = \vec{F}_E + \vec{F}_B$$

$$\vec{F}_E = q\vec{E} = qE\hat{j}$$

$$\vec{F}_B = q(\vec{v} \times \vec{B}) = q(v\hat{i} \times B\hat{k})$$

$$= -qvB\hat{j}$$

$$\therefore \vec{F} = q(E - vB)\hat{j}$$

If $v = E/B$, $F_{\text{net}} = 0$
Charge moves in a straight line

Thus, electric and magnetic forces are in opposite directions as shown in figure.

Force on current carrying Conductor in magnetic field

consider a small element $d\vec{l}$ of the conductor

$$d\vec{F} = I(d\vec{l} \times \vec{B})$$

To find resulting force

$$\int d\vec{F} = \int I(d\vec{l} \times \vec{B})$$

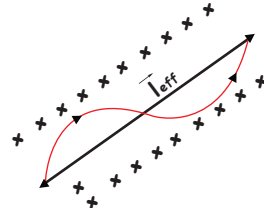
$$= I \int d\vec{l} \times \vec{B}$$

In uniform field

$$\vec{F} = I \left(\int d\vec{l} \right) \times \vec{B} = I(\vec{l} \times \vec{B})$$

The above equation can be used in the following situations

1)



$$\vec{F} = I(\vec{l}_{\text{eff}} \times \vec{B})$$

$$= I l_{\text{eff}} B \sin \theta$$

2) $\vec{l}_{\text{eff}}$ perpendicular to $\vec{B}$

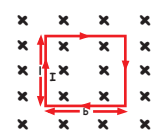
$$\theta = 90^\circ$$

$$F_{\max} = I l_{\text{eff}} B$$

For closed Loop in uniform field

$$\text{Here, } l_{\text{eff}} = 0$$

$$\therefore \vec{F} = 0$$



Lorentz force

A charge q in an electric field $\vec{E}$ experiences the electric force

$$\vec{F} = q\vec{E}$$

The magnetic force experienced by the charge q moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$ is given by

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

The total force, or the Lorentz force, experienced by the charge q due to both electric and magnetic field is given by

$$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

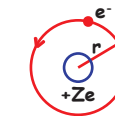
Magnetic dipole moment of a revolving electron

$$I = \frac{e}{T}$$

T is the time period of revolution

$$T = \frac{2\pi r}{v}$$

$$I = \frac{eV}{2\pi r}$$



There will be a magnetic moment, usually denoted by μ , associated with this circulating current

$$\mu = I \pi r^2 = \frac{eVr}{2}$$

$$\mu = \frac{e}{2m_e} (m_e v r) = \frac{e}{2m_e} \ell$$

$$\vec{\ell} = \vec{r} \times \vec{p}$$

$$= r m v \sin \theta$$

$$= m v r$$

In vector form

$$\vec{\mu} = \frac{-e}{2m_e} \vec{\ell}$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment

The ratio of magnetic moment to the angular momentum is called gyromagnetic ratio

$$\frac{\mu}{\ell} = \frac{e}{2m_e}$$

Its value is a constant and is equal to $8.8 \times 10^{10} \text{ C/kg}$ for an electron

According to Bohr's quantization condition, angular momentum assumes a discrete set of values namely.

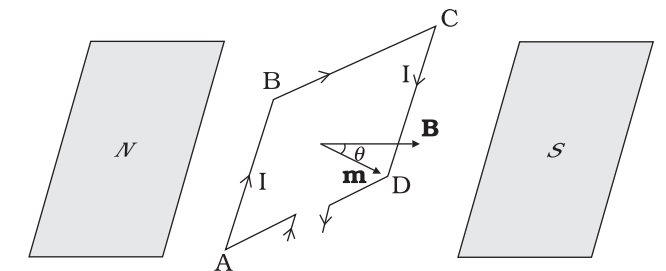
$$\ell = \frac{nh}{2\pi} \text{ where } n = 1, 2, 3, \dots$$

$h \rightarrow \text{plank's constant}$

Taking $n=1$, we get,

$$(\mu)_{\min} = \frac{e}{4\pi m_e} h = 9.27 \times 10^{-24} \text{ Am}^2$$

Torque on a current loop in a uniform magnetic field



$$\text{Then } \tau = NIAB \sin \theta$$

Special cases:

i) When $\theta = 0^\circ$

$\tau = 0$, i.e. the torque is minimum when the plane of the loop is perpendicular to the magnetic field

ii) When $\theta = 90^\circ$

$\tau = NIAB$ i.e., the torque is maximum when the plane of the loop is parallel to the magnetic field. Thus,

$$\tau_{\max} = NIAB$$

Note:

1) The constant, $\frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.

2) Figure of merit of a galvanometer

$$G = \frac{I}{\Phi} = \frac{K}{NAB}$$

Moving Coil Galvanometer (MCG)

$$\tau = NIAB \sin \theta$$

Sensitivity of a Galvanometer

$$\text{Current sensitivity, } I_s = \frac{\Phi}{I} = \frac{NAB}{K}$$

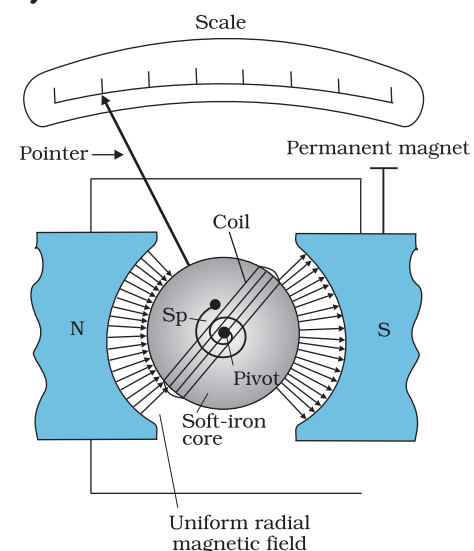
Where $K \rightarrow$ torsional constant

Voltage Sensitivity

$$\text{Voltage Sensitivity, } V_s = \frac{\Phi}{V} = \frac{\Phi}{IR}$$

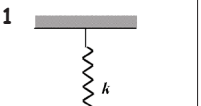
$$= \frac{NAB}{KR}$$

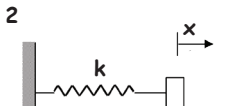
$$\text{Voltage Sensitivity} = \frac{\text{Current Sensitivity}}{R}$$

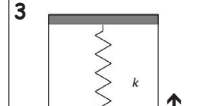


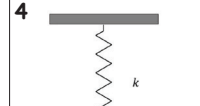
OSCILLATIONS 01

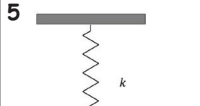
TIME PERIOD OF S.H.M

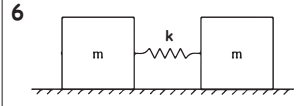
1 
 $F = -kx$
 $T = 2\pi \sqrt{\frac{m}{k}}$

2 
 $F = -kx$
 $T = 2\pi \sqrt{\frac{m}{k}}$

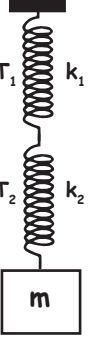
3 
 $F = -kx$
 $T = 2\pi \sqrt{\frac{m}{k}}$

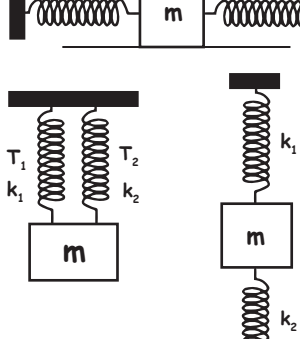
4 
 $F = -kx$
 $T = 2\pi \sqrt{\frac{m}{k}}$

5 
 $T = 2\pi \sqrt{\frac{m}{k + \rho H g}}$

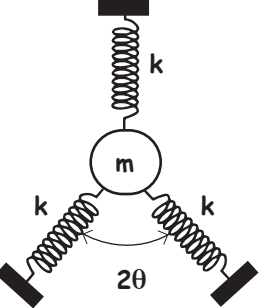
6 
 $F = -kx$
 $T = 2\pi \sqrt{\frac{m_{rel}}{k}}$
 $m_{rel} = \frac{m_1 m_2}{m_1 + m_2}$

COMBINATIONS OF SPRINGS

1) Series 
 $k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$
 $T = 2\pi \sqrt{\frac{m}{k_{eq}}}$
 $T_s^2 = T_1^2 + T_2^2$
 $T_s = \sqrt{T_1^2 + T_2^2}$

2) Parallel 
 $k_{eq} = k_1 + k_2$
 $T = 2\pi \sqrt{\frac{m}{k_{eq}}}$
 $\frac{1}{T_p^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$

SPECIAL CASE


 $k_{eq} = k + 2k \cos^2 \theta$
 $T = 2\pi \sqrt{\frac{m}{k_{eq}}}$
eg:
if $\theta = 120^\circ, \theta = 60^\circ$
 $k_{eq} = k + \frac{2k}{4} = \frac{3k}{2}$
 $T = 2\pi \sqrt{\frac{2m}{3k}}$

CUTTING OF SPRINGS

$k \propto \frac{1}{L}$

$k_1 L_1 = k_2 L_2$

$L = L_1 + L_2$

$k_1 = \frac{kL}{L_1}$ $k_2 = \frac{kL}{L_2}$

$k_1 = \frac{k(L_1 + L_2)}{L_1}$ $k_2 = \frac{k(L_1 + L_2)}{L_2}$

eg:

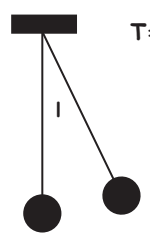
$L_1 : L_2 : L_3 = 1 : 2 : 3$

$k_1 = \frac{k(L_1 + L_2 + L_3)}{L_1} = \frac{k6}{1} = 6k$

$k_2 = \frac{k6}{2} = 3k$

$k_3 = \frac{k6}{3} = 2k$

SIMPLE PENDULUM


 $T = 2\pi \sqrt{\frac{l}{g}}$

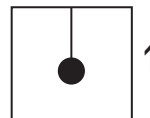
• Second's pendulum
 $T = 2$ second
 $l = 1$ meter

Concept of $g_{effective}$

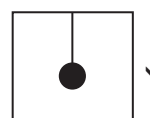
$$T = 2\pi \sqrt{\frac{l}{g_{eff}}}$$

Pendulum in lift

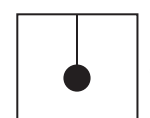
Case 1 - Moving up with constant acceleration 'a'


 $\uparrow a$ $g_{eff} = g + a$
 $T = 2\pi \sqrt{\frac{l}{g+a}}$

Case 2 - Moving down with constant acceleration 'a'


 $\downarrow a$ $g_{eff} = g - a$
 $T = 2\pi \sqrt{\frac{l}{g-a}}$

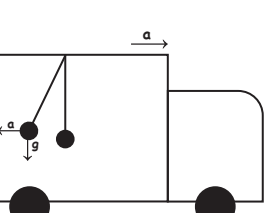
Case 3 - Moving with constant velocity


 $\downarrow a$ $a = 0$
 $T = 2\pi \sqrt{\frac{l}{g}}$

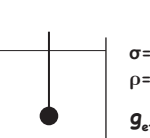
Case 4 - Free fall

$a = g$
 $g_{eff} = g - a = g - g = 0$ $T \rightarrow \infty$

Pendulum in a truck moving with constant acceleration


 $g_{eff} = \sqrt{g^2 + a^2}$
 $T = 2\pi \sqrt{\frac{l}{g^2 + a^2}}$

Pendulum in Water


 $\sigma = \text{Density of Bob}$
 $\rho = \text{Density of liquid}$
 $g_{eff} = g \left(1 - \frac{\rho}{\sigma}\right)$
 $T = 2\pi \sqrt{\frac{l}{g \left(1 - \frac{\rho}{\sigma}\right)}}$

DIFFERENTIAL EQUATION OF S.H.M

$$ma = -kx$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = \frac{-k}{m} x$$

$$\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\omega^2 = \frac{k}{m} \quad \omega = \sqrt{\frac{k}{m}}$$

A = Amplitude of SHM

Φ_0 = Initial phase difference

Eg: $\begin{bmatrix} m = 4. \text{ Kg} \\ K = 320 \text{ N/m} \end{bmatrix}$

$$4 \frac{d^2x}{dt^2} + 320 x = 0$$

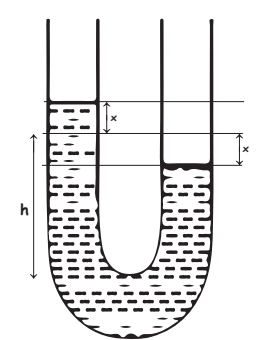
$$\frac{d^2x}{dt^2} + 80 x = 0$$

$$\omega^2 = 80 \quad \omega = \sqrt{80}$$

$$\frac{2\pi}{T} = \sqrt{80}$$

$$T = \frac{2\pi}{\sqrt{80}}$$

OSCILLATION OF LIQUID COLUMN

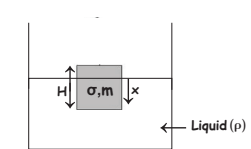


$$T = 2\pi \sqrt{\frac{m}{2\rho g A}}$$

$$T = 2\pi \sqrt{\frac{h}{g}}$$

ρ = Density of liquid
 A = Area of U-tube

OSCILLATION OF FLOATING BODY

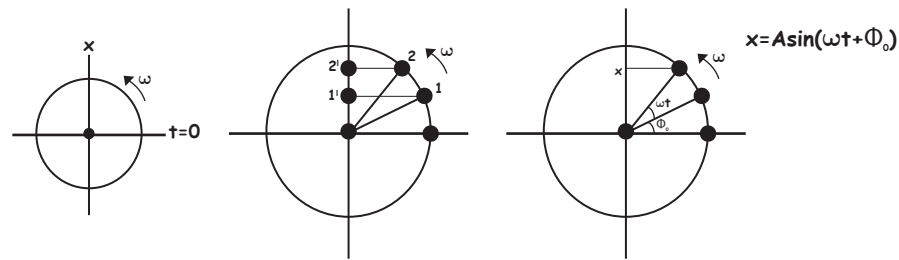


$$T = 2\pi \sqrt{\frac{m}{\rho g A}}$$

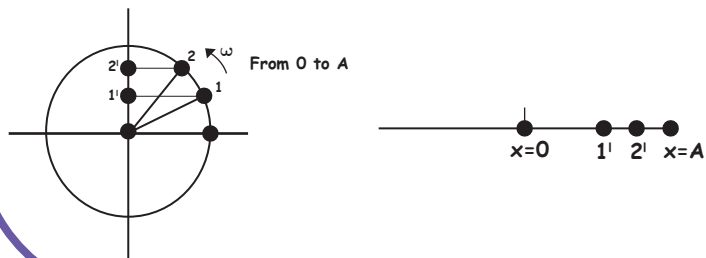
$$T = 2\pi \sqrt{\frac{\sigma H}{\rho g}}$$

σ = Density of Body
 ρ = Density of liquid

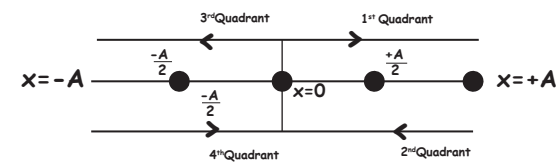
PROJECTION OF CIRCULAR MOTION



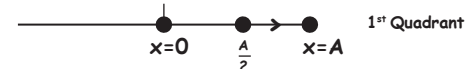
Projection/shadow of uniform circular motion in y axis is SHM



INITIAL PHASE FROM POSITION & DIRECTION



Eg: Particle lies at $x = \frac{A}{2}$ [$t=0$] and move towards A



1st Quadrant

Start from mean position
 $x = A \sin(\omega t + \Phi_0)$

$$1) X = \frac{A}{2} \quad \frac{A}{2} = A \sin \Phi_0$$

$$t=0 \quad \Phi_0 = 30^\circ = \frac{\pi}{6}$$

$$x = A \sin(\omega t + \frac{\pi}{6})$$

$$2) X = \frac{A}{\sqrt{2}} \quad \frac{A}{\sqrt{2}} = A \sin \Phi_0$$

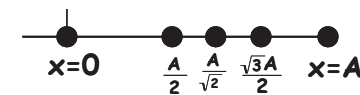
$$t=0 \quad \Phi_0 = 45^\circ = \frac{\pi}{4}$$

$$x = A \sin(\omega t + \frac{\pi}{4})$$

$$3) X = \frac{\sqrt{3}A}{2} \quad \frac{\sqrt{3}A}{2} = A \sin \Phi_0$$

$$t=0 \quad \Phi_0 = 60^\circ = \frac{\pi}{3}$$

$$x = A \sin(\omega t + \frac{\pi}{3})$$



2nd Quadrant

Start from extreme position
 $x = A \cos(\omega t + \Phi_0)$

$$1) X = \frac{A}{2} \quad \frac{A}{2} = A \cos \Phi_0$$

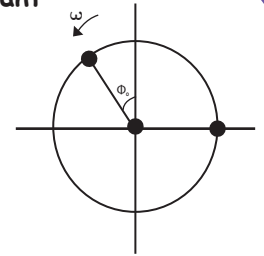
$$t=0 \quad \Phi_0 = 60^\circ = \frac{\pi}{3}$$

$$x = A \cos(\omega t + \frac{\pi}{3})$$

$$2) X = \frac{A}{\sqrt{2}} \quad \frac{A}{\sqrt{2}} = A \cos \Phi_0$$

$$t=0 \quad \Phi_0 = 45^\circ = \frac{\pi}{4}$$

$$x = A \cos(\omega t + \frac{\pi}{4})$$



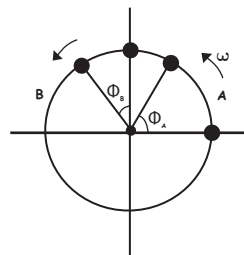
OSCILLATIONS 02

Two particles executing SHM meet at $X = \frac{\sqrt{3}A}{2}$

for particle A

$$X = \frac{\sqrt{3}A}{2} \quad \frac{\sqrt{3}A}{2} = A \sin \Phi_A$$

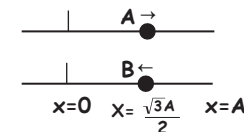
$$t=0 \quad \Phi_A = 60^\circ = \frac{\pi}{3}$$



for particle B

$$X = \frac{\sqrt{3}A}{2} \quad \frac{\sqrt{3}A}{2} = A \cos \Phi_B$$

$$t=0 \quad \Phi_B = 30^\circ = \frac{\pi}{6}$$

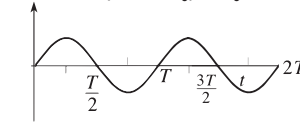


Phase difference

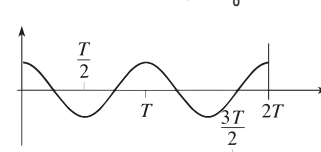
$$\Phi = 30^\circ + 30^\circ = 60^\circ \text{ or } 300^\circ$$

Graphical Representation of Displacement

Start from mean position
 $x = A \sin(\omega t + \Phi_0)$, $\Phi_0 = 0$



Start from extreme position
 $x = A \cos(\omega t + \Phi_0)$, $\Phi_0 = 0$



Velocity of SHM

$$x = A \sin(\omega t + \Phi_0)$$

$$v = A \omega \cos(\omega t + \Phi_0)$$

$$v = \omega \sqrt{A^2 - x^2}$$

$$a) x=0 \quad v_{\max} = A \omega$$

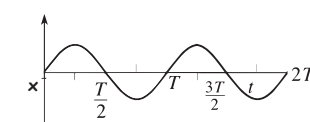
$$b) x = \frac{A}{2} \quad v = \frac{\sqrt{3}A}{2} \omega$$

$$c) x = \frac{A}{\sqrt{2}} \quad v = \frac{A \omega}{\sqrt{2}}$$

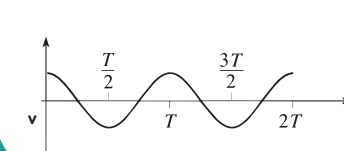
$$d) x = A \quad v = 0$$

Graphical Representation of Velocity

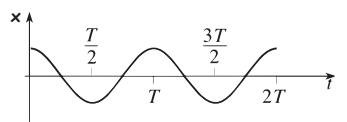
Start from mean position
 $x = A \sin(\omega t + \Phi_0)$, $\Phi_0 = 0$



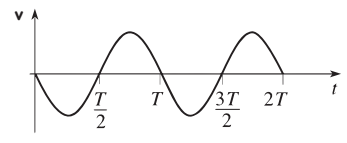
$$v = A \omega \cos(\omega t + \Phi_0)$$



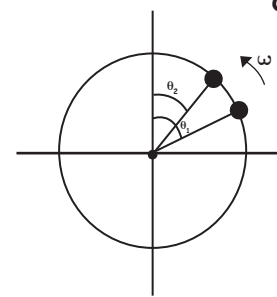
Start from extreme position
 $x = A \cos(\omega t + \Phi_0)$, $\Phi_0 = 0$



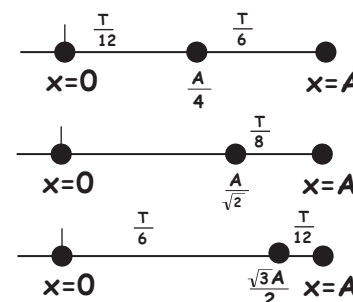
$$v = -A \omega \sin(\omega t + \Phi_0)$$



Calculation of time



$$\Delta t = \frac{\Delta \theta}{\omega} = \left(\frac{\theta_2 - \theta_1}{2\pi} \right) T$$



$$A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} \quad \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

OSCILLATIONS 03

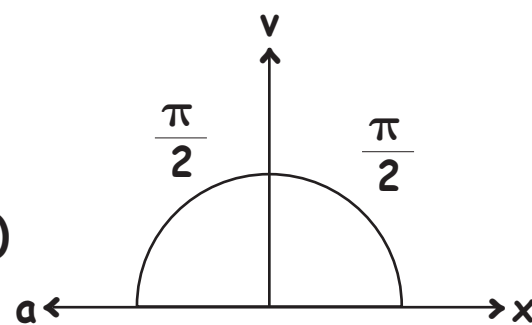
Acceleration of SHM

$$x = A \sin(\omega t + \Phi_0)$$

$$v = A \omega \cos(\omega t + \Phi_0)$$

$$a = -A \omega^2 \sin(\omega t + \Phi_0)$$

$$a_{\max} = -A \omega^2$$



Phase difference between x and $v = \frac{\pi}{2}$

Phase difference between v and $a = \frac{\pi}{2}$

Phase difference between x and $a = \pi$

Calculation of Time period and amplitude

$$v_{\max} = A \omega$$

$$a_{\max} = A \omega^2$$

$$\omega = \frac{a_{\max}}{v_{\max}}$$

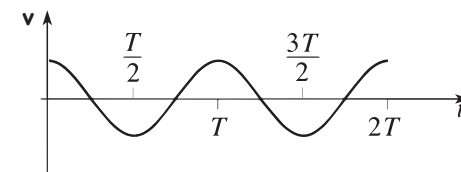
$$A = \frac{v_{\max}^2}{a_{\max}}$$

$$\frac{2\pi}{T} = \frac{a_{\max}}{v_{\max}}$$

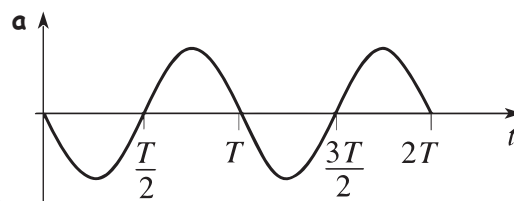
$$T = 2\pi \frac{v_{\max}}{a_{\max}}$$

Graphical Representation of Acceleration

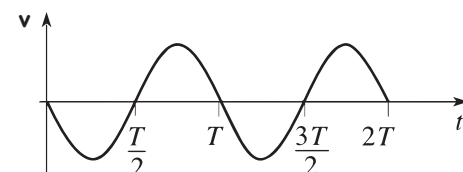
Start from mean position
 $v = A \omega \cos(\omega t + \Phi_0)$, $\Phi_0 = 0$



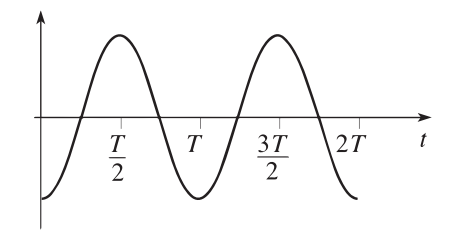
$$a = -A \omega^2 \sin(\omega t + \Phi_0)$$



Start from extreme position
 $v = -A \omega \sin(\omega t + \Phi_0)$, $\Phi_0 = 0$



$$a = -A \omega^2 \cos(\omega t + \Phi_0)$$



Energy of SHM

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 (A^2 - x^2)$$

$$KE=0 \quad KE_{\max} = \frac{1}{2} m \omega^2 A^2 \quad KE=0$$

$$x=-A \quad x=0 \quad x=A$$

$$P.E = \frac{1}{2} m \omega^2 x^2$$

$$PE_{\max} = \frac{1}{2} m \omega^2 A^2 \quad PE=0 \quad PE_{\max} = \frac{1}{2} m \omega^2 A^2$$

$$x=-A \quad x=0 \quad x=A$$

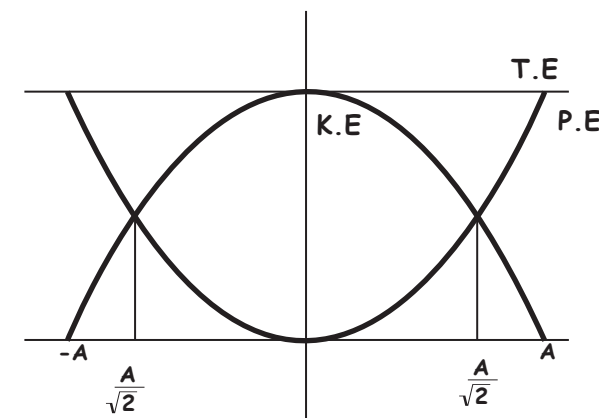
$$\text{Total mechanical energy} = \frac{1}{2} m \omega^2 A^2$$

$$1) x = \frac{A}{\sqrt{2}} \quad K.E = P.E = \frac{E}{2}$$

$$2) x = \frac{A}{2} \quad K.E = \frac{3E}{4} \quad P.E = \frac{E}{4}$$

$$3) x=0 \quad K.E = E \quad P.E = 0$$

$$4) x=A \quad K.E = 0 \quad P.E = E$$

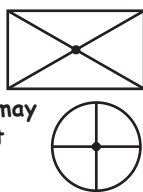


$$\text{Total mechanical energy}(E) = \frac{1}{2} m \omega^2 A^2$$

Note: In SHM, if particle oscillate with frequency ω , then the K.E & P.E oscillate with 2ω

Centre of Mass

- Avg. position of all the parts of the system, weighted according to their mass
- For homogeneous object, centre of mass lie at their geometric centres
- Centre of mass, may or may not lies inside the object
- Centre of mass, may change it's location



Centre of Mass For System of n Particles

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

For n particles

General Equation

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

In terms of Cartesian co-ordinates

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \dots + m_n z_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

Centre Of Mass For Non Point Mass

$$\vec{r}_{cm} = \frac{\int dm \cdot \vec{r}}{\int dm}$$

λ = mass per unit length
 σ = mass per unit area
 ρ = mass per unit volume

- Mass distributed over length $\Rightarrow dm = \lambda dl$
- Mass distributed over area $\Rightarrow dm = \sigma dA$
- Mass distributed over volume $\Rightarrow dm = \rho dV$

Centre of mass for various shapes

Uniformly distributed mass centre of mass

1) Rod

$$(0,0) \rightarrow \left(\frac{L}{2}, 0\right)$$

2) Square Lamina

$$\left(\frac{L}{2}, \frac{L}{2}\right)$$

3) Semicircular Ring

$$(0, \frac{2R}{\pi})$$

4) Hemispherical shell

$$(0, \frac{R}{2})$$

5) Solid circular cone

$$(0, \frac{h}{4})$$

6) Solid hemispherical

$$(0, \frac{3R}{8})$$

Motion centre of mass

velocity of centre of mass

$$\vec{V}_{cm} = \frac{M_1 \vec{V}_1 + M_2 \vec{V}_2 + M_3 \vec{V}_3 + \dots}{M_1 + M_2 + M_3 + \dots}$$

Acceleration of centre of mass

$$\vec{a}_{cm} = \frac{M_1 \vec{a}_1 + M_2 \vec{a}_2 + M_3 \vec{a}_3 + \dots}{M_1 + M_2 + M_3 + \dots}$$

Isolated System

- No external forces acting on the system
- They can have mutual force of attraction

$M_1 \vec{r}_1 = M_2 \vec{r}_2$ $a_{cm} = 0, v_{cm} = \text{constant}$

Moment of Inertia

$I = Mr^2$
 m = Mass of body
 r = Perpendicular distance from the axis of rotation

Moment of Inertia

Tensor Quantity $I = Mr^2$ Rotational analogous of mass

Two Point Mass

$$I_{COM} = M_1 r_1^2 + M_2 r_2^2$$

$$r_1 = \frac{m_2 r}{m_1 + m_2}, \quad r_2 = \frac{m_1 r}{m_1 + m_2}$$

$$I_{com} = m_{red} \cdot r^2, \quad m_{red} = \frac{m_1 m_2}{m_1 + m_2}$$

Factors Affecting Moment of Inertia

Mass of the body Axis of rotation Mass distribution

ROTATIONAL MOTION 01

Moment of Inertia

i) For discrete system of particles

$$I = \frac{M_1 r_1^2 + M_2 r_2^2 + M_3 r_3^2 + \dots + M_n r_n^2}{M_1 + M_2 + M_3 + \dots + M_n}$$

For n particles

$$I = \frac{M_1 r_1^2 + M_2 r_2^2 + M_3 r_3^2 + \dots + M_n r_n^2}{M_1 + M_2 + M_3 + \dots + M_n}$$

r_i = $\perp$ distance from axis of rotation

ii) For non-point mass

$$I = \int dI = \int r^2 dm = \int dm r^2$$

Parallel Axis Theorm

Conditions:-

- Axis where considered to be parallel to each other
- One of the axis must pass through centre of mass

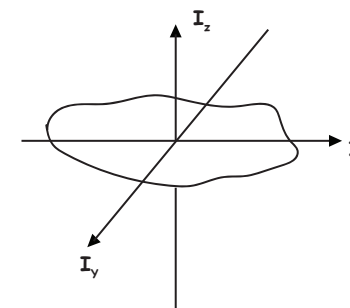
$$I_{parallel} = I_{COM} + Ma^2$$

Theorm of Perpendicular axis

$$I_z = I_x + I_y \quad (\text{Only valid for laminar bodies})$$

Note:-

- X and Y axis lie in the plane of body
- Z- axis $\perp$ need to the plane of the body
- Axis need not pass through center of mass



Moment Of Inertia For Various Objects

Disc

$$I = \frac{1}{2} MR^2$$

Hollow cylinder

$$I = MR^2$$

I = Moment of Inertia along the center of mass

Ring

$$I = MR^2$$

Solid cylinder

$$I = \frac{1}{2} MR^2$$

I_t = Moment of Inertia along the tangent perpendicular to the plane
 $I_t' =$ Moment of Inertia along the tangent parallel to the plane

Thin Rod

$$I = \frac{1}{12} ml^2$$

Hollow sphere

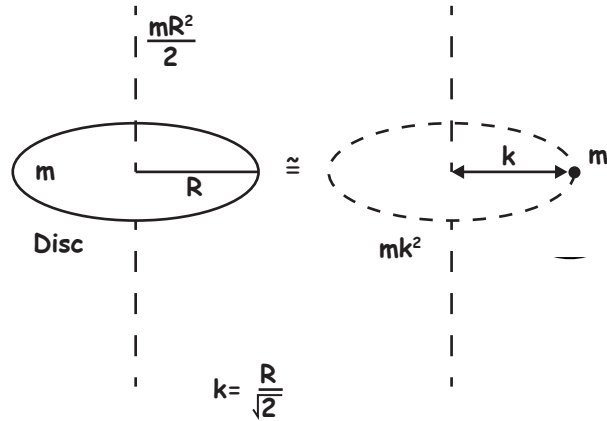
$$I_t = \frac{2}{3} MR^2$$

Solid sphere

$$I_t = \frac{7}{5} MR^2$$

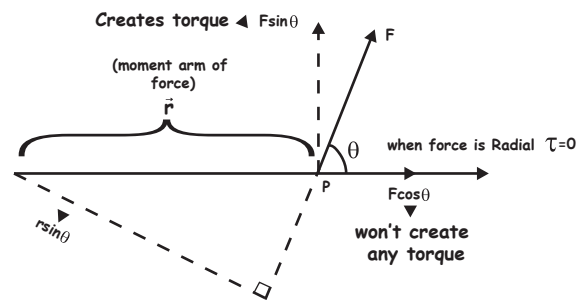
Radius of Gyration

Definition: The distance from the axis of a point mass whose mass is equal to the mass of whole body and whose moment of inertia is equal to moment of inertia of the body about the axis



k is the radius of gyration

Torque



Torque $\tau = r F \sin \theta$
 $= F \sin \theta \times r = F_{\perp} r$ (1)
 $= F \times r \sin \theta = F r_{\perp}$ (2)
 $\vec{\tau} = \vec{r} \times \vec{F}$ (Vector form)

If force is radial ie $\theta = 0^\circ$ or 180°
 Torque $\tau = 0$

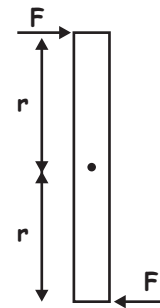
If force is tangential and $\perp$ to radius vector $\theta = 90^\circ$

Torque, $\tau = \tau_{\max} = rF$

Equilibrium

For translational equilibrium

$F_{\text{net}} = 0$
 τ_{net} may or maynot be zero



Rotational Equilibrium

$\tau_{\text{net}} = 0$
 F_{net} may or maynot be zero

Also called principle of moments



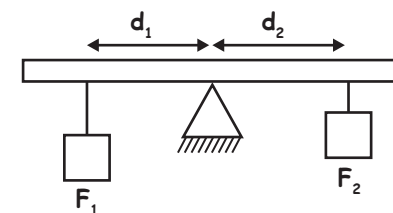
Static Equilibrium

Combination of both translational and rotational equilibrium

$F_{\text{net}} = 0 \Rightarrow$ Forces are balanced
 $\tau_{\text{net}} = 0 \Rightarrow \tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$

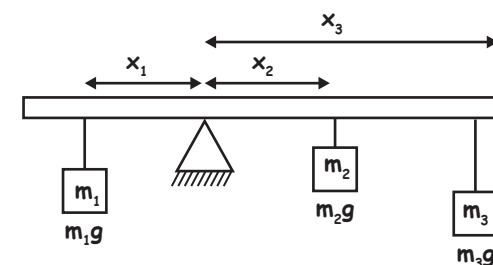
Principle of moments

When a body is in rotational equilibrium sum of clock wise moments about any point is equal to sum of anticlockwise moments about that point

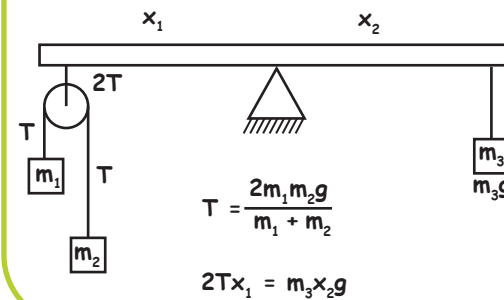


$F_1 \times d_1 = F_2 \times d_2$

Load $\times$ load arm = Effort $\times$ effort arm



$m_1 g x_1 = m_2 g x_2 + m_3 g x_3$



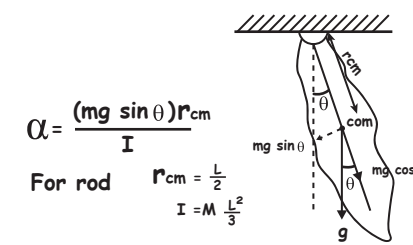
$T = \frac{2m_1 m_2 g}{m_1 + m_2}$
 $2T x_1 = m_3 x_2 g$

Angular acceleration

$\tau = I \alpha$
 τ - torque
 I moment of inertia
 α angular acceleration

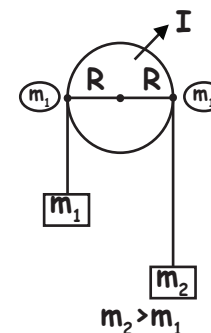
Initial angular acceleration when a rod is released

Initial angular acceleration when a body is released from an angle θ



Translatory - rotation combination

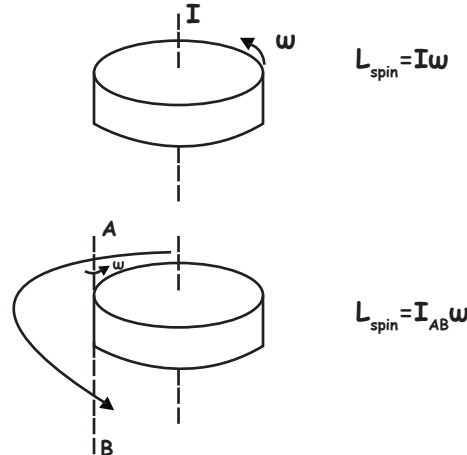
$\alpha = \frac{\tau_{\text{applied}} - \tau_{\text{opposition}}}{\text{Total } I}$
 $\alpha = \frac{m_2 R g - m_1 R g}{m_1 R^2 + m_2 R^2 + I}$



1) When $\theta = 90^\circ$

$|\vec{L}| = r p \sin \theta$
 $= r p$
 $= L_{\max}$

Spin angular momentum



Conservation of Angular momentum

If there is no external torque
 Angular momentum is conserved

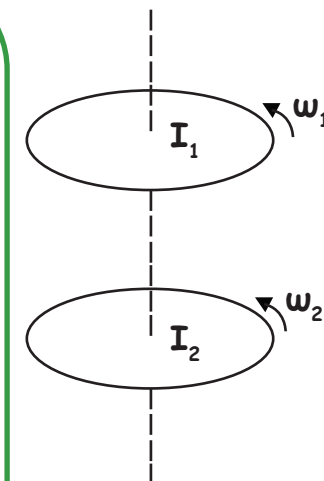
$\tau = \frac{dL}{dt}$
 If $\tau = 0 \Rightarrow \frac{dL}{dt} = 0$
 $L = \text{constant}$
 $I \omega = \text{constant}$

- $I_1 \omega_1 = I_2 \omega_2$
- If moment of inertia increases angular velocity decreases and if moment of inertia decreases angular velocity increases

Moment of inertia when two discs joined:

same direction:-

$\omega_f = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2}$



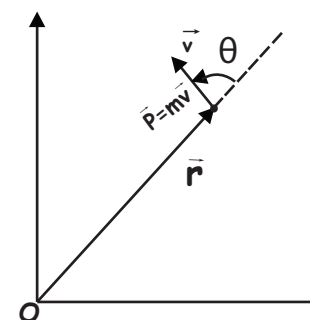
Angular momentum & its conservation

Angular momentum of a point mass:-

Angular momentum about origin
 $\vec{L} = \vec{r} \times \vec{p}$
 $= m(\vec{r} \times \vec{v})$
 $\vec{p} = m\vec{v}$

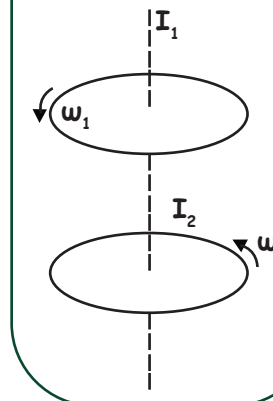
- When $\theta = 0^\circ$ or 180°
 $L_o = mvr \sin 180 = 0$
 $mvr \sin 0 = 0$

Angular momentum is minimum

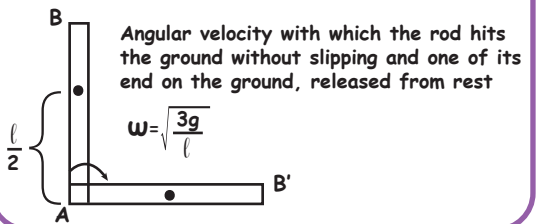


Opposite direction:-

$\omega_f = \frac{I_1 \omega_1 - I_2 \omega_2}{I_1 + I_2}$



Mechanical energy conservation



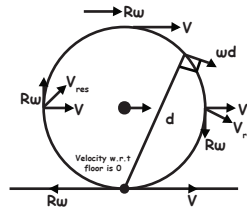
Rolling Motion

Translatory + Rotatory = Rolling

Velocity in rolling

- Condition for rolling without slipping:- $V = R\omega$
- Velocity of any point on rolling object, V_p
 $= \omega d = \frac{v d}{R}$

d is the distance from point content



Energy in rolling motion

1) Translatory Motion

$T_{K.E} = \frac{1}{2} m v^2$

2) Spinning motion/rotational motion

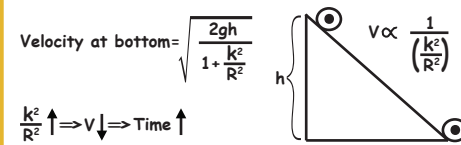
$R_{K.E} = \frac{1}{2} I \omega^2 = \frac{1}{2} m k^2 \frac{V^2}{R^2} = \frac{1}{2} m v^2 \times \left(\frac{k^2}{R^2} \right)$

3) Rolling motion

$T_{K.E} + R_{K.E} = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \times \frac{k^2}{R^2} = \frac{1}{2} m v^2 \left(1 + \frac{k^2}{R^2} \right)$

$\frac{k_{\text{total}}}{k_{\text{trans}}} = \left(1 + \frac{k^2}{R^2} \right)$

Motion on an inclined plane



Velocity: solid sphere > Disc > Hollow > Sphere > Ring

Time to reach bottom: Ring > Hollow sphere > Disc > solid sphere

Value of velocity:-

1) Ring/Hollow cylinder = $\sqrt{gh}$

2) Disc/Solid cylinder = $\sqrt{\frac{4}{3}gh}$

3) Hollow sphere = $\sqrt{\frac{6}{5}gh}$

4) Solid sphere = $\sqrt{\frac{10}{7}gh}$

Acceleration

$a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$ $a \propto \frac{1}{1 + \frac{k^2}{R^2}}$

Time of descend:-

$t = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g} \left(1 + \frac{k^2}{R^2} \right)}$

$t \propto \sqrt{1 + \frac{k^2}{R^2}}$

Ring > Hollow sphere > Disc > solid sphere

TEMPERATURE SCALE

Result $\rightarrow \frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5}$ (celcius-fahrenheit-kelvin conversion)

any scale conversion formula

$$\frac{\text{Reading on any scale} - \text{lower fixed point}}{\text{Upper fixed point} - \text{lower fixed point}} = \text{constant}$$

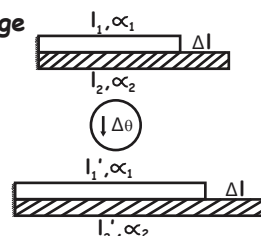
THERMAL EXPANSION

A-LINEAR

- $\Delta l = \alpha l \Delta \theta$
- $l' = l(1 + \alpha \Delta \theta)$
- $\alpha = \frac{\Delta l}{l \Delta \theta} \rightarrow \text{unit} \rightarrow /^{\circ}\text{C or } /^{\circ}\text{K, dimension-}K^{-1}$

Whatever be the change in temperature, the difference in length remains constant

$$l_1 \alpha_1 = l_2 \alpha_2$$



APPLICATIONS OF LINEAR EXPANSION

Pendulum clock

Fact $\rightarrow$ When temperature increases, time period increases, clock runs slow
 $\rightarrow$ When temperature decreases, time period decreases, clock runs fast

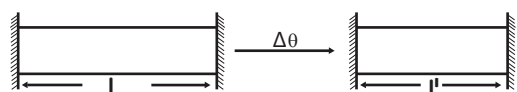
- Loss of time in any given time interval t ,

$$\Delta t = \frac{1}{2} \alpha \Delta \theta t$$

- Time lost by clock in a day

$$\Delta t = \frac{1}{2} \alpha \Delta \theta t = \frac{1}{2} \alpha \Delta \theta 86400 = 43200 \alpha \Delta \theta$$

Thermal Stress in a rigidly fixed rod



$$\text{Thermal Stress} = Y \alpha \Delta \theta$$

$$\text{Thermal Force} = Y A \alpha \Delta \theta$$

Y - Young's Modulus

α - coefficient of linear expansion

$\Delta \theta$ - temperature change

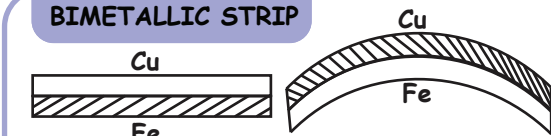
A - Area of rod

ERROR IN SCALE READING DUE TO EXPANSION OR CONTRACTION

Result $\rightarrow$ (1) At $\theta' > \theta$ True value $>$ Scale reading
 At $\theta' < \theta$ True value $<$ Scale reading

$$\text{True value} = \text{Scale reading} (1 + \alpha \Delta \theta)$$

BIMETALLIC STRIP



$\alpha_{Cu} > \alpha_{Fe} \rightarrow$ So when temperature increases
 $\rightarrow \Delta l \text{ of Cu} > \Delta l \text{ of Fe}$
 $\rightarrow$ strip with higher value of α will be on convex side

EXPANSION OF CAVITY

Area of hole increases $\rightarrow$ body expands on heating. Expansion of area of body is independent of shape and size of hole

SUPERFICIAL/AREA EXPANSION

- $\Delta A = A \beta \Delta \theta$
- $A' = A(1 + \beta \Delta \theta)$
- $\beta = \frac{\Delta A}{A \Delta \theta} \rightarrow \text{unit} \rightarrow /^{\circ}\text{C or } /^{\circ}\text{K, dimension-}[K^{-1}]$
- $\beta = 2\alpha$

CUBICAL EXPANSION/VOLUME EXPANSION

- $\Delta V = V \gamma \Delta \theta$
- $V' = V(1 + \gamma \Delta \theta)$
- $\gamma = \frac{\Delta V}{V \Delta \theta} \rightarrow \text{unit} \rightarrow /^{\circ}\text{C or } /^{\circ}\text{K, dimension-}[K^{-1}]$
- $\gamma = 3\alpha$

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

Variation of density with temperature

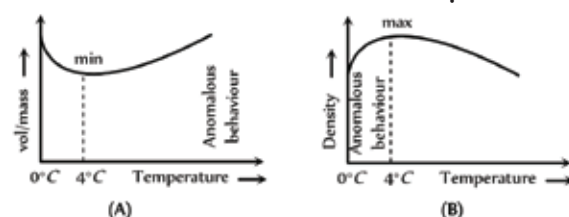
$$\text{Density} \propto \frac{1}{\text{Volume}}$$

$$V' = V(1 + \gamma \Delta \theta)$$

$$\text{then } \rho' = \rho(1 - \gamma \Delta \theta)$$

ANOMALOUS EXPANSION OF WATER

- Water has maximum density at 4°C (minimum volume)
- On heating, $0^{\circ}\text{C} \rightarrow 4^{\circ}\text{C}$ water contracts
- Graphs $4^{\circ}\text{C} \rightarrow$ above water expands



REAL AND APPARENT EXPANSION OF LIQUID

- Apparent Expansion of liquid $\rightarrow$ Real expansion of liquid - Expansion of solid in which liquid is contained
- Apparent change in volume

$$1. \Delta V_{\text{apparent}} = V_0 \gamma_{\text{apparent}} \Delta \theta$$

$$2. \Delta V_{\text{apparent}} = V_0 (\gamma_l - \gamma_s) \Delta \theta$$

$$3. \Delta V_{\text{apparent}} = V_0 (\gamma_l - 3\alpha_s) \Delta \theta$$

$$4. \gamma_{\text{apparent}} = \gamma_l - 3\alpha_s$$

γ_l - Real expansion of liquid

α_s - coefficient of linear expansion of solid

CALORIMETRY

$$1 \text{ calorie} = 4.2 \text{ J}$$

Heat Supplied (ΔQ)

change temperature of body

- $\Delta Q = ms \Delta T$
 S - specific heat capacity
 SI unit - $\frac{\text{Joule}}{\text{Kg Kelvin}} \rightarrow \text{J Kg}^{-1} \text{K}^{-1}$
- $S_{\text{water}} = 1 \frac{\text{cal}}{\text{g}^{\circ}\text{C}} = 4.2 \frac{\text{J}}{\text{g}^{\circ}\text{C}} = 4200 \frac{\text{J}}{\text{kg}^{\circ}\text{C}}$
 [or use Kelvin instead of $^{\circ}\text{C}$]
 $S_{\text{ice}} = \frac{1}{2} \frac{\text{cal}}{\text{g}^{\circ}\text{C}} = 2.1 \frac{\text{J}}{\text{g}^{\circ}\text{C}} = 2100 \frac{\text{J}}{\text{kg}^{\circ}\text{C}}$

change state of body

Melting
 $\Delta \theta : m L_f$
 L_f - Latent heat of fusion

Boiling
 $\Delta \theta : m L_v$
 L_v - Latent heat of vapourisation

- $L_f = L_{\text{ice}} = 80 \frac{\text{cal}}{\text{g}} = 80 \times 4.2 \frac{\text{J}}{\text{g}} = 80 \times 4200 \frac{\text{J}}{\text{kg}}$
- $L_v = L_{\text{steam}} = 540 \frac{\text{cal}}{\text{g}} = 540 \times 4.2 \frac{\text{J}}{\text{g}} = 540 \times 4200 \frac{\text{J}}{\text{kg}}$

HEAT CAPACITY

Heat capacity = mass $\times$ specific heat capacity

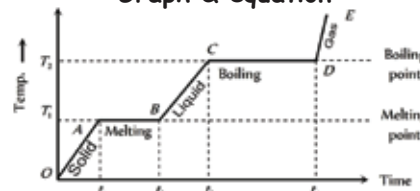
$$\text{Unit} = \frac{\text{cal}}{^{\circ}\text{C}} \Rightarrow \text{SI unit } \frac{\text{J}}{\text{K}}$$

WATER EQUIVALENT

The mass of water that will absorb or lose as same quantity of heat as substance for the same changes in temperature

$$m_w s_w = m_b s_b \quad \begin{matrix} w = \text{water} \\ b = \text{body} \end{matrix}$$

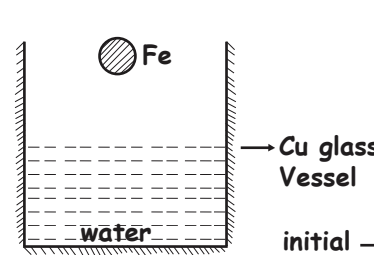
Heat supplied at constant rate
 Graph & equation



$$\frac{msT_1}{\Delta t_1} = \frac{mL_f}{\Delta t_{12}} = \frac{ms(T_2 - T_1)}{\Delta t_{23}} = \frac{mL_v}{\Delta t_{34}}$$

$$\text{if specific heat is variable } S = f(T) \quad T_1 \rightarrow T_2 \quad \Delta Q = \int_{T_1}^{T_2} msdT$$

PRINCIPLE OF CALORIMETRY



Heat lost by the hotter body = Heat gained by colder bodies

$$Q_3 = Q_1 + Q_2$$

Final equilibrium temperature,

$$T_{\text{eq}} = \frac{m_1 s_1 T_1 + m_2 s_2 T_2 + m_3 s_3 T_3}{m_1 s_1 + m_2 s_2 + m_3 s_3} = \frac{\sum msT}{\sum ms}$$

Facts :
 Calorimeter - A device in which the measurement of heat can be done.

ICE-WATER SYSTEM

Problem solving methodology

- $m_1 \text{ g ice } [-\theta_1, ^{\circ}\text{C}]$ mixed with $m_2 \text{ g water } [-\theta_2, ^{\circ}\text{C}]$
- Convert $-\theta_1, ^{\circ}\text{C}$ ice $\rightarrow 0^{\circ}\text{C}$ ice
 $\Delta Q_1 = m_1 s_{\text{ice}} \theta_1$
- Convert 0°C ice $\rightarrow 0^{\circ}\text{C}$ water
 $\Delta Q_2 = m_1 L_f$
- Convert $\theta_2, ^{\circ}\text{C}$ water $\rightarrow 0^{\circ}\text{C}$ water
 $\Delta Q_3 = m_2 s_{\text{water}} \theta_2$

check

$$\Delta Q_3 =, > \text{ or } < \Delta Q_1 + \Delta Q_2$$

- | | |
|---|---|
| $\Delta Q_3 > \Delta Q_1 + \Delta Q_2$ <ol style="list-style-type: none"> Whole ice melts into water Additional heat is used to increase the temperature of system from 0°C Final temperature can be found out by
 $\Delta Q' = M_{\text{total}} s_{\text{water}} T$
 where $\Delta Q' \rightarrow$ additional heat | $\Delta Q_3 < \Delta Q_1 + \Delta Q_2$ <ol style="list-style-type: none"> Only m' g of ice melts Mass of ice melts can be found by
 $m' L_f = Q$ Final temperature is 0°C |
|---|---|

CONVERSION OF MECHANICAL ENERGY TO HEAT ENERGY

- Potential energy to Heat energy

$$\Delta U = mgh \Rightarrow \Delta Q = m' L_f$$

When equating multiply with 4200 for ΔQ (if L_f is in $\frac{\text{calorie}}{\text{g}}$)
 ie, $mgh \Rightarrow m' L_f \times 4200$

- Kinetic energy to Heat energy

$$K.E. = \frac{1}{2} mv^2 \Rightarrow \Delta Q = m' L_f$$

if L_f is in $\frac{\text{calorie}}{\text{g}}$

$$\text{then } \frac{1}{2} mv^2 \Rightarrow m' L_f \times 4200$$

HEAT TRANSFER

- Conduction

Heat flows from hot end to cold end, medium is necessary, slow process

$$\frac{\Delta Q}{\Delta t} = K A \frac{\Delta \theta}{l}$$

$$\text{Unit of 'K'} = \frac{\text{watt}}{\text{metre}^{\circ}\text{C}} \text{ or } \frac{\text{watt}}{\text{metre K}}$$

'K' depends on the nature of material

$\frac{\Delta Q}{\Delta t}$ = Rate of flow of heat

A = Area of cross section

$\frac{\Delta \theta}{l}$ = Temperature gradient

K = coefficient of thermal conductivity

THERMAL PROPERTIES OF MATTER

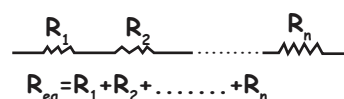
OHM'S LAW OF CONDUCTION

Electrical Conduction

- 1) current, $I = \frac{dq}{dt}$
- 2) $I = \frac{\Delta V}{R}$ ($\Delta V = V_{\text{high}} - V_{\text{low}}$)
- 3) electrical resistance, $R = \frac{\rho l}{A}$
- 4) $I = \frac{V_1 - V_2}{R} = \frac{(V_1 - V_2)A}{l\rho} = \sigma \frac{A}{l} (V_1 - V_2)$

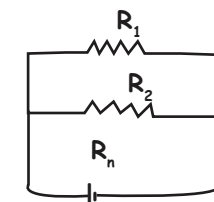
5) Combination of resistors

i) Series Combination



Here 'I' is same

ii) Parallel Combination



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

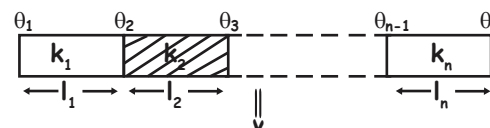
Here $(V_1 - V_2)$ is same

Thermal Conduction

- 1) Heat current, $H = \frac{dQ}{dt}$
- 2) $H = \frac{\theta_1 - \theta_2}{R} = \frac{\Delta\theta}{R}$
- 3) Thermal resistance, $R = \frac{l}{KA}$
- 4) $H = \frac{\theta_1 - \theta_2}{R} = \frac{\theta_1 - \theta_2}{(l/K_A)} = \frac{KA}{l} (\theta_1 - \theta_2)$

5) Combination of conductors

i) Series Combination



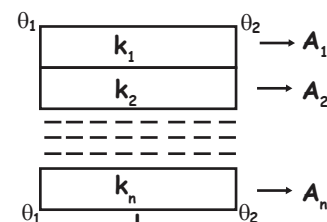
replace with resistors

$$R_1 = \frac{l_1}{K_1 A}, \quad R_2 = \frac{l_2}{K_2 A} \quad \text{Find } R_{eq} = R_1 + R_2 + \dots$$

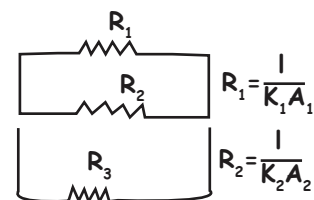
From that find ' K_{eq} '

Here 'H' is same

ii) Parallel Combination



replace to resistors



$$\text{Find } \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

from that find K_{eq}

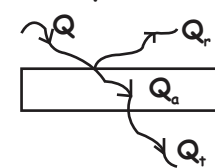
Here, Temp Difference same

CONVECTION

- Requires a medium, actual movement of fluid, occurs naturally or forced.
- Natural convection takes place due to the effect of gravity
- Sea Breeze: Wind blows from sea to land during day time
- Land Breeze: Wind blows from land to sea during night

RADIATION

Absorptive, reflective and Transmitted power



$$\text{Absorptive power}(a) = \frac{Q_a}{Q} = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

$$\text{Reflective power}(r) = \frac{Q_r}{Q} = \frac{\text{Energy reflected}}{\text{Energy incident}}$$

$$\text{Transmitted power}(t) = \frac{Q_t}{Q} = \frac{\text{Energy transmitted}}{\text{Energy incident}}$$

EMISSIVE POWER/INTENSITY OF THERMAL RADIATION

$$\text{Emissive power}(E) = \frac{\text{Energy radiated}}{\text{area} \times \text{time}} \quad \text{unit} \rightarrow \frac{\text{Watt}}{\text{m}^2}$$

$$\text{Spectral emissive power}(E_\lambda) = \frac{\text{Energy radiated}}{\text{area} \times \text{time} \times \text{wavelength}} \quad \text{unit} \rightarrow \frac{\text{Watt}}{\text{m}^3}$$

$$\text{Relation between } E \text{ \& } E_\lambda \implies E = \int_0^\infty E_\lambda d\lambda$$

EMISSIVITY (E)

$$e = \frac{\text{Energy radiated by a general body}}{\text{Energy radiated by a black body}}$$

value of $e \implies 0 < e < 1$

If $e=0$, means general body radiates no energy

If $e=1$, it indicates a perfect black body

KIRCHHOFF'S LAW

Ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

$$\frac{E_1}{a_1} = \frac{E_2}{a_2} = \dots = \frac{E}{A} = E$$

STEFAN'S LAW

Emissive power of a black body $\propto$ fourth power of absolute temperature

$$E = \sigma T^4, \quad \frac{\Delta Q}{\Delta t} = \sigma AT^4$$

$\sigma \rightarrow$ Stefan's constant $\frac{\Delta Q}{\Delta t} \rightarrow$ Radiant power

value of $\sigma \rightarrow 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

Dimension $\rightarrow [\sigma] = \text{MT}^{-3}\text{K}^{-4}$

For ordinary body $E = e\sigma T^4$

$$\frac{\Delta Q}{\Delta t} = eA\sigma T^4 \quad e = \text{emissivity}$$

In the presence of a surrounding (T_0) (black body)

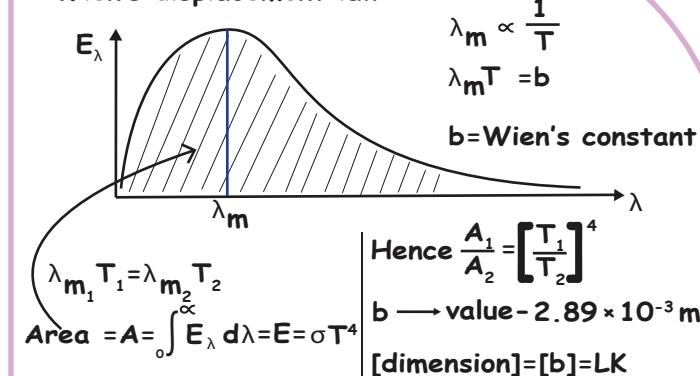
$$E = \sigma (T^4 - T_0^4) \quad \frac{\Delta Q}{\Delta t} = \sigma A (T^4 - T_0^4)$$

In the presence of a surrounding T_0 (general body)

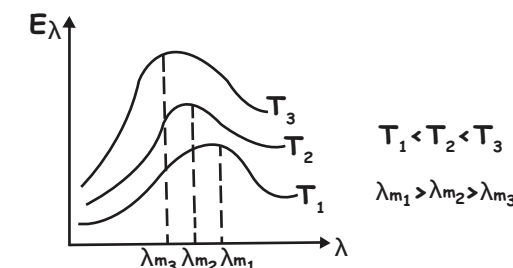
$$E = \sigma e (T^4 - T_0^4) \quad \frac{\Delta Q}{\Delta t} = \sigma e A (T^4 - T_0^4)$$

WIEN'S LAW

Wien's displacement law



"As the temperature of the body increases, the wavelength at which the spectral intensity (E) is maximum shift towards left."



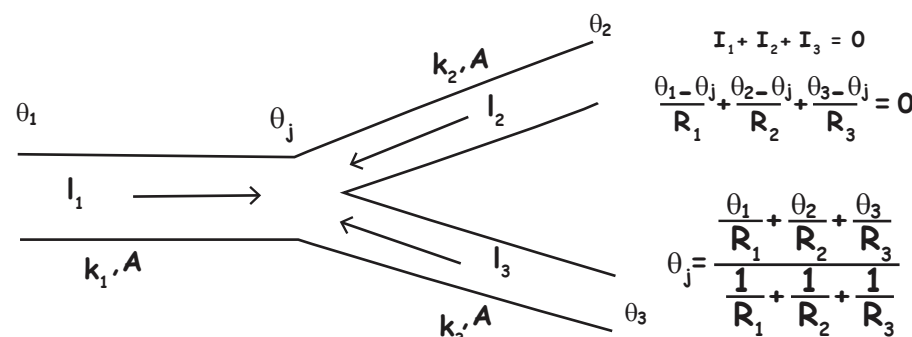
NEWTON'S LAW OF COOLING

Rate of cooling directly proportional to excess of temperature of the body over that of surrounding.

$$\frac{-dT}{dt} \propto (T - T_0)$$

$T =$ Temperature of body
 $T_0 =$ Temperature of surrounding

TEMPERATURE OF INTERMEDIATE JUNCTION

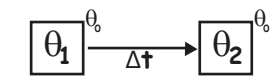


THERMAL PROPERTIES OF MATTER

NEWTON'S LAW OF COOLING

$\rightarrow$ EQUATION FOR PROBLEM SOLVING

$$\frac{-[\theta_2 - \theta_1]}{\Delta t} = K \left[\left(\frac{\theta_2 + \theta_1}{2} \right) - \theta_0 \right]$$



$\theta_1 > \theta_2$

$\Delta t = \text{time}$

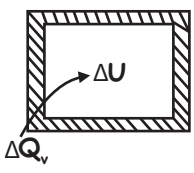
$\theta_0 \rightarrow$ surrounding temperature

Internal Energy(U)

$$(1) \Delta U = \Delta Q_v = nC_v \Delta T = n \frac{f}{2} R \Delta T$$

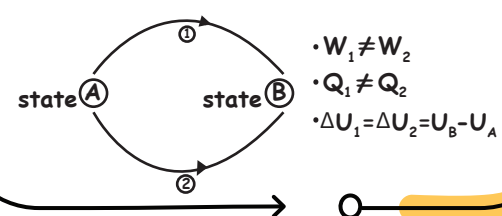
$$= \frac{nR \Delta T}{\gamma - 1} = \frac{\Delta(PV)}{\gamma - 1}$$

$$= \frac{P_f V_f - P_i V_i}{\gamma - 1}$$



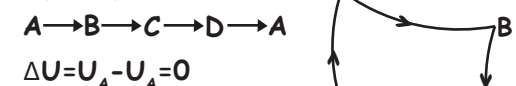
(Internal Energy is a function of temperature)

- First law of T.D $\Rightarrow \Delta Q = \Delta U + \Delta W$
- $\Delta Q, \Delta W \Rightarrow$ path functions
- $\Delta U \Rightarrow$ state function



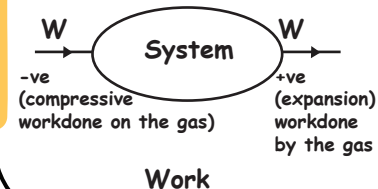
Isothermal process $\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$

Cyclic process



Work done:
path function
 $\Delta W = \int P dv$
• 1 cal = 4.2 Joule

Sign Convention



Heat: path function
 $\Rightarrow$ unit: calorie/Joule

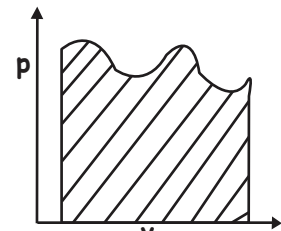


Adiabatic process $\Rightarrow \Delta Q = 0$ [No heat transfer]

At constant volume $\Rightarrow \Delta Q_v = \Delta U = nC_v \Delta T$

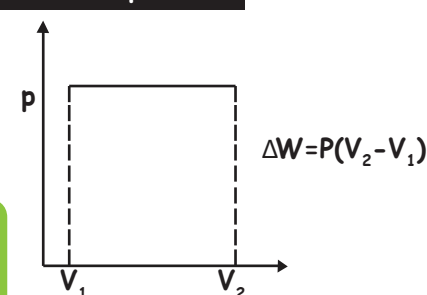
At constant pressure $\Rightarrow \Delta Q_p = \Delta U + \Delta W = nC_p \Delta T$

Work done from P-V Graph

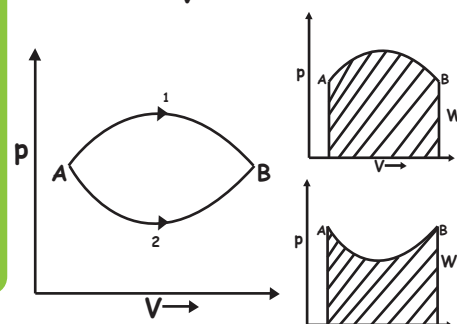
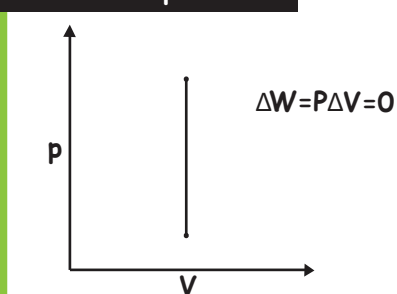


Area under P-V diagram gives work done by the gas

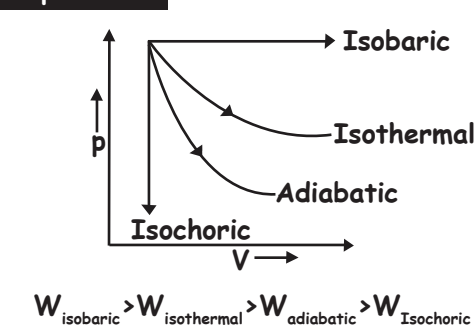
Isoobaric process



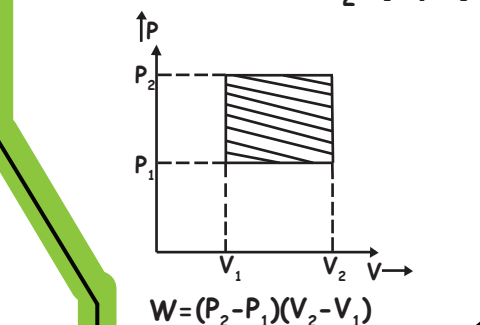
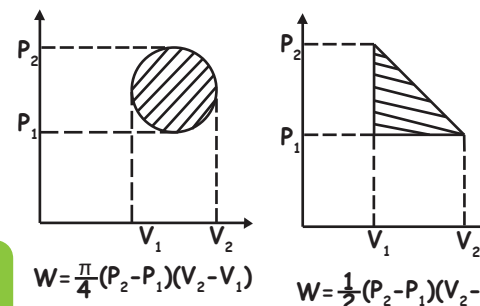
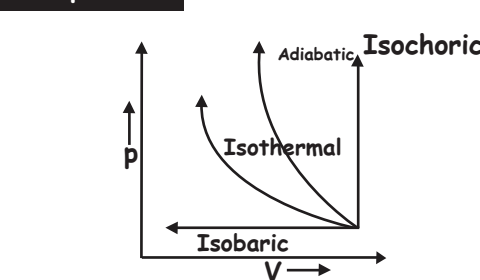
Isochoric process



Expansion



Compression



Thermodynamic process

01 $\llcorner$

Adiabatic process

• $\Delta Q = 0$ [no exchange of heat]

• Rapid or spontaneous process/insulated vessel

• FLOT $\Rightarrow \Delta Q = \Delta U + \Delta W$

$\Delta Q = 0 \quad \Delta U = -\Delta W$

Compression

$\Delta W = -ve \quad \Delta U = +ve$

$\Delta U \uparrow \Rightarrow$ Temperature $\uparrow$
 $\Rightarrow$ Pressure $\uparrow$

Expansion

$\Delta W = +ve \quad \Delta U = -ve$

$\Delta U \downarrow \Rightarrow$ Temperature $\downarrow$
 $\Rightarrow$ Pressure $\downarrow$

Equation of state

$PV^\gamma = a \text{ constant}$

$TV^{\gamma-1} = a \text{ constant}$

$$P \propto T^{\frac{\gamma}{\gamma-1}}$$

Workdone by the gas

$$\Delta W = -\Delta U = nC_v(T_1 - T_2)$$

$$= n \frac{f}{2} R(T_1 - T_2)$$

$$\Delta W = \frac{nR}{\gamma-1}(T_1 - T_2)$$

$$\Delta W = \frac{P_1 V_1 - P_2 V_2}{\gamma-1}$$

Slope of adiabatic process
= γ slope of isothermal process
specific heat of gas $\Rightarrow C = 0$

$$C = \frac{\Delta Q}{\Delta T} \rightarrow \Delta Q = 0 \quad C = 0$$

Isothermal process

$\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$

eg:- perfectly conducting slow process

FLOT $\Rightarrow \Delta Q = \Delta U + \Delta W$

$$\Delta Q = \Delta W$$

equation of states $\Rightarrow P_1 V_1 = P_2 V_2$

Workdone by the gas

$$W = 2.303 nRT \log \left(\frac{V_2}{V_1} \right)$$

$$W = 2.303 nRT \log \left(\frac{P_1}{P_2} \right)$$

Slope $\Rightarrow$

Slope of adiabatic process

= γ slope of isothermal process

specific heat $C = \infty$

03 $\llcorner$

Isoobaric process

$\Delta P = 0$

FLOT $\Rightarrow \Delta Q = \Delta U + \Delta W$

equation of states $\Rightarrow V \propto T$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Workdone by the gas

$$\Delta W = P\Delta V = P(V_2 - V_1) = nR(T_2 - T_1)$$

specific heat of gas

$$\Rightarrow C_p = \left(1 + \frac{f}{2} \right) R = \frac{\gamma R}{\gamma - 1}$$

04 $\llcorner$

Isochoric process

$\Delta V = 0$ or $V = \text{constant}$

equation of states $\Rightarrow P \propto T \quad \frac{P_1}{P_2} = \frac{T_1}{T_2}$

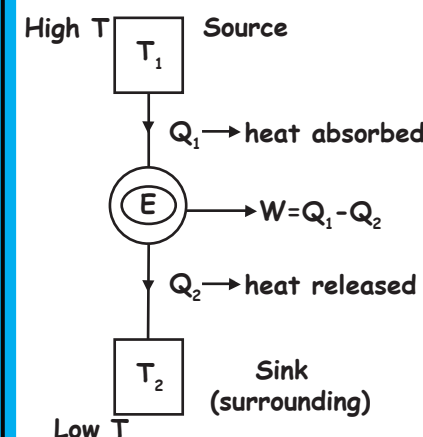
Workdone by the gas

$\Delta V = 0 \quad \Delta W = 0$

specific heat of gas $\Rightarrow C_v = \frac{f}{2} R = \frac{R}{\gamma - 1}$

Heat Engine

'Devices that convert heat into work'



efficiency(η)

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

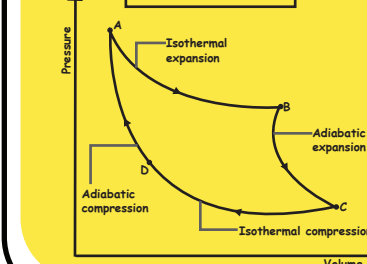
$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

$\eta_{\text{max}} \Rightarrow$ Where $Q_2 = 0$ or $T_2 = 0K$ (not possible)

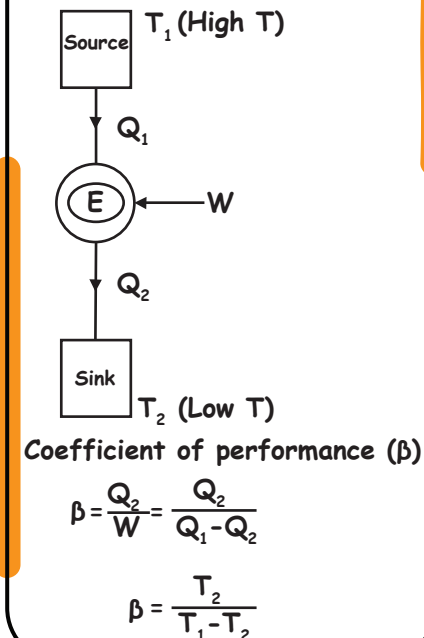
Carnot Engine

$\rightarrow$ Ideal engine

$$\eta = \frac{T_1 - T_2}{T_1}$$



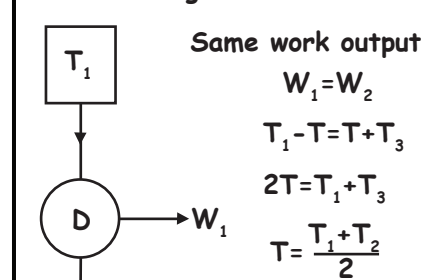
Refrigerator



Relationship between η & β

$$\beta = \frac{1 - \eta}{\eta}$$

Cascaded engine



Same efficiency

$$1 - \frac{T_3}{T_1} = 1 - \frac{T_3}{T_2} \cdot \frac{T_2}{T_1}$$

$$T_2^2 = T_1 T_3$$

$$T_2 = \sqrt{T_1 T_3}$$

THERMODYNAMICS

UNITS & MEASUREMENTS

MEASUREMENT OF MASS & TIME

MASS

• Unified atomic mass unit (amu) is used to measure mass of atoms & molecules

• $1\text{amu} = (1/12)^{\text{th}}$ mass of one ^{12}C atom

• $1\text{amu} = 1.66 \times 10^{-27} \text{ kg}$

• Electron mass - 10^{-30} kg

• Earth mass : 10^{25} kg

• Observable Universe 10^{55} kg

TIME

• SI unit is second (based on caesium clock with an uncertainty less than 1 part in 10^{-13} ie, 3µs loss every year)

• Timespan of unstable particle: 10^{-24} s

• Age of universe: 10^{17} s

MEASUREMENT OF LENGTH

• Large distance is measured by parallax method

• Parallax angle = $\frac{\text{BASIS}}{\text{DISTANCE}} = \frac{b}{x}$

• $1^\circ = 1.745 \times 10^{-2} \text{ rad}$

• $1' = 2.91 \times 10^{-4} \text{ rad}$

• $1'' = 4.85 \times 10^{-6} \text{ rad}$

• For very small sizes, optical microscope, tunneling microscope, electron microscope were used.

• $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$

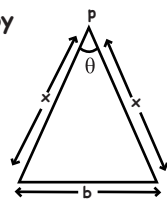
• $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$

• $1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$

• Size of proton: 10^{-15} m

• Radius Of Earth: 10^7 m

• Distance to Boundary Of Observable Universe : 10^{26} m

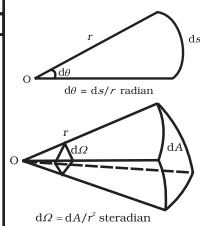


SI SYSTEM

7 Base units and 2 supplementary units

NO.	Base Units		
	Quantity	Unit	Symbol
1	Length	meter	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol

NO.	Supplementary Units		
	Quantity	Unit	Symbol
1	Plane angle	radian	rad
2	Solid angle	steradian	sr



Q. Unit of permittivity of free space ϵ_0 is

- (a) coulomb/newton-metre
(b) newton-metre² /coulomb²
(c) coulomb²/newton-metre²
(d) coulomb²/(newton-metre)²

SIGNIFICANT FIGURES

The digits in a measured quantity which are reliable and confidence in our measurement + the digit which is uncertain.

RULES FOR SIGNIFICANT FIGURES

1. All non-zero digits are significant. For example, 42.3 has three significant figures; 243.4 has four significant figures; and 24.123 has five significant figures.

2. A zero becomes significant figure if it appears between two non-zero digits. For example, 5.03 has three significant figures; 5.604 has four significant figures; and 4.004 has four significant figures.

3. Leading zeros or the zeros placed to the left of the number are never significant. For example, 0.543 has three significant figures; 0.045 has two significant figures; and 0.006 has one significant figure.

4. Trailing zeros or the zeros placed to the right of the number are significant. For example, 4.330 has four significant figures; 433.00 has five significant figures; and 343.000 has six significant figures.

5. In exponential notation, the numerical portion gives the number of significant figures. For example, 1.32×10^{-2} has three significant figures and 1.32×10^4 has three significant figures.

RULES FOR ROUNDING OF A MEASUREMENT

1. If the digit to be dropped is less than 5, then the preceding digit is left unchanged. For example, $x = 7.82$ is rounded off to 7.8 and $x = 3.94$ is rounded off to 3.9.

2. If the digit to be dropped is more than 5, then the preceding digit is raised by one. For example, $x = 6.87$ is rounded off to 6.9 and $x = 12.78$ is rounded off to 12.8.

3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one. For example, $x = 16.351$ is rounded off to 16.4 and $x = 6.758$ is rounded off to 6.8.

4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is even, is left unchanged. For example, $x = 3.250$ becomes 3.2 on rounding off and $x = 12.650$ becomes 12.6 on rounding off.

5. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is odd, is raised by one. For example, $x = 3.750$ is rounded off to 3.8, again $x = 16.150$ is rounded off to 16.2.

RULES FOR ROUNDING OF A MEASUREMENT

ADDITION & SUBTRACTION

In addition or subtraction, the final result should be reported to the same number of decimal Places as that of the original number with minimum number of decimal places

$$\begin{array}{r} 3.1421 \\ 0.241 \\ +0.09 \\ \hline 3.4731 \end{array}$$

← (has two decimal places)
← (Answer should be reported to two decimal places after rounding off)

Answer = 3.47

MULTIPLICATION & DIVISION

When numbers are multiplied or divided, the number of significant figures in the answer equals the smallest number of significant figures in any of the original numbers

$$\begin{array}{r} 51.028 \\ \times 1.31 \\ \hline 66.84668 \end{array}$$

← (Three significant figures)
← (Answer should have three significant figures after rounding off)

Answer = 66.8

If $L = 2.331 \text{ cm}$, $B = 2.1 \text{ cm}$, then $L+B = ?$

- (a) 4.431 cm (b) 4.43 cm
(c) 4.4 cm (d) 4 cm

Dimensional Analysis

Dimensions of a physical quantity are power to which units of base quantity are raised. Eg: $[M]^a [L]^b [T]^c [A]^d [K]^e$

APPLICATIONS

checking the correctness of various formulae
Eg: If $Z = A+B$, $[Z] = [A] = [B]$

conversion of one system of unit into another

$n_1 u_1 = n_2 u_2$
Eg: $n_1 [M_1^A L_1^B T_1^C] = n_2 [M_2^A L_2^B T_2^C]$

$n_1 = n_2 \left[\frac{M_2}{M_1} \right]^A \left[\frac{L_2}{L_1} \right]^B \left[\frac{T_2}{T_1} \right]^C$

Deducing relation among physical quantity

DIMENSIONAL FORMULA

- 1) Pressure = stress = Young's modulus = $ML^{-1} T^{-2}$
- 2) Work = Energy = Torque = $M L^2 T^{-2}$
- 3) Power $P = M L^2 T^{-3}$
- 4) Gravitational constant $G = M^{-1} L^3 T^{-2}$
- 5) Force constant = Spring constant = $M T^{-2}$
- 6) Coefficient of viscosity = $M L^{-1} T^{-1}$
- 7) Latent heat $L = L^2 T^{-2}$
- 8) Electric potential = $\frac{P}{I} = M L^2 T^{-3} A^{-1}$
- 9) Resistance = $\frac{V_0}{I_0} = M L^2 T^{-3} A^{-2}$
- 10) Capacitance = $M^{-1} L^{-2} T^{-4} A^2$
- 11) Permittivity $\epsilon_0 = M^{-1} L^{-3} T^4 A^2$
- 12) Angular momentum = planck's constant = $M^1 L^2 T^{-1}$

$$\text{Time period } \frac{T}{R} = RC = \sqrt{LC}$$

DIMENSIONLESS QUANTITIES

- 1) Strain
- 2) Refractive index
- 3) Relative density
- 4) Plane angle
- 5) Solid angle

In SI Units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is:

- a) $A^{-1} T M L^3$ b) $A T^2 M^{-1} L^{-1}$ c) $A T^{-3} M L^{3/2}$ d) $A^2 T^3 M^{-1} L^{-2}$

INSTRUMENTS

Least Count:
Smallest quantity an instrument can measure

mm scale vernier scale screw gauge
1mm 0.1mm 0.01mm

VERNIER CALIPERS

Least Count = 1 MSD - 1 VSD

If n VSD Coincides with (n-1) MSD, then (n-1) MSD = n VSD
∴ 1VSD = $\frac{n-1}{n}$ MSD

Least Count = 1MSD - $\frac{n-1}{n}$ MSD = $\frac{1}{n}$ MSD

Total Reading = Main Scale Reading + coinciding Vernier Scale division $\times$ least count

In a vernier calipers, one main scale division is x cm & n division of vernier scale coincide with n-1 divisions of the main scale. the least count (in cm) of the calipers is.

- a) $\left(\frac{n-1}{n}\right)x$ b) $\frac{nx}{(n-1)}$ c) $\frac{x}{(n-1)}$ d) $\frac{x}{n}$

SCREW GAUGE

Pitch = $\frac{\text{Main Scale Reading}}{\text{No. of rotations}}$

Least Count = $\frac{\text{pitch}}{\text{Total no. of divisions on circular scale}}$

Total Reading = Linear Scale Reading + circular scale reading $\times$ least count

The least count of the main scale of a screw gauge is 1mm the minimum no. of divisions on its circular scale required to measure 5µm diameter of wire is

- a) 200 b) 50 c) 400 d) 100

ERRORS IN MEASUREMENT

Difference between true value & measured value of a quantity

Systematic Errors

Errors which tend to occur only in one direction, either positive or negative

Random Errors

Irregular and at random in magnitude & direction

Instrumental
Due to inbuilt defect of measuring instrument

Experimental
Limitations in experimental technique

Personal
Due to individual bias, Lack of proper setting of apparatus

• Least count error is the smallest value that can be measured by instrument (occurs with random & systematic errors)

• Absolute Error :- $\Delta a = |a - a_{\text{mean}}|$, $a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$

• Relative Error :- $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$; $\Delta a_{\text{mean}} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \dots + \Delta a_n}{n}$

• Percentage Error :- $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100$

COMBINATION OF ERRORS

Operations	Formula Z	Absolute error ΔZ	Relative error $\Delta Z/Z$	Percentage error $100 \times \Delta Z/Z$
Sum	$A+B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A+B}$	$\frac{\Delta A + \Delta B}{A+B} \times 100$
Difference	$A-B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A-B}$	$\frac{\Delta A + \Delta B}{A-B} \times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A + A\Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Power	A^n	$n A^{n-1} \Delta A$	$n \frac{\Delta A}{A}$	$n \frac{\Delta A}{A} \times 100$
Root	$A^{1/n}$	$\frac{1}{n} A^{1/n-1} \Delta A$	$\frac{1}{n} \frac{\Delta A}{A}$	$\frac{1}{n} \frac{\Delta A}{A} \times 100$

General rule:

If $Z = \frac{A^p B^q}{C^r}$, Then the maximum fractional relative error in Z will be:

$$\frac{\Delta Z}{Z} = p \frac{\Delta A}{A} + q \frac{\Delta B}{B} + r \frac{\Delta C}{C}$$

In an expirement four quantities a,b,c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows:

$$P = \frac{a^2 b^2}{cd} \text{ then percentage error in P is}$$

- (a) 14% (b) 10%
(c) 7% (d) 4%

Waves

wave is a disturbance which propagates energy and momentum from one place to another without the transport of medium.

⇒ The medium should have elasticity, inertia and uniform density

Characteristics of Wave

⇒ The particles of the medium are executing simple harmonic motion.

⇒ The phase of vibration of the particle keep on changing.

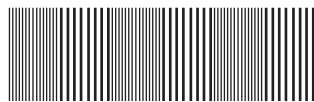
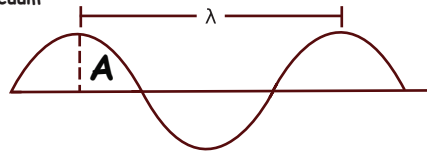
⇒ Wave carries energy and momentum.

⇒ The velocity of the particle is not equal to velocity of wave.

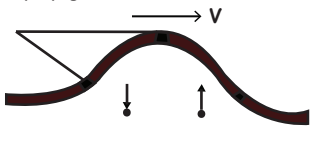
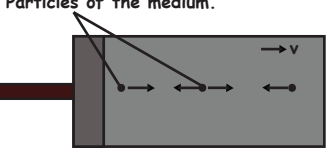
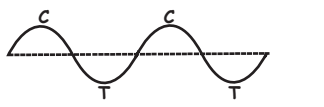
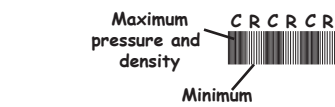
Classification of waves

According to

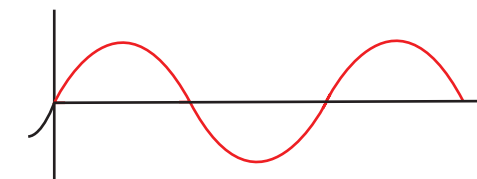
Necessity of medium	Energy propagation	Dimension	Vibration of particles
Mechanical waves	Progressive waves	One Dimensional	Transverse waves
Non-mechanical waves(EMW)	Stationary waves	Two Dimensional	Longitudinal waves
can travel in vacuum		Three Dimensional	



Classification of waves based on vibration of particles

Transverse waves	Longitudinal waves
Particles of the medium vibrates in a direction perpendicular to the direction of propagation of wave.	Particles of the medium vibrates in the direction of wave motion.
	
Transverse wave on a string	Longitudinal wave in a fluid
It travels in the form of crests (C) and troughs (T).	It travels in the form of compression (C) and rarefaction (R).
	
Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted through liquids and gases.	These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.
	
Transverse-wave in a rod	Longitudinal wave in a rod
Medium should posses the property of rigidity	Medium should posses the property of elasticity
Transverse waves can be polarised	Longitudinal waves can not be polarised
Movement of string of a sitar or violin, movement of the membrane of a tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.	Sound waves travel through air, vibration of air column in organ pipes vibration of air column above the surface of water in the tube of resonance apparatus.

All travelling wave satisfy a differential equation called wave equation.



$$\frac{d^2y}{dx^2} = \frac{1}{v^2} \frac{d^2y}{dt^2}$$

$$\frac{dy}{dt} = -\frac{\omega}{k} \times \frac{dy}{dx}$$

Velocity of particle = -Velocity wave × Slope of the graph

Equation of progressive wave

(i) $y = A \sin(\omega t - kx)$

(ii) $y = A \sin(\omega t - \frac{2\pi}{\lambda}x)$

(iii) $y = A \sin 2\pi [\frac{t}{T} - \frac{x}{\lambda}]$

(iv) $y = A \sin \frac{2\pi}{\lambda} (vt - x)$

(v) $y = A \sin \omega (t - \frac{x}{v})$

The general equation of a plane progressive wave with initial phase is

$$y(x, t) = a \sin(\omega t \pm kx \pm \phi_0)$$

Labels in the diagram:

- Displacement: $y(x, t)$
- Amplitude: a
- Oscillating term: $\sin(\omega t \pm kx \pm \phi_0)$
- Phase: ϕ_0
- Initial phase: ϕ_0
- Direction: $\pm$
- Propagation: x
- Angular frequency: ω

Important Terms

A Amplitude	v Frequency
λ wave length	T Time period
ω Angular frequency	K Wave Constant
$\omega = \frac{2\pi}{T}$ or $2\pi\nu$	$K = \frac{2\pi}{\lambda}$

WAVES 1

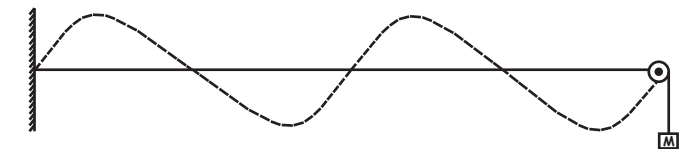
Rate of Energy Transmission

$$\frac{dK}{dt} = \frac{dU}{dt} = \frac{1}{4} \mu V \omega^2 A^2$$

Rate of an power Transmission

$$\frac{1}{2} \mu V \omega^2 A^2$$

Velocity of Transmission wave in a string.



$V = \sqrt{\frac{T}{\mu}}$

$T = Mg$

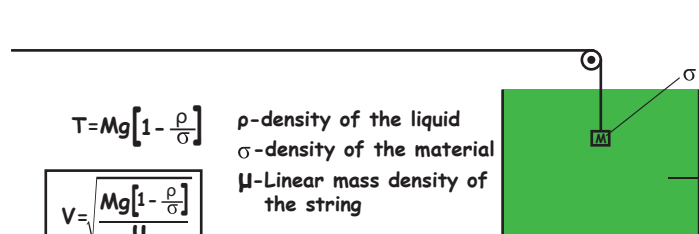
$\mu = \frac{m}{l}$

$\mu = \rho A$

M-Mass of block
m-mass of string
ρ-density of material of string

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} \times \frac{r_2}{r_1}$

Case 1

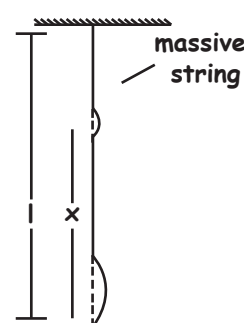


$T = Mg [1 - \frac{\rho}{\sigma}]$

ρ-density of the liquid
σ-density of the material
μ-Linear mass density of the string

$V = \sqrt{\frac{Mg[1 - \frac{\rho}{\sigma}]}{\mu}}$

Case 2



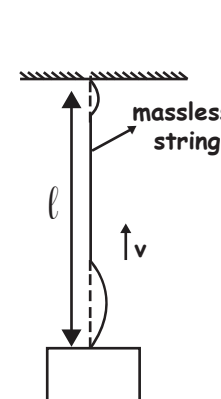
Velocity at any Point

$$V = \sqrt{gx}$$

Time taken to reach the pulse at Top

$$T = 2\sqrt{\frac{l}{g}}$$

Case 3



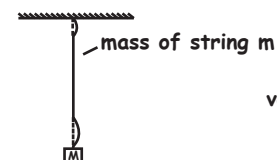
Time taken to reach the pulse at top

Velocity, $v = \sqrt{\frac{T}{\mu}}$

$$\text{Time taken, } t = \frac{l}{v} = l \sqrt{\frac{\mu}{T}}$$

T= Tension in the string
μ= Linear mass density

Case 4



velocity at bottom

$$V_1 = \sqrt{\frac{Mg}{\mu}}$$

velocity at Top

$$V_2 = \sqrt{\frac{(M+m)g}{\mu}}$$

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{M}{M+m}}$$

Velocity of Longitudual Wave

$$V = \sqrt{\frac{E}{\rho}} \quad (E = \text{Elasticity of the medium; } \rho = \text{density of the medium})$$

(1) As solids are most elastic while gases least i.e. $E_s > E_L > E_g$. So the velocity of sound is maximum in solids and minimum in gases

Velocity Sound (air)

$$V = \sqrt{\frac{P}{\rho}} \quad \text{Newton}$$

$$V = \sqrt{\frac{\gamma P}{\rho}} \quad \text{Laplace}$$

$$v = \frac{C_p}{C_v}$$

Mono atomic	$\gamma = 5/3$
diatomic	$\gamma = 7/5$

Factors affecting velocity of sound

→ Pressure

• Velocity of sound in air is independent of pressure

→ Density

$$V \propto \frac{1}{\sqrt{\rho}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

→ Temp

$$V = \sqrt{\frac{RT}{m}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

Temp Coefficient(α)

Increase in velocity of sound for 1°C or 1K rise in temperature of gas

$$\text{Value of } \alpha = 0.608 \frac{\text{m/s}}{^\circ\text{C}} = 0.61$$

Humidity

Humidity ↑

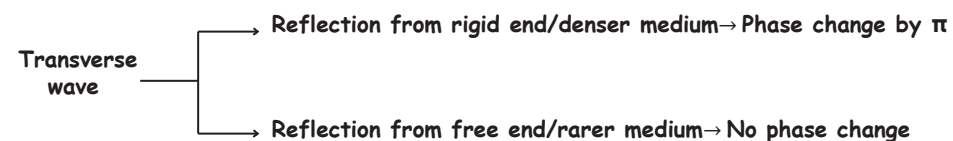
Speed of sound ↑

Sound travels faster in moist air than in dry air

Relation between Δx and ΔΦ

$$\Delta\Phi = \frac{2\pi}{\lambda} \Delta x$$

Reflection of Mechanical waves



Principle of superposition

The displacement at any time due to number of waves meeting simultaneously at a point in a medium is the vector sum of individual displacements due each one of the waves at that point of same time

Superposition

Interference

- Constructive
- Destructive

Beat

Standing wave

- In a string
- In an open pipe
- In a closed pipe

Interference of sound wave

Condition:-

- Two waves of same frequency, same wavelength, same velocity
- Resultant intensities will be different from the sum of intensities of each wave separately
- This is due to the interference of waves

$$y_1 = a_1 \sin \omega t, y_2 = a_2 \sin(\omega t + \phi)$$

ϕ - Phase difference between two waves

$$\bar{y} = \bar{y}_1 + \bar{y}_2 \Rightarrow y = A \sin(\omega t + \theta)$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$$

$$\tan \theta = \frac{a_2 \sin \phi}{a_1 + a_2 \cos \phi}$$

Intensity $\propto A^2$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 \quad \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

i) For Constructive interference:-

$$\phi = 0, 2\pi, 4\pi, \dots, 2\pi n \text{ when } n = 0, 1, 2, \dots$$

$$x = 0, \lambda, 2\lambda, \dots, n\lambda, \text{ when } n = 0, 1, \dots$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2 = (A_1 + A_2)^2$$

ii) For Destructive interference:-

$$\text{when } \phi = \pi, 3\pi, 5\pi, \dots, (2n-1)\pi; \text{ where } n = 1, 2, 3, \dots$$

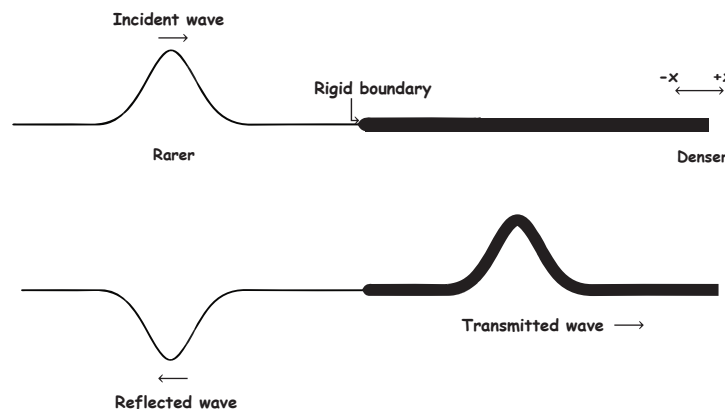
$$x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \dots, (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \dots$$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$\Rightarrow I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 \propto (A_1 - A_2)^2$$

Wave combination of string

1) From rarer to denser medium



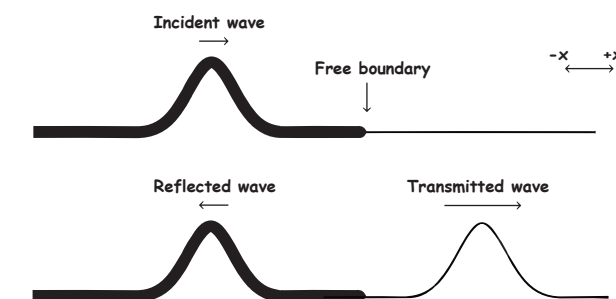
$$\text{Incident wave } y_i = a_i \sin(\omega t - k_1 x)$$

$$\text{Reflected wave } y_r = a_r \sin(\omega t - k_1(-x) + \pi)$$

$$= -a_r \sin(\omega t + k_1 x)$$

$$\text{Transmitted wave } y_t = a_t \sin(\omega t - k_2 x)$$

2) From rarer to denser medium



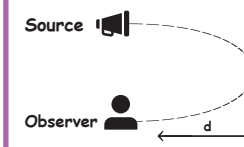
$$\text{Incident wave } y_i = a_i \sin(\omega t - k_1 x)$$

$$\text{Reflected wave } y_r = a_r \sin(\omega t - k_1(-x) + 0)$$

$$= a_r \sin(\omega t + k_1 x)$$

$$\text{Transmitted wave } y_t = a_t \sin(\omega t - k_2 x)$$

Echo



Source at distance "d" from screen
 $t = \frac{d}{v} + \frac{d}{v} = \frac{2d}{v}$

Persistence of hearing for human ear is 0.1 sec

Conditions for echo:

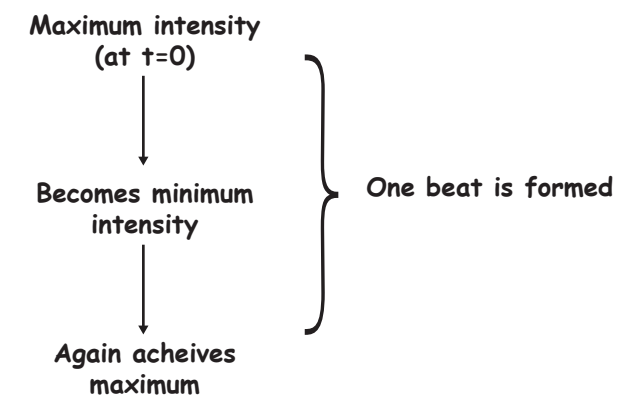
$$\text{if } t > 0.1 \Rightarrow \frac{2d}{v} > 0.1 \Rightarrow d > \frac{v}{20}$$

Beats:-

- sound waves travelling in same medium with slightly different frequencies superimpose on each other.
- The intensity of resultant sound at particular position rises and falls regularly with time.
- The phenomenon of variation of intensity of sound with time at a particular position is called beats.

Point to remember:-

1) One beat:-



Beat period:-

Time interval between two successive beats (ie. two successive maximum of sound) is called beat period.

Beat frequency:-

No. of beats produced per second

Beat frequency:-

$$n = |n_1 - n_2|$$

$$\text{Beat period:- } T = \frac{1}{\text{Beat frequency}} = \frac{1}{|n_1 - n_2|}$$

Determination of Unknown Frequency

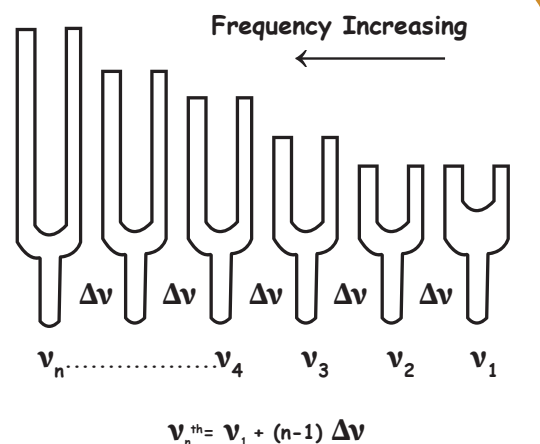
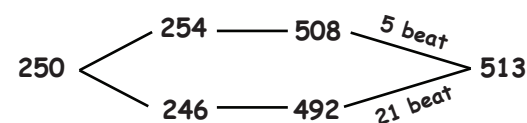
Let n_2 is the unknown frequency of tuning fork B, and this tuning fork B produce x beats per second with another tuning fork of known frequency n_1 .
As number of beat/sec is equal to the difference in frequencies of two sources, therefore $n_2 = n_1 \pm x$

By loading	By filing
If B is loaded with wax so its frequency decreases	If B is filed, its frequency increases

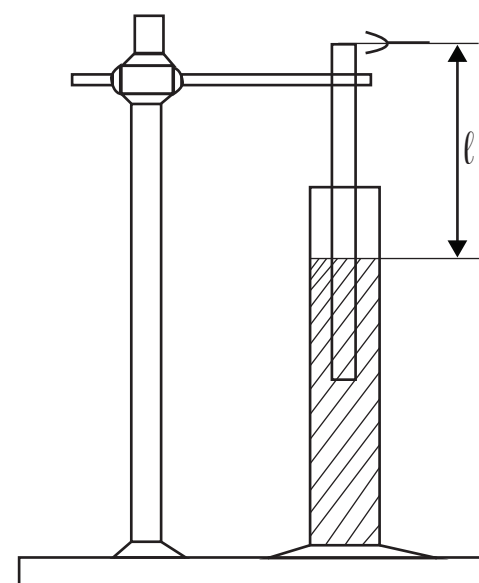
? A Source of unknown frequency produces 4 beat/s when sounded with a source of Known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 beat/s when Sounded with a source of frequency 513 Hz. The unknown frequency is?

- a) 254 Hz b) 246 Hz c) 240 Hz d) 260 Hz

Solution:

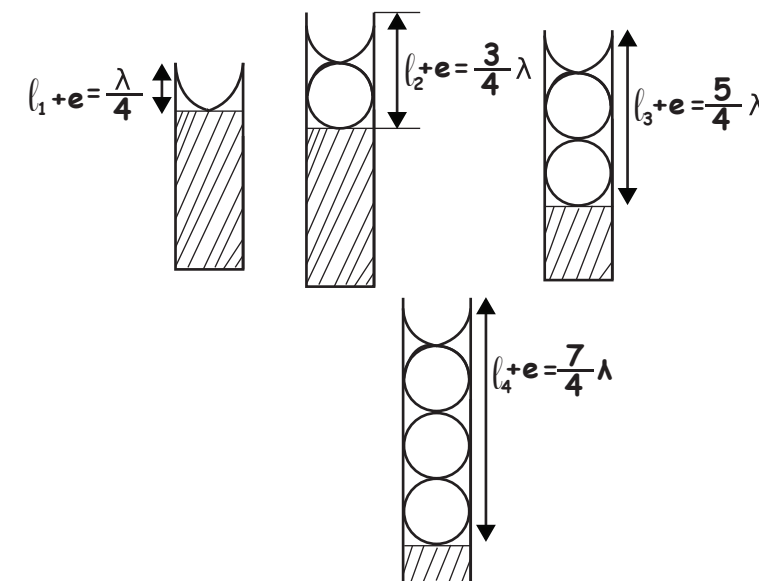


Resonance tube experiment



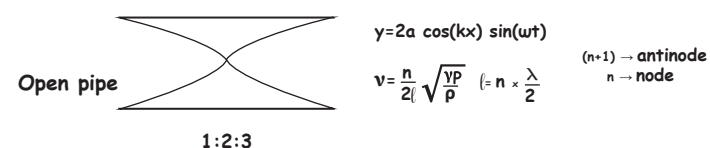
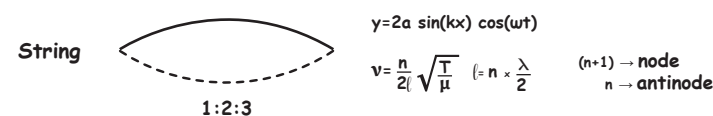
End correction:-

$$e = \frac{1}{2}(l_2 - 3l_1)$$

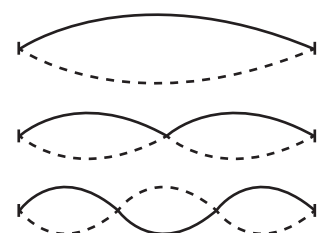


Standing Waves:

When two progressive waves (both longitudinal and transverse) having same amplitude, time period, frequency moving along a straight line in opposite direction superpose a new wave is formed. It is called stationary Or standing wave.



Note



Distance between two adjacent node & antinode is $\frac{\lambda}{2}$

Distance between two adjacent node & antinode is $\frac{\lambda}{4}$

Phase difference between 2 particle at both sides of node is 180° or π

Strain and pressure is maximum at node and minimum at antinode

Octave: The tone whose frequency is double the fundamental frequency is defined as Octave.

(i) If $n_2 = 2n_1$ it means n_2 is an octave higher than n_1 or n_1 is an octave lower than n_2 .

(ii) If $n_2 = 2^3 n_1$ it means n_1 is 3-octave higher or n_1 is 3-octave lower.

(iii) Similarly if $n_2 = 2^n n_1$, it means n_2 is n-octave higher n_1 is n octave lower.

Unison: If the two frequencies are equal then vibrating bodies are said to be in unison.

Resonance: The phenomenon of making a body vibrate with it's natural frequency under the influence of another vibrating body with the same frequency is called resonance.

Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. NO	Parameter	Stretched string	Open organ pipe	Closed organ pipe
1	Fundamental frequency or 1 st harmonic	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$
2	Frequency of or 2 nd harmonic 1 st overtone	$n_2 = 2n_1$	$n_2 = 2n_1$	Missing
3	Frequency of or 3 rd harmonic 2 nd overtone	$n_3 = 3n_1$	$n_3 = 3n_1$	$n_3 = 3n_1$ 1 st overtone
4	Frequency ratio of overtones	2:3:4.....	2:3:4.....	3:5:7.....
5	Frequency ratio of harmonics	1:2:3:4.....	1:2:3:4.....	1:3:5:7.....
6	Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary

WAVES 3

Relation between loudness and intensity

$$L \propto \log_{10} \text{Intensity}$$

$$\text{unit(dB)} \quad \text{unit } W/m^2$$

$$dB = 10 \times \log_{10} \frac{I}{I_0}$$

$$L_1 = 10 \times \log_{10} \frac{I_1}{I_0}$$

$$L_2 = 10 \times \log_{10} \frac{I_2}{I_0}$$

$$L_2 - L_1 = 10 \left[\log_{10} \left(\frac{I_2}{I_0} \right) - \log_{10} \left(\frac{I_1}{I_0} \right) \right]$$

$$\Delta L = 10 \log_{10} \left(\frac{I_2}{I_1} \right)$$

$$I_0 = 10^{-12} W/m^2$$

I_0 = Threshold intensity

ΔL = change in loudness

ΔI = change in intensity

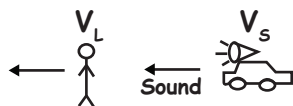
$$I_1 \rightarrow L_1$$

$$I_2 \rightarrow L_2$$

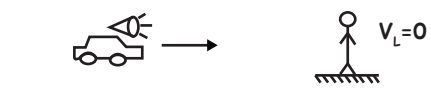
Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

General equation
(when both source & listener are moving)

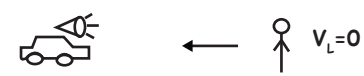
$$v' = \left(\frac{V \pm V_L}{V \pm V_S} \right) v$$


Case 1
(listener is stationary & source is approaching the listener)



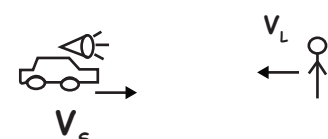
$$v' = \left(\frac{V}{V - V_S} \right) v$$

Case 2
(The source is stationary & listener is approaching the source)



$$v' = \left(\frac{V + V_L}{V} \right) v$$

Case 3
(source & listener are approaching each other)



$$v' = \left(\frac{V + V_L}{V - V_S} \right) v$$

Case 4
(source is stationary, listener is moving away from the source)



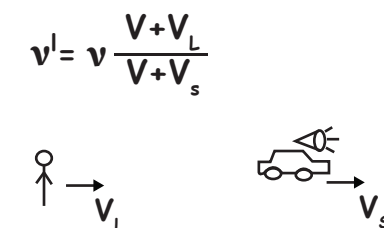
$$v' = \left(\frac{V - V_L}{V} \right) v$$

Case 5
(source is moving away from the listener, listener is stationary)



$$v' = \left(\frac{V}{V + V_S} \right) v$$

Case 6
(source and listener moving in same direction)



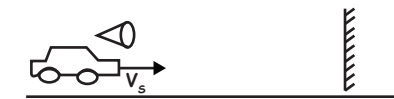
$$v' = v \frac{V + V_L}{V + V_S}$$

Case 7
(source and listener moving in opposite direction)



$$v' = \left(\frac{V - V_L}{V + V_S} \right) v$$

Case 8
(source approaching a stationary wall)



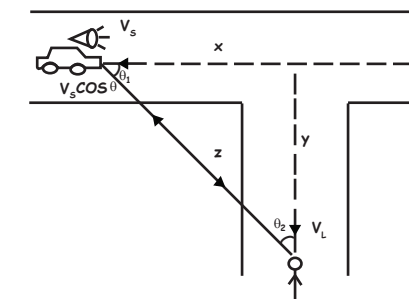
$$v' = \frac{V + V_S}{V - V_S} v$$

Case 9
(source is moving away from stationary wall)



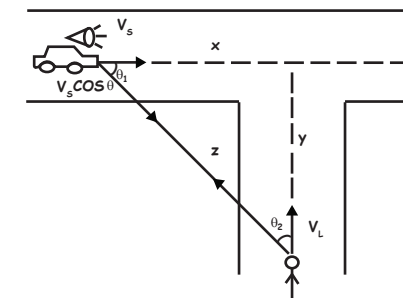
$$v' = \left(\frac{V - V_S}{V + V_S} \right) v$$

Case 10



$$v' = \left(\frac{V - V_L \cos \theta_2}{V + V_S \cos \theta_1} \right) v$$

Case 11

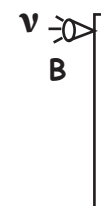
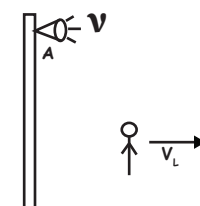


$$v' = \left(\frac{V + V_L \cos \theta_2}{V - V_S \cos \theta_1} \right) v$$

$$\cos \theta_1 = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos \theta_2 = \frac{y}{\sqrt{x^2 + y^2}}$$

Note:



$$v'_A = \left(\frac{V - V_L}{V} \right) v$$

$$v'_B = \left(\frac{V + V_L}{V} \right) v$$

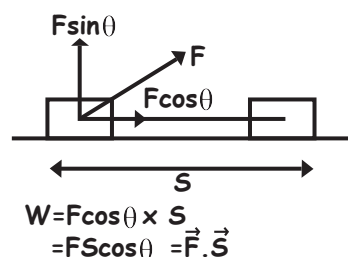
$$\text{Beat}(\Delta v) = v'_B - v'_A = \frac{v}{V} [V + V_L - V + V_L]$$

$$\Delta v = \frac{2V_L v}{V}$$

WORK

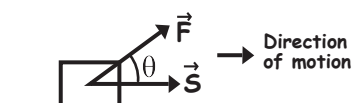
Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force

WORK DONE BY CONSTANT FORCE

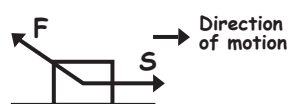


NATURE OF WORK DONE

1) Positive work ($0^\circ \leq \theta \leq 90^\circ$)



2) Negative work ($90^\circ \leq \theta \leq 180^\circ$)



2) Zero work

Work done becomes 0 for three conditions

1. Force is perpendicular to displacement
2. if there is no displacement
3. if there is no force acting on the body

WORK DONE BY VARIABLE FORCE

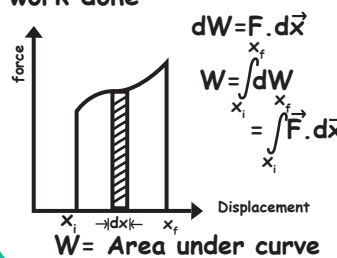
$$dW = \vec{F} \cdot d\vec{s}$$
$$W = \int \vec{F} \cdot d\vec{s} = \int F ds \cos \theta$$

in terms of rectangular components

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$
$$d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$$

$$W = \int F_x dx + \int F_y dy + \int F_z dz$$

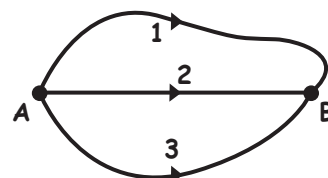
Graphical representation of work done



WORK DONE BY CONSERVATIVE & NON CONSERVATIVE FORCE

Conservative: work done does not depend on path followed

Non-conservative: work depends on the path followed



$$W_{A \rightarrow B} (\text{Path 1}) = W_{A \rightarrow B} (\text{Path 2}) = W_{A \rightarrow B} (\text{Path 3})$$

(for conservative force)

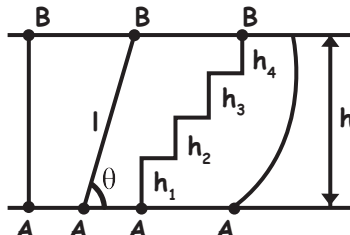
$$W_{A \rightarrow B} (\text{Path 1}) \neq W_{A \rightarrow B} (\text{Path 2}) \neq W_{A \rightarrow B} (\text{Path 3})$$

(for non conservative force)

Note:

Work done for a complete cycle by a conservative force is zero

WORK DONE BY DIFFERENT FORCES



$$W_1 = mgh = mgh$$

$$W_2 = mgx \sin \theta = mgx \times \frac{h}{l} = mgh$$

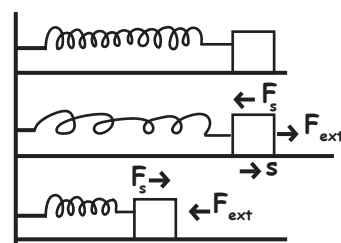
$$W_3 = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4 = mgh$$

work done by static friction $\rightarrow 0$

work done by kinetic friction $\rightarrow -ve$

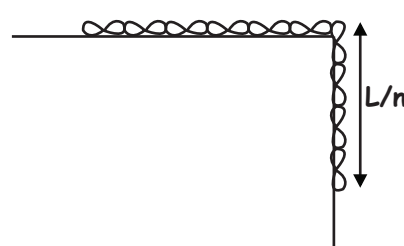
$$W_{fk} = f_k \cdot S = f_k S \cos 180^\circ = -f_k S$$

work done by spring force
magnitude of spring force $= -kx$



$$W_s = \int \vec{F}_s \cdot d\vec{s} = \int -kx dx = -\frac{1}{2} k(x_f^2 - x_i^2)$$

WORK DONE IN PULLING THE CHAIN



$L \rightarrow$ Total length

$(1/n)^{\text{th}}$ Part of length hanging

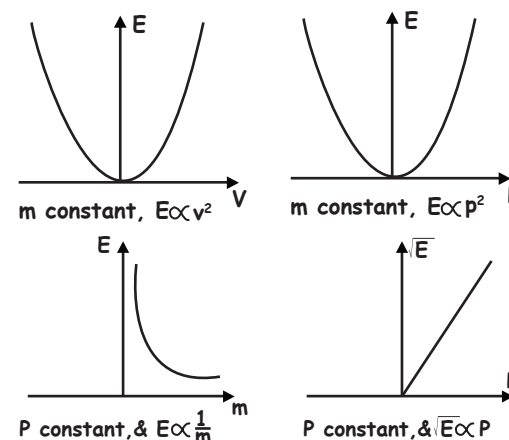
$M \rightarrow$ Mass of chain

$$\text{Work done in pulling, the hanging portion on the table } W = \frac{MgL}{2n^2}$$

RELATION OF KINETIC ENERGY WITH OTHER QUANTITIES

Linear momentum: $P = \sqrt{2mE}$

Variation of graph of kinetic Energy



P constant, $E \propto \frac{1}{m}$

P constant, $E \propto P$

POTENTIAL ENERGY

- Defined only for conservative force
 - Energy possessed by a body by virtue of its position
 - Can either be positive, negative or zero according to point of reference
 - Body always move from higher potential to lower potential
- Identifying forces with potential energy

1) Attractive force:-

On increasing x , if U increases

$$\frac{dU}{dx} = \text{positive}$$

(BC portion of graph)

2) Repulsive force:-

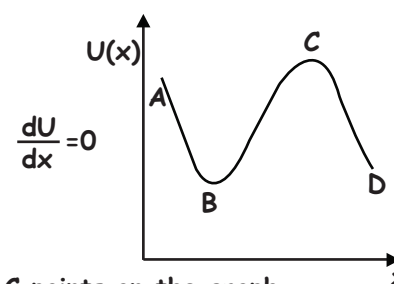
On increasing x , if U decreases

$$\frac{dU}{dx} = \text{negative}$$

(AB portion of graph)

3) Zero force:-

On increasing x , if U does not change



B, C points on the graph

Types of Potential Energy

- Elastic Potential Energy
- Electric Potential Energy
- Gravitational Potential Energy

Types of equilibrium

If net force acting on a particle is zero it is said to be in equilibrium

STABLE

If particle displaced from equilibrium position force acting will try to bring back to the initial position

Potential energy is minimum

$$F = -\frac{dU}{dx} = 0$$

$$\frac{d^2U}{dx^2} = \text{positive}$$



UNSTABLE

If particle displaced from equilibrium position force acting on it tries to displace further away from equilibrium position

Potential energy is maximum

$$F = -\frac{dU}{dx} = 0$$

$$\frac{d^2U}{dx^2} = \text{negative}$$



NUETRAL

If particle is slightly displaced from equilibrium, then it doesn't experience a force or continues in equilibrium

Potential energy is constant

$$F = -\frac{dU}{dx} = 0$$

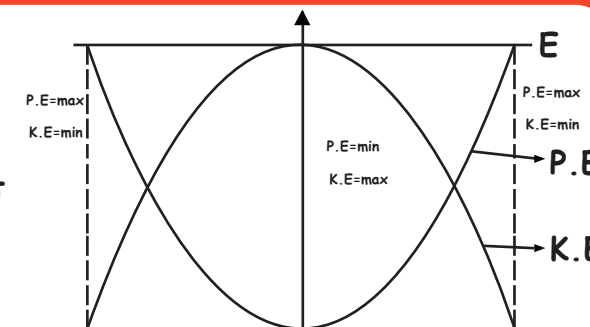
$$\frac{d^2U}{dx^2} = 0$$



CONSERVATION OF ENERGY

For an isolated system for a body in presence of conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion

$$K.E + P.E = \text{constant}$$



POWER

• Rate at which body can do work

$$\text{Average power } (P_{av}) = \frac{\Delta W}{\Delta t}$$

$$\text{Instantaneous power } (P_{inst}) = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Relation between units:

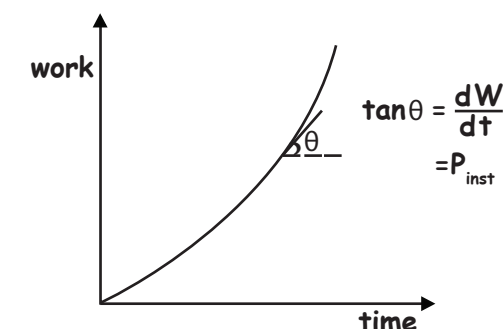
- 1 watt = 1 joule/sec = 10^7 erg/sec
- 1 HP = 746 watt, 1 MW = 10^6 watt
- 1 KW = 10^3 watt

• If work done by two bodies is same then power $\propto \frac{1}{\text{time}}$

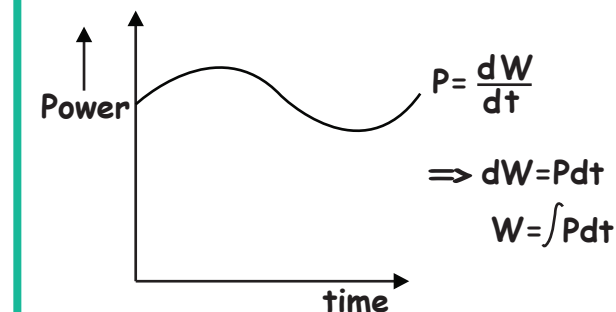
• Unit of power multiplied by time always give work

$$1 \text{ KWh} = 3.6 \times 10^6 \text{ Joules}$$

• Slope of work-time curve gives instantaneous power



• Area under power time graph gives work done



$$W = \text{Area under } P-t \text{ graph}$$

Position and velocity in terms of power:-

1) Velocity, $V = \left[\frac{2Pt}{m} \right]^{1/2}$

2) Position, $S = \left[\frac{8P}{9m} \right]^{1/2} t^{3/2}$

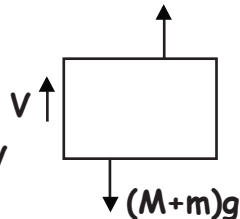
Power delivered by an elevator

$a=0, T=(M+m)g$

$\vec{P} = \vec{T} \cdot \vec{V}$

$= TV$

Power, $P = (M+m)gV$



Power of a water drawing pump

• Power, $P = \frac{dW}{dt} = \frac{dm}{dt} \left[gh + \frac{V^2}{2} \right]$

• h = height of water level

$\frac{dm}{dt} \Rightarrow$ mass flow rate of pump

$V \rightarrow$ velocity of the water outlet

• Power required to just lift water, $V=0$

$P = gh \left(\frac{dm}{dt} \right)$

Efficiency of pump

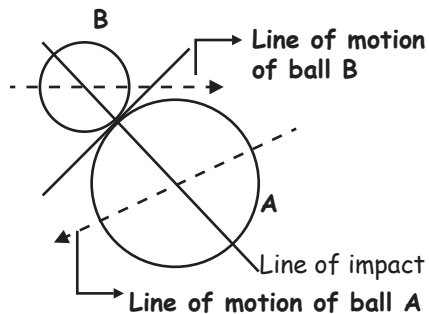
$\mu = \frac{\text{Output Power}}{\text{Input Power}}$

WORK ENERGY & POWER

Event in which impulsive force acts between two or more bodies which result in change of their velocities.

Line of impact

Line passing through common normal to surfaces in contact during impact



Coefficient of restitution (e)

$$e = \frac{\text{Velocity of separation along the line of impact}}{\text{Velocity of approach along the line of impact}}$$

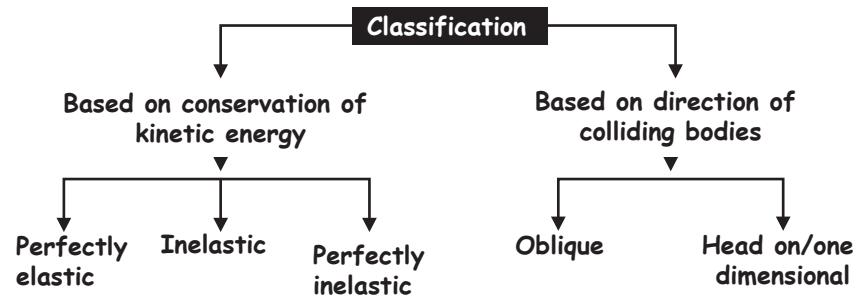
$$= \frac{\text{Relative velocity after collision along the line of impact}}{\text{Relative velocity before collision along the line of impact}}$$

Conditions

1. For elastic collision: $e=1$

2. For inelastic collision: $e < 1$

3. For perfectly elastic collision: $e=0$



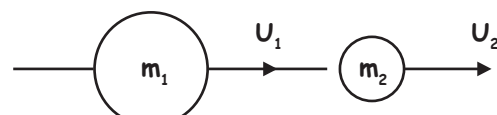
Perfectly elastic collision

K.E before and after collision is same

Inelastic collision

K.E after collision is not equal to K.E before collision then it is said to be inelastic collision

Head on collision / One dimensional collision



Before collision

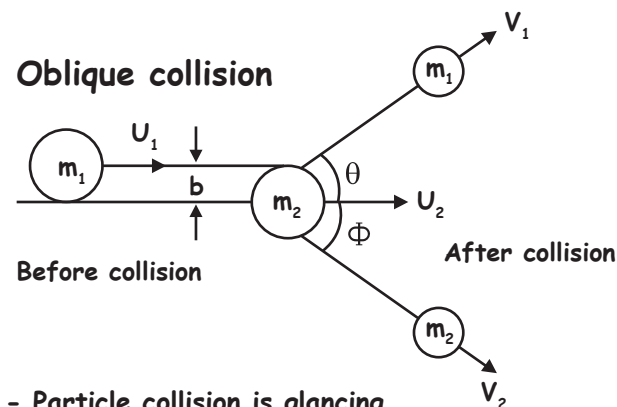


After collision

Impact parameter $b=0$

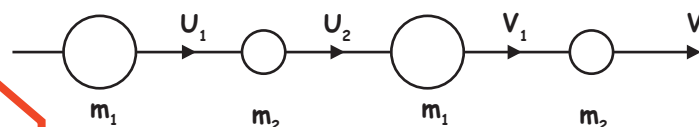
COLLISION

Oblique collision



- Particle collision is glancing
- Direction of motion after collision are not along initial line of motion
- If they collide in same plane, collision is 2 dimensional otherwise 3 dimensional
- Impact parameter $0 < b < (r_1 + r_2)$ r_1, r_2 are radii of colliding bodies

Perfectly elastic Head on collision



Velocity after collision

$$V_1 = U_1 \left[\frac{m_1 - m_2}{m_1 + m_2} \right] + \frac{2m_2 U_2}{m_1 + m_2}$$

$$V_2 = U_2 \left[\frac{m_2 - m_1}{m_1 + m_2} \right] + \frac{2m_1 U_1}{m_1 + m_2}$$

Special cases:

1) Projectile and target having same mass $m_1 = m_2$
 $v_1 = u_2, v_2 = u_1$, the velocities get interchanged

2) If massive projectile collide with a light target
ie $m_1 \gg m_2$ $v_1 = u_1, v_2 = -u_2 + 2u_1$

3) If the light projectile collides with a very heavy target,
 $m_1 \ll m_2$ $v_1 = -u_1 + 2u_2, v_2 = u_2$

Energy transfer from projectile to target

1) Fractional decrease in kinetic energy
(If target is at rest)

$$\frac{\Delta K}{K} = \frac{4m_1 m_2}{(m_1 - m_2)^2 + 4m_1 m_2}$$

Greater the difference in masses, less will be transfer of K.E and vice versa

If $m_2 = nm_1$ $\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$

Inelastic collision

$$e = \frac{V_2 - V_1}{U_1 - U_2} \quad \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

Velocity after collision $V_1 = \frac{(1+e)m_2 U_2}{m_1 + m_2} + \frac{(m_1 - em_2)U_1}{m_1 + m_2}$

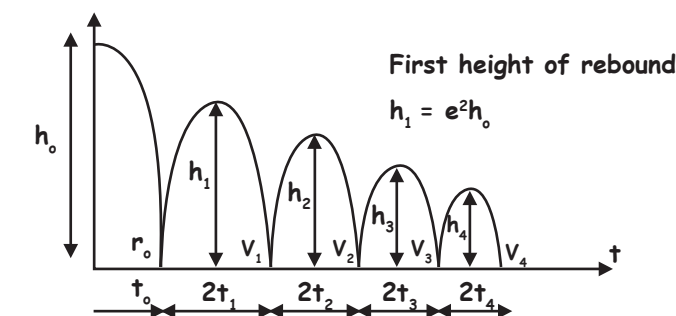
$V_2 = \frac{(1+e)m_1 U_1}{m_1 + m_2} + \frac{(m_2 - em_1)U_2}{m_1 + m_2}$

Ratio of velocities

$$\frac{V_1}{V_2} = \frac{1-e}{1+e}$$

Loss in kinetic energy $\Delta K = \frac{1}{2} \left[\frac{m_1 m_2}{m_1 + m_2} \right] (1-e^2) (U_1 - U_2)^2$

Rebounding of ball



Total distance travelled by the ball before it stops bouncing

$$H = h_0 \left[\frac{1+e^2}{1-e^2} \right]$$

Total time taken by the ball to stop bouncing $T = \left(\frac{1+e}{1-e} \right) \frac{2h_0}{g}$

Perfectly inelastic collision

colliding bodies are moving in the same direction



Loss in kinetic energy $\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 - U_2)^2$

Colliding bodies are moving in the opposite direction

$$V = \frac{m_1 U_1 - m_2 U_2}{m_1 + m_2} \quad \text{Change in kinetic energy } \Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 + U_2)^2$$

Collision in two dimension

If the initial velocities of two colliding bodies are not along the line of impact, then the collision is said to be oblique collision or collision in two dimension.

ALTERNATING CURRENT

"If the direction of current in a resistor or any other element changes alternately, the current is called an alternating current"

AVERAGE AND RMS VALUE OF AC

If the current or voltage is said to be sinusoidal, it can be expressed as

$$i = i_0 \sin(\omega t + \phi) \quad v = v_0 \sin(\omega t + \phi)$$

$i_0 \rightarrow$ Peak current or current amplitude

$v_0 \rightarrow$ Peak voltage or voltage amplitude

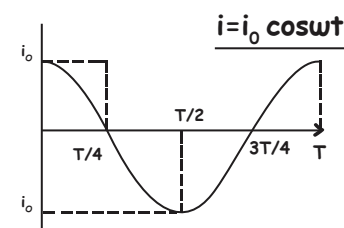
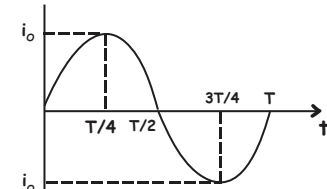
$$\omega = \frac{2\pi}{T} = 2\pi f \quad T: \text{Time period}$$

f : frequency (Hz or cycle/sec)

$(\omega t + \phi)$: Total phase

GENERAL GRAPH

if $i = i_0 \sin \omega t$



$\Rightarrow$ for measuring ac hot wire instruments are used

ROOT MEAN SQUARE CURRENT

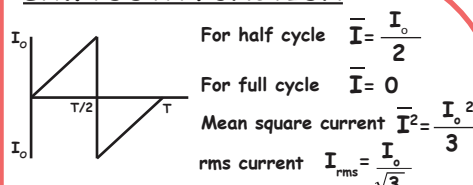
$$I_{rms} = \sqrt{I^2} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} \quad V_{rms} = \frac{V_0}{\sqrt{2}}$$

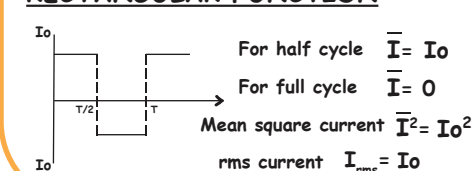
Average value of ac is defined for positive or negative half cycle

$$\bar{I} = \frac{2I_0}{\pi} \quad \bar{V} = \frac{2V_0}{\pi}$$

SAWTOOTH FUNCTION



RECTANGULAR FUNCTION



AVERAGE HEAT PRODUCED DURING A CYCLE OF AC

$$H_{av} = \frac{1}{2} I_0^2 R = I_{rms}^2 R$$

Keep in mind

$\Rightarrow$ rms value also called virtual value or effective value

$\Rightarrow$ AC ammeter and voltmeter always measure rms value

$\Rightarrow$ Values printed on ac circuits are rms values

$\Rightarrow$ In houses ac is supplied at 220v which is the rms of voltage

$\Rightarrow$ Peak value is $220/\sqrt{2} = 311V$

$\Rightarrow$ Frequency in general is 50Hz

$\Rightarrow \omega = 2\pi f = 100\pi \text{ rad/sec (314 rad/sec)}$

AVERAGE VALUE OF AC FOR ONE TIME PERIOD

$$\bar{I} = \frac{\int_0^T I dt}{\int_0^T dt} = \frac{\int_0^T I_0 \sin \omega t dt}{\int_0^T dt} = \frac{\text{area of } I-t \text{ graph}}{\text{time}}$$

$\bar{I} = 0$ for $0 \rightarrow T$ for a ac wave.

The average value of sin or cos function for one time period or n time periods ($n=1,2,\dots$) is zero

Keep in mind

Long period is equivalent to one time period

$\Rightarrow$ Mean square current for one time period

$$\bar{I}^2 = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{I_0^2}{2}$$

Remember

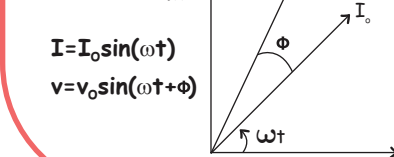
The average value of square of sin or cosine function for one time period is $\frac{1}{2}$

$$I = \int_0^T \sin^2 k\omega t = \frac{1}{2} \times T$$

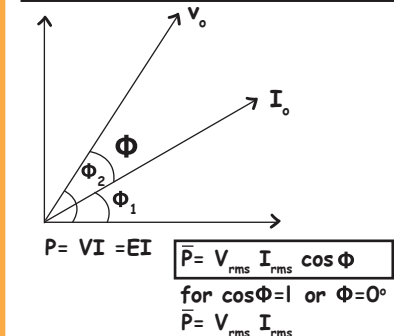
$$I = \int_0^{nT} \sin^2 k\omega t = \left(\frac{n}{2}\right) \times T$$

PHASOR DIAGRAM

Diagram representing ac voltage or current as vectors with phase between them.

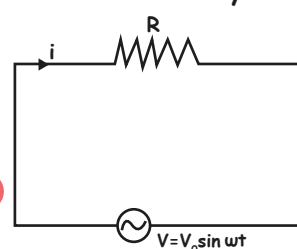


AVERAGE POWER CONSUMPTION

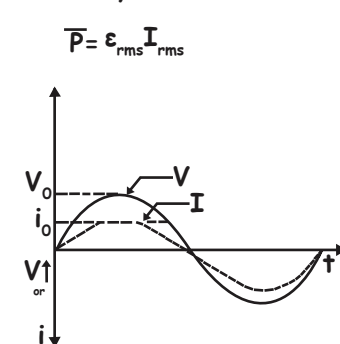


SINGLE COMPONENT CIRCUITS

Resistor only

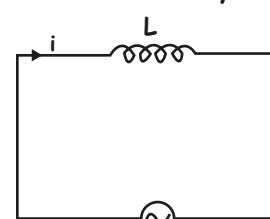


1. $V = V_0 \sin \omega t$
2. $i = i_0 \sin \omega t$
3. V & i are in phase
4. $\rightarrow \quad \rightarrow$
 I
5. $\phi = 0, \cos \phi = 1$

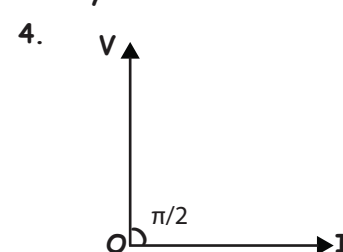


$$6. i_0 = \frac{V_0}{R}$$

Inductor only

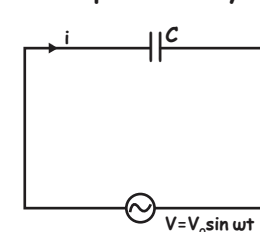


1. $V = V_0 \sin \omega t$
2. $i = i_0 \sin(\omega t - \pi/2)$
3. Voltage leads the current or current lags the voltage by $\pi/2$

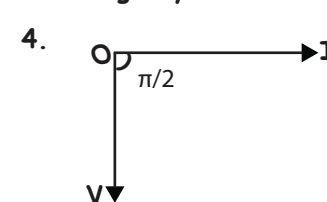


5. $\phi = \pi/2, \cos \phi = 0$
6. $\bar{P} = 0$ (wattless circuits)
7. Inductive reactance (X_L)
 $X_L = L\omega$
Unit-ohm(Ω)
plays role of resistance
8. $i_0 = \frac{V_0}{X_L}$

Capacitor only



1. $V = V_0 \sin \omega t$
2. $i = i_0 \sin(\omega t + \pi/2)$
3. Current leads to the voltage by $\pi/2$



5. $\phi = \pi/2, \cos \phi = 0$
6. $\bar{P} = 0$ (wattless circuits)
7. Inductive reactance
 $X_C = \frac{1}{C\omega}$
Unit-ohm(Ω)
plays role of resistance
8. $i_0 = \frac{V_0}{X_C}$

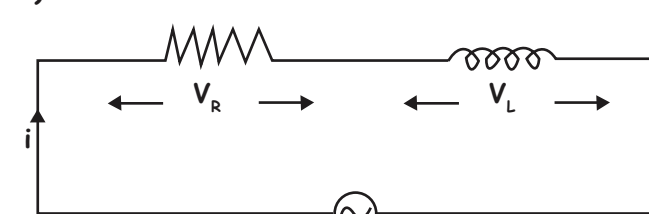
SUMMARY

	Z (Impedance)	ϕ
1. R only	R	0
2. L only	$X_L = \omega L$	$-\pi/2$
3. C only	$X_C = \frac{1}{C\omega}$	$\pi/2$

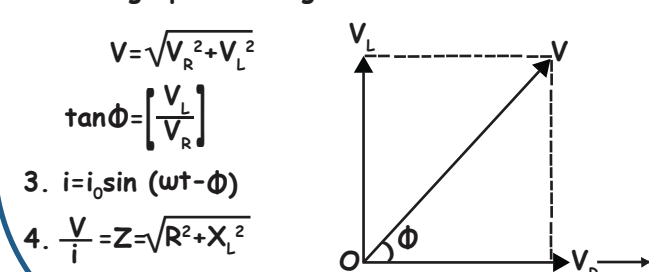
ALTERNATING CURRENT

SERIES AC CIRCUITS

1) R-L CIRCUITS



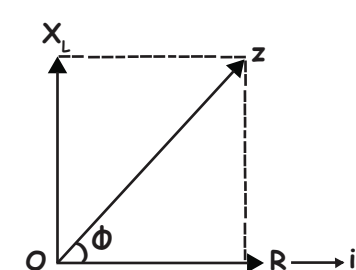
1. $V = V_0 \sin \omega t$ $V_R = iR$, $V_L = iX_L$
2. Voltage phasor diagram $\rightarrow$



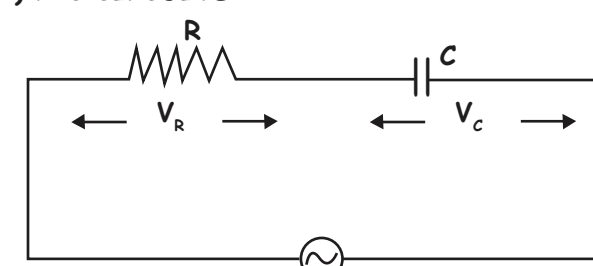
5. Impedance phasor

$$\tan \phi = \frac{X_L}{R}$$

$$6. i_0 = \frac{V_0}{Z}$$



2) R-C CIRCUITS

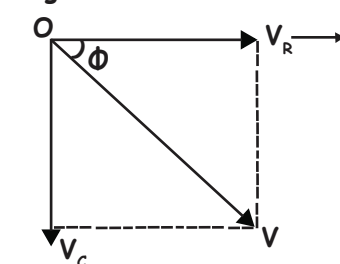


1. $V = V_0 \sin \omega t$ $V_R = iR$, $V_C = iX_C$

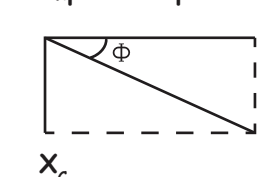
2. Voltage phasor diagram

$$V = \sqrt{V_R^2 + V_C^2}$$

$$\tan \phi = \left[\frac{V_C}{V_R} \right]$$



Impedance phasor



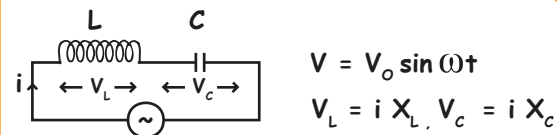
$$i = i_0 \sin(\omega t + \phi)$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$\tan \phi = \left(\frac{X_C}{R} \right)$$

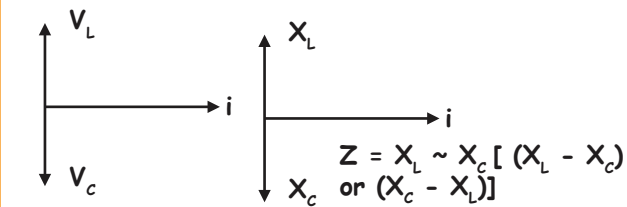
$$i_0 = \left(\frac{V_0}{Z} \right)$$

3) L-C CIRCUIT



Voltage phasor diagram

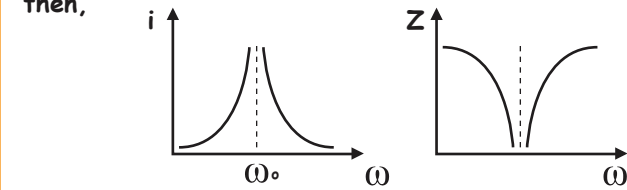
$$V = V_L \sim V_C [\text{ie, } (V_L - V_C) \text{ or } (V_C - V_L)]$$



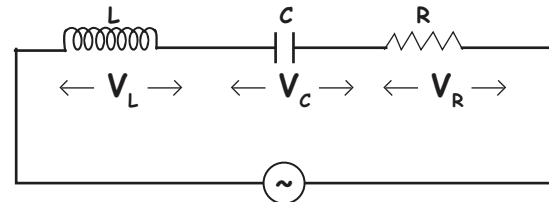
if $X_L > X_C$, Voltage leads the current by $\frac{\pi}{2}$

if $X_C > X_L$, current leads the voltage by $\frac{\pi}{2}$

if $X_L = X_C$, $Z = 0$, $i = \infty$ then,



L-C-R Series Circuit

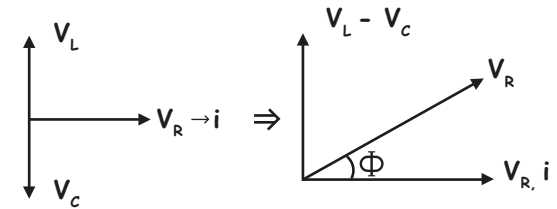


$$V = V_o \sin(\omega t)$$

$$V_R = iR, V_L = iX_L, V_C = iX_C$$

Assuming $V_L > V_C$ for drawing phasor

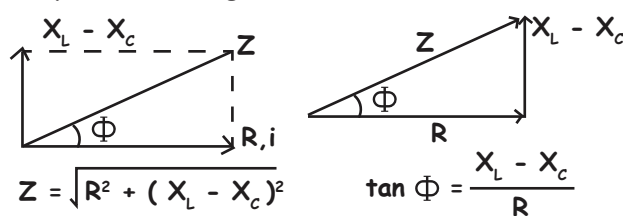
Voltage phasor diagram



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Here $i = i_o \sin(\omega t - \Phi)$
 (since V_L is leading)

Impedance Triangle



RESONANCE IN LCR SERIES CIRCUIT

In series resonance, impedance of circuit is minimum & equal to resistance $Z = R$ and current is maximum

Condition for resonance

$$X_L = X_C \Rightarrow L\omega = \frac{1}{C\omega}$$

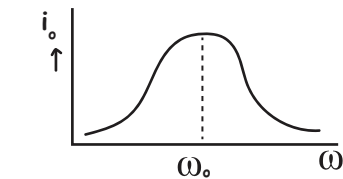
$$\omega = \omega_o = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$\omega_o \rightarrow$ resonant frequency (angular)

$$f = f_o = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$f_o =$ resonant frequency

GRAPH



Variation of peak current with applied frequency

$\omega < \omega_o$, $X_C > X_L$ current leads

$\omega > \omega_o$, $X_L > X_C$ current lags

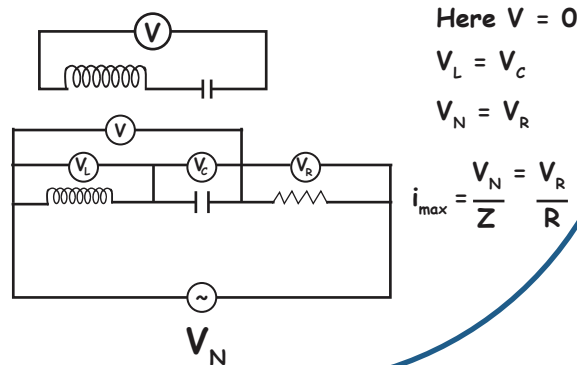
In resonance

$V = V_R$ (applied voltage = voltage across resistance)

$Z = R$ (impedance is minimum and equal to resistance)

Voltmeter connected across V_L & V_C will show the same reading

Voltmeter connected commonly across inductor & capacitor shows no reading



APPLICATION OF RESONANT CIRCUIT

Tuning mechanism of a radio or TV set

1. Antenna of radio accepts signals

2. Signal acts as a source in tuning the radio

3. In tuning, capacitance of capacitor is varied such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received.

QUALITY FACTOR

$$Q = \frac{\omega_o L}{R} \text{ or } \frac{1}{\omega_o C R}$$

$$\text{or } Q = \left[\frac{\text{Voltage across } C \text{ or } L}{\text{applied voltage}} \right]_{\text{resonance}}$$

Less sharp the resonance, less is the selectivity of the circuit. If the Quality factor is large, R is low or L is large, the circuit is more selective.

Sharpness of Resonance

Sharpness = $\frac{\omega_o}{2\Delta\omega}$; $2\Delta\omega$ - bandwidth
 smaller $\Delta\omega$, sharper or narrower the resonance.

POWER IN AC CIRCUIT

$$\text{Average Power } \bar{P} = VI \cos \Phi$$

$$\text{Case 1 } \bar{P} = I^2 Z \cos \Phi$$

Resistive circuit - $\Phi = 0$, $\cos \Phi = 1$

Maximum power dissipation

Case 2

Purely inductive or capacitive circuit -

$$\Phi = 90^\circ \quad \cos \Phi = 0$$

No power is dissipated even though a current is flowing in the circuit

Case 3

LCR Series circuit

Φ non zero in R-L, C-R, or CLR circuit.

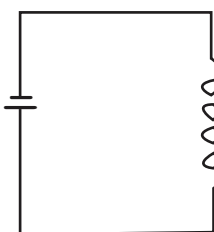
Case 4

Power dissipation at resonance

$$X_L - X_C = 0 \text{ or } \Phi = 0 \Rightarrow \cos \Phi = 1$$

$P = I^2 Z$ or $I^2 R$ means maximum power is dissipated in a circuit at resonance.

LC OSCILLATIONS



Capacitor $\rightarrow$ stores electrical energy

Inductor $\rightarrow$ stores magnetic energy

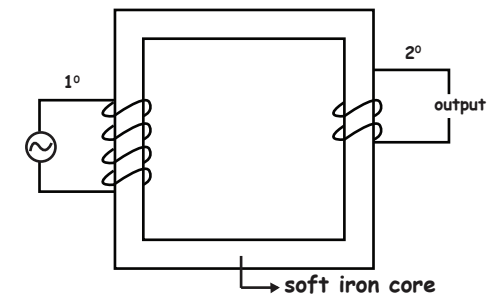
When connected, charge on the capacitor and current on the inductor exhibit electrical oscillations.

COMPARISON OF OSCILLATION OF A MASS SPRING SYSTEM

Mass spring system	LC Circuit
Displacement (x)	Charge (q)
Velocity (v)	Current (I)
Acceleration (a)	Rate of change of current $= \left(\frac{dI}{dt} \right)$
Mass (m), (inertia)	Inductance (L)
Momentum $p = mv$	Magnetic flux $\Phi = LI$
Retarding force $-m \frac{dv}{dt}$	Self induced emf $(-L \frac{dI}{dt})$
Differential equation $\frac{d^2x}{dt^2} + \omega^2 x = 0$	$\frac{d^2q}{dt^2} + \omega^2 q = 0$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{1}{LC}}$
Force constant K	capacitance C
K.E. $= \frac{1}{2} mv^2$	Magnetic energy $= \frac{1}{2} LI^2$
Elastic $U = \frac{1}{2} kx^2$	Elastic $U = \frac{q^2}{2C}$

TRANSFORMERS

"Devices which raises or lowers voltage in ac circuits through mutual induction". Transformer can increase or decrease voltage or current but not both simultaneously.



EQUATIONS

$$1) \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

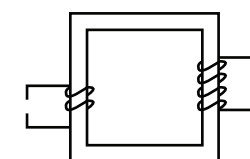
$$2) \text{Efficiency } \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{V_s I_s}{V_p I_p}$$

$$3) \text{For ideal transformer, } \eta = 1$$

V_s - Voltage in secondary
 V_p - Voltage in primary
 N_s - No of turns in secondary
 N_p - No of turns in primary
 I_p - Current in primary
 I_s - Current in secondary

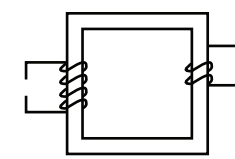
TRANSFORMER TYPES

step up transformer



$$\begin{matrix} N_p \downarrow & N_s \uparrow \\ V_p \downarrow & V_s \uparrow \\ I_p \uparrow & I_s \downarrow \\ R_p \downarrow & R_s \uparrow \end{matrix}$$

step down transformer



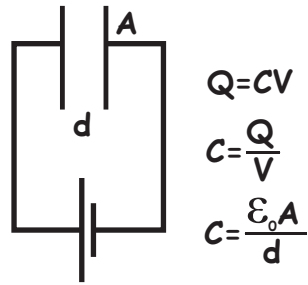
$$\begin{matrix} N_p \uparrow & N_s \downarrow \\ V_p \uparrow & V_s \downarrow \\ I_p \downarrow & I_s \uparrow \\ R_p \uparrow & R_s \downarrow \end{matrix}$$

LOSSES IN TRANSFORMER

- 1) Cu loss ($I^2 R$ loss)
 $\rightarrow$ To minimise, windings are made of thick Cu wires (high resistance)
- 2) Eddy current loss
 $\rightarrow$ To minimise Cores are laminated
- 3) Hysteresis loss
 $\rightarrow$ select material of narrow hysteresis loop
 $\rightarrow$ Cores of transformer is made of soft iron
- 4) Magnetic flux linkage
 $\rightarrow$ To minimise, secondary winding is kept inside the primary winding
- 5) Humming loss

CAPACITANCE

CAPACITANCE



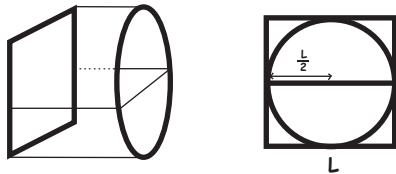
Capacitance Depends on

1. Distance between plates
2. Area of plates
3. Medium b/w plates

Capacitance Independent of

1. Charge
2. Potential difference

Unit Of Capacitance = Farad



$$A = \pi \left(\frac{L}{2}\right)^2 = \pi \frac{L^2}{4}$$

$$C = \frac{\epsilon_0 A_{\text{eff}}}{d} = \frac{\pi \epsilon_0 L^2}{4d}$$

ENERGY STORED

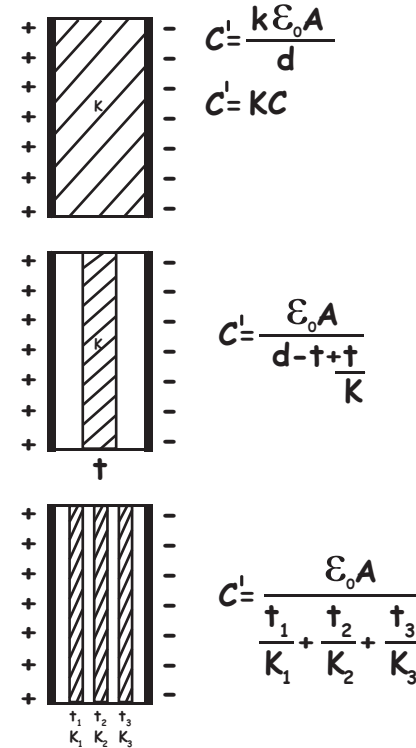
Work done by battery (QV) 100%

$$\begin{aligned} & \text{50\% Energy stored} \\ & = \frac{1}{2} QV = \frac{CV^2}{2} \\ & = \frac{Q^2}{2C} \\ & \text{50\% Energy dissipated} \\ & = \frac{1}{2} QV = \frac{CV^2}{2} \\ & = \frac{Q^2}{2C} \end{aligned}$$

Variation with plate separation

1. $Q = CV = \frac{\epsilon_0 AV}{d}$
2. $C = \frac{\epsilon_0 A}{d} \quad C \propto \frac{1}{d}$
3. $V = \frac{Q}{C} = \frac{Qd}{\epsilon_0 A}$
4. $E = \frac{Q}{A\epsilon_0} = \frac{V}{d}$
5. $F = \frac{Q^2}{2A\epsilon_0} = \frac{CV^2}{2d}$
6. $U = \frac{CV^2}{2} = \frac{Q^2}{2C}$

DIELECTRIC IN CAPACITOR



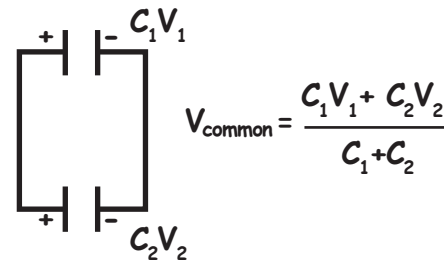
Battery removed

1. $C' = KC$
2. $Q' = Q$
3. $V' = \frac{V}{K}$
4. $E' = \frac{E}{K}$
5. $U' = \frac{U}{K}$

Battery remains connected

1. $C' = KC$
2. $Q' = KQ$
3. $V' = V$
4. $E' = E$
5. $U' = KU$

Connecting two capacitor - Case 1



Initial Energy

$$U_i = \frac{C_1 V_1^2}{2} + \frac{C_2 V_2^2}{2}$$

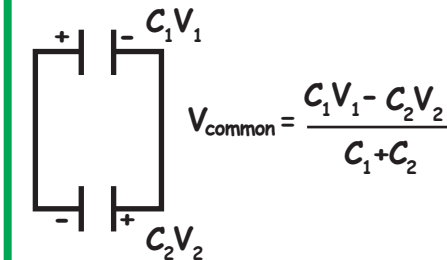
Heat loss

Final Energy

$$U_f = \frac{1}{2} (C_1 + C_2) \left(\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right)^2$$

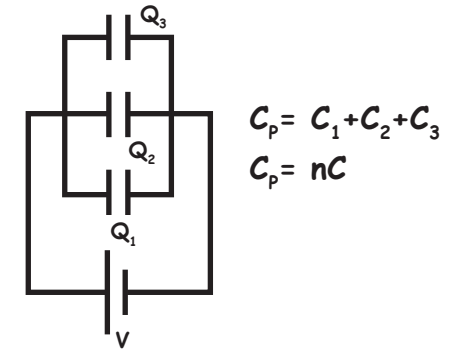
$$\text{Energy loss} = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$$

Connecting two capacitor - Case 2



$$\text{Energy loss} = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 + V_2)^2$$

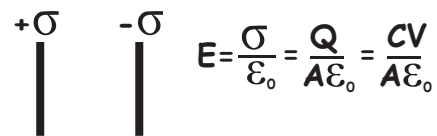
2. Parallel combination



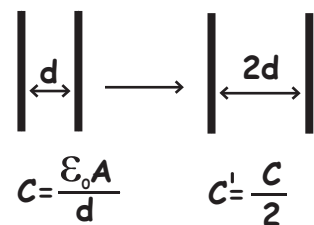
Charge divider rule

$$\begin{aligned} Q_1 &= \left(\frac{C_1}{C_1 + C_2} \right) Q \\ Q_2 &= \left(\frac{C_2}{C_1 + C_2} \right) Q \end{aligned}$$

ELECTRIC FIELD



when plate separation doubled



POTENTIAL

$$V = Ed = \frac{Qd}{A\epsilon_0}$$

Force b/w parallel plate capacitor

$$\begin{aligned} F &= Q \times E \text{ Due to one plate} \\ &= Q \times \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0} \times Q \\ &= \frac{Q^2}{2A\epsilon_0} = \frac{C^2 V^2}{2A\epsilon_0} = \frac{\epsilon_0 A}{d} \times \frac{CV^2}{2A\epsilon_0} \\ &= \frac{CV^2}{2d} \end{aligned}$$

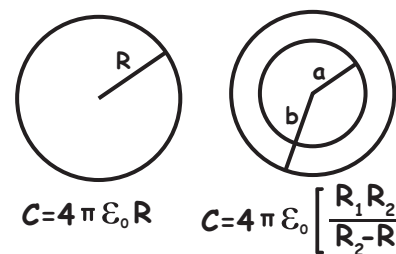
Battery connected
 $d \rightarrow 2d$

1. $V = \text{Constant}$
2. $C' = \frac{C}{2}$
3. $Q = CV, Q' = \frac{Q}{2}$
4. $E = \frac{V}{d}, E' = \frac{E}{2}$
5. $F = \frac{CV^2}{2d}, F' = \frac{F}{4}$
6. $U = \frac{CV^2}{2}, U' = \frac{U}{2}$

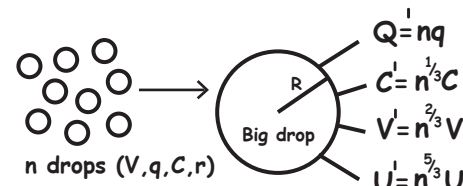
Battery Disconnected
 $d \rightarrow 2d$

1. $Q = \text{Constant}$
2. $C' = \frac{C}{2}$
3. $V = \frac{Q}{C}, V' = 2V$
4. $E = \frac{Q}{A\epsilon_0} = \text{Constant}$
5. $F = \frac{Q^2}{2A\epsilon_0} = \text{Constant}$
6. $U = \frac{Q^2}{2C}, U' = 2U$

SPHERICAL CAPACITOR

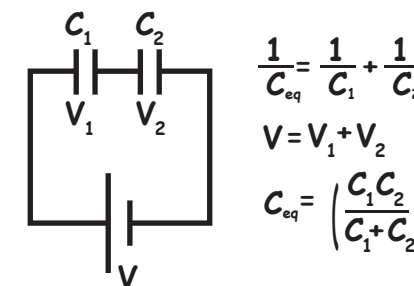


REDISTRIBUTION OF CHARGE



Grouping of capacitors

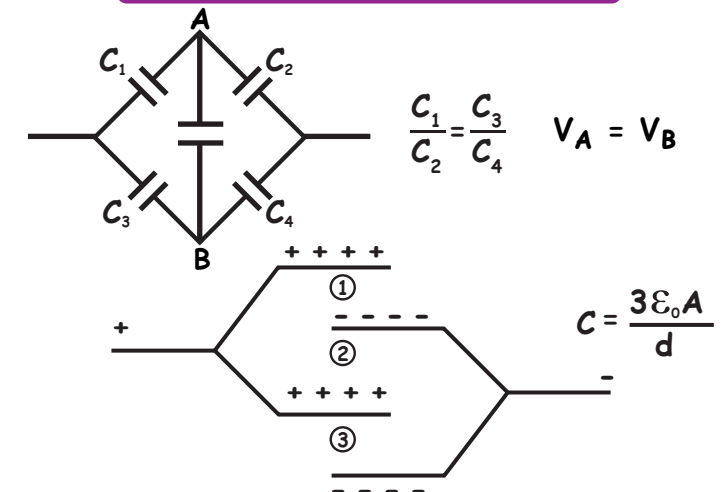
1. Series combination



Voltage divider rule

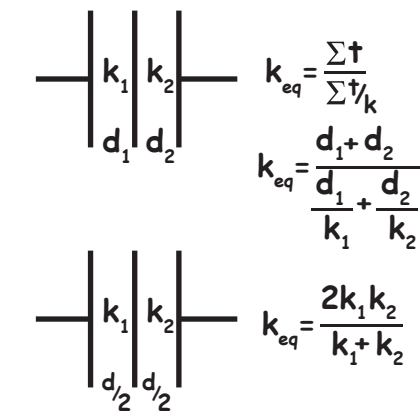
$$V \propto \frac{1}{C} \quad V_1 = \left(\frac{C_2}{C_1 + C_2} \right) V \quad V_2 = \left(\frac{C_1}{C_1 + C_2} \right) V$$

WHEATSTONE'S BRIDGE

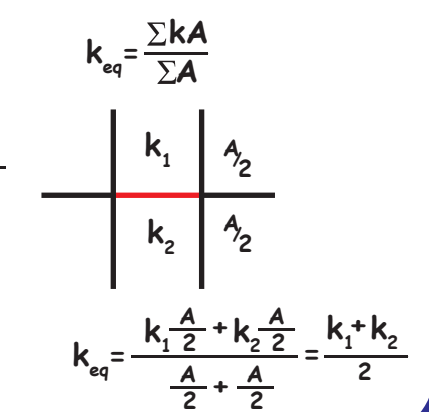


MULTIPLE DIELECTRICS

1. Series combination

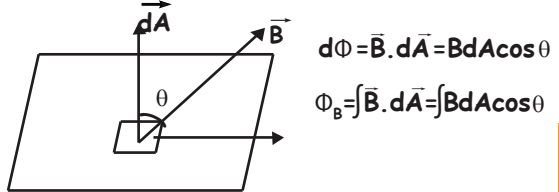


2. Parallel combination



ELECTROMAGNETIC INDUCTION

MAGNETIC FLUX $\propto$ no of lines of force



$$d\Phi = \vec{B} \cdot d\vec{A} = B dA \cos \theta$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B dA \cos \theta$$

for uniform field, $\vec{B}$ & $\vec{A}$ $\Phi_B = \vec{B} \cdot \vec{A}$
 $\Phi_B = BA \cos \theta$

• Scalar quantity $\rightarrow$ Unit $\rightarrow$ Weber(Wb)

• $[\Phi_B] = ML^2T^{-2}A^{-1}$

EMF

Average value

$$\mathcal{E}_{ind} = -\frac{\Delta\Phi}{\Delta t}$$

$$[\Delta\Phi = \Phi_2 - \Phi_1]$$

Induced current

$$I_{ind} = \frac{\mathcal{E}_{ind}}{R} = -\frac{1}{R} \frac{\Delta\Phi}{\Delta t}$$

Induced charge

$$q_{flow} = \frac{\Delta\Phi}{R}$$

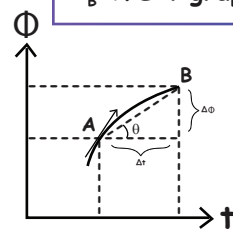
Charge flown due to induced current depends only on change in flux

Instantaneous value

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt}$$

$$I_{ind} = \frac{\mathcal{E}_{ind}}{R} = -\frac{1}{R} \frac{d\Phi}{dt}$$

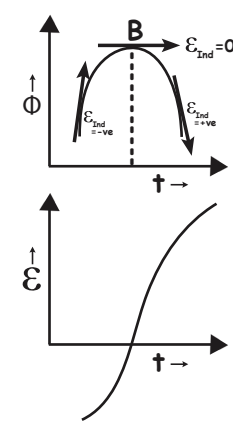
References from Φ_B V/S t graph



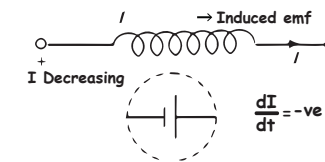
Slope of chord in $\Phi-t$ Graph $= \mathcal{E}_{ind} = \frac{\Delta\Phi}{\Delta t}$

Slope of the tangent in the $\Phi-t$ Graph $= \mathcal{E}_{ind} = \frac{d\Phi}{dt}$

CHANGING $\Phi-t$ GRAPH INTO OTHER GRAPH



CASE 2



self inductance of a long solenoid

$$B = \frac{\mu_0 NI}{\ell} \quad \Phi_{B1} = \frac{\mu_0 NIA}{\ell}$$

$$E_L = -\frac{d\Phi_{B1}}{dt} = -N \frac{d\Phi_{B1}}{dt} = -\frac{d}{dt} \left(\frac{\mu_0 N^2 IA}{\ell} \right) = -\frac{\mu_0 N^2 A}{\ell} \frac{dI}{dt}$$

$$E = -L \frac{dI}{dt} \quad L = \mu_0 N^2 A / \ell$$

L = Coefficient of self inductance $L = \mu_0 N^2 A / \ell$

$$L = \mu_0 n^2 A \ell$$

Unit of inductance $[H] = ML^2T^{-2}A^{-2}$

CHARGE FLOW DUE TO INDUCED CURRENT

$$q_{flow} = \frac{1}{R} \int d\Phi \text{ (independent of time)}$$

$$q_{flow} = \frac{1}{R} \Delta\Phi_{total} = \frac{L}{R} \Delta I$$

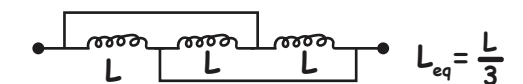
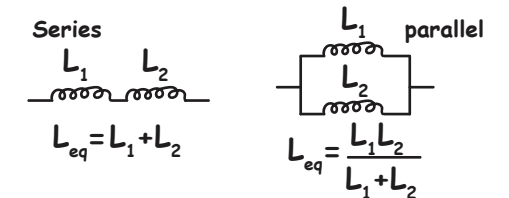
ENERGY STORED IN INDUCTOR

$$U_B = \frac{1}{2} LI^2$$

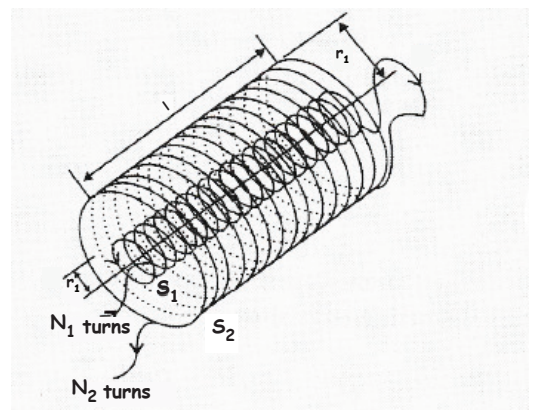
MAGNETIC ENERGY STORED PER UNIT VOLUME

$$u = \frac{U_B}{V} = \frac{U_B}{Al} = \frac{\mu_0 n^2 I^2}{2} = \frac{B^2}{2\mu_0}$$

SERIES & PARALLEL COMBINATION



MUTUAL INDUCTANCE OF TWO COILS



I_2 = Current through outer coil

$$B = \mu_0 n_2 I_2$$

$$\Phi_{12} = \mu_0 n_2 I_2 \times \pi r_1^2 n_1 l$$

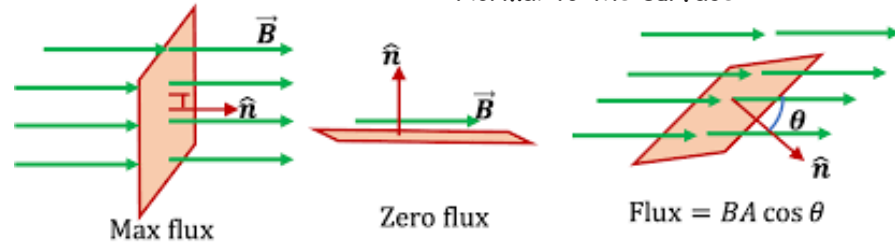
$$M = M_{12} = \mu_0 n_1 n_2 \times \pi r_1^2 l$$

$$= \frac{\mu_0 N_1 N_2}{l} \pi r_1^2$$

ELECTROMAGNETIC INDUCTION

In uniform $\vec{B}$

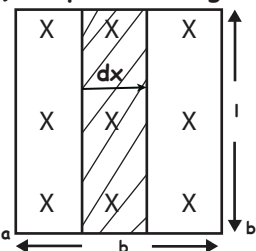
$\hat{n}$ = Normal to the surface



$$\Phi_B = BA \text{ (maximum)}$$

FLUX IN NON-UNIFORM FIELD

1) Steps/or solving



1. Take a small strip 'dx'
2. flux $d\Phi = B dx$ [dA = l dx]
3. Total flux $\Phi = \int d\Phi = \int_0^b B dx$

FARADAY'S LAW

1) Whenever the amount of magnetic flux with a circuit changes, an emf is induced in the circuit

2) The induced EMF is given by rate of change of magnetic flux linked with the circuit

$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt}$$

Negative sign indicates that induced emf opposes the cause of flux change

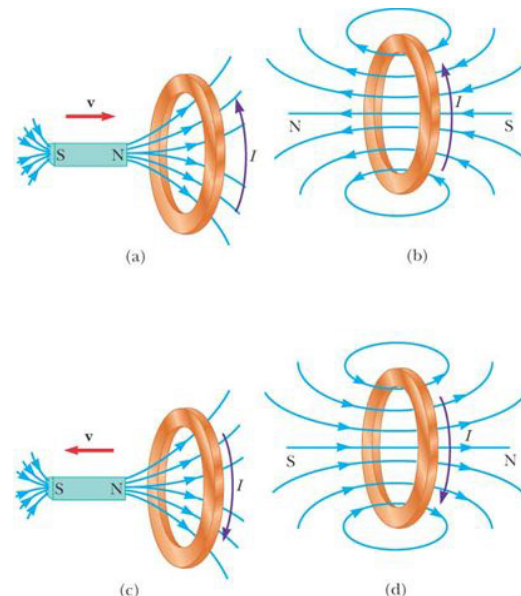
$$\mathcal{E}_{ind} = -\frac{Nd\Phi_B}{dt}$$

LENZ'S LAW & CONSERVATION OF ENERGY

The direction of any magnetic induction effect is such as to oppose the change that produced it

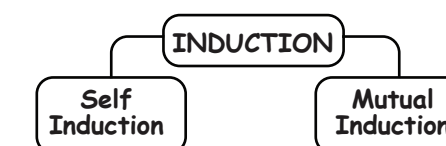
DIRECTION OF INDUCED CURRENT

1. If flux is decreasing the magnetic field due to induced current will be along the existing magnetic field
2. If flux is increasing the magnetic field due to induced current will be opposite to existing magnetic field

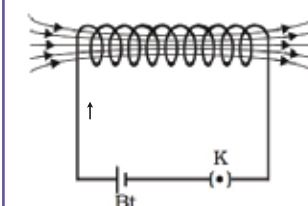


INDUCTANCE

- Scalar quantity
- Unit of inductance (H)
- Dimension : $ML^2T^{-2}A^{-2}$



Self Induction



Current I in the coil changes due to external source

Causes change in magnetic field inside the coil

Results in change in magnetic flux inside the coil

EMF is induced which opposes the changing magnetic flux

Creates an induced current which is opposing in nature

NATURE OF INDUCED CURRENT DUE TO SELF INDUCTION

CASE 1

MUTUAL INDUCTANCE OF SOME STANDARD CASES

$R \gg r$

$$\Phi = BA \quad B = \frac{\mu_0 I}{2R}$$

$$\Phi = \frac{\mu_0 I \times \pi r^2}{2R} = LI$$

$$L = \frac{\mu_0 \times \pi r^2}{2R}$$

$l \gg b$

$$\Phi = BA$$

$$B = \frac{\mu_0 2\sqrt{2}}{\pi l} I$$

$$\Phi = \frac{\mu_0 2\sqrt{2}}{\pi} I \times b^2$$

$$L = \frac{\mu_0 2\sqrt{2}}{\pi l} \times b^2$$

$l \gg b$

$$\Phi = BA$$

$$B = \frac{9}{2} \frac{\mu_0 I}{\pi l}$$

$$\Phi = BA = \frac{9}{2} \frac{\mu_0 I}{\pi l} \times \frac{\sqrt{3}}{4} b^2$$

$$L = \frac{9}{2} \frac{\mu_0}{\pi l} \times \frac{\sqrt{3}}{4} b^2$$

Relation between mutual inductance & self inductance

$$M = K \sqrt{L_1 L_2} \quad 0 \leq K \leq 1$$

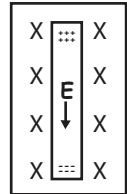
$K \rightarrow$ coefficient of coupling
If $K=1$, Perfect flux linkage

$$M = \sqrt{L_1 L_2}$$

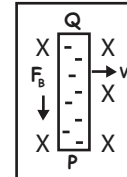
Otherwise $\rightarrow$ imperfect linkage

$$K < 1$$

DYNAMIC MOTIONAL OF EMI DUE TO TRANSLATORY MOTION

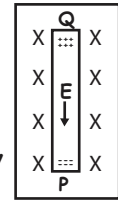


At steady state



$$F_B = F_e \quad \frac{V_{PQ}}{l} = vB$$

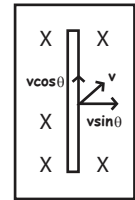
$$E = vB \quad V_{PQ} \text{ or } \mathcal{E} = Blv$$



Direction to find the positive terminal
By using right hand rule,
Thumb - velocity
Fingers - Magnetic field
Palm - Positive terminal of the battery

Modification

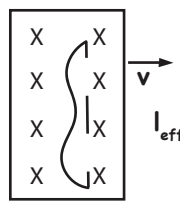
i) Velocity is not perpendicular



$$\mathcal{E} = Blv \sin \theta$$

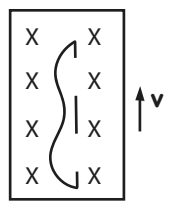
ii) Conductor of arbitrary shape

Perpendicular to effective length

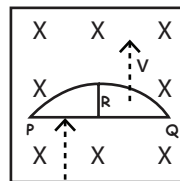


$$\mathcal{E}_{ind} = Bl_{eff} v$$

Parallel to effective length



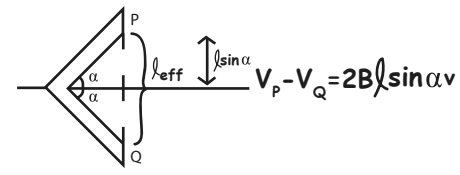
$$\mathcal{E}_{ind} = 0$$



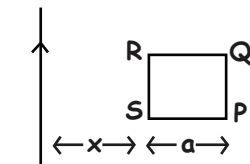
Semi circular loop in a magnetic field
 $l_{eff} = 2R$
 $\mathcal{E} = Bl_{eff} v$
 $\mathcal{E} = B \times 2Rv$
from right hand rule,
P is at higher potential
Q is at lower potential

TRANSLATORY MOTION OF METALLIC FRAME IN UNIFORM / NON UNIFORM MAGNETIC FIELD

Metal frame of different shapes moving in uniform magnetic field



MOVING METAL IN NON UNIFORM MAGNETIC FIELD

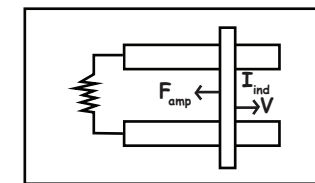


$$B_1 = \frac{\mu_0 I}{2\pi x}, \quad \mathcal{E}_1 = \frac{\mu_0 I}{2\pi x} av$$

$$B_2 = \frac{\mu_0 I}{2\pi(x+a)}, \quad \mathcal{E}_2 = \frac{\mu_0 I}{2\pi(x+a)} av$$

$$\mathcal{E}_{net} = \mathcal{E}_1 - \mathcal{E}_2 = \frac{\mu_0 I a^2 v}{2\pi x(x+a)}$$

INDUCED CURRENT AND AMPERIAN FORCE



● Amperian force

$$F_{amp} = \frac{B^2 l^2 v}{R} \rightarrow \text{Opposes motion}$$

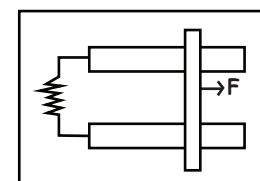
● Work done by amperian force

$$W = \Delta KE = 0 - \frac{1}{2} mv_0^2$$

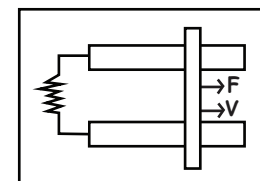
● Power developed in circuit

$$P = I^2 R \quad P = \frac{B^2 l^2 v^2}{R}$$

Terminal velocity



$t=0$
slider at rest



$$t=t \quad F_{amp} = \frac{B^2 l^2 v}{R}$$

INSTANTANEOUS VALUE OF INDUCED EMF

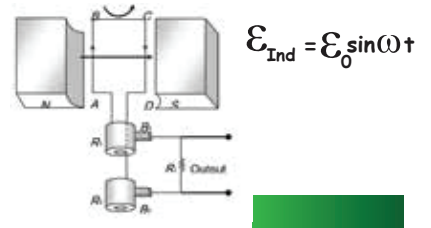
$$\mathcal{E}_{Ind} = NBA \omega \sin(\omega t)$$

$$\mathcal{E}_0 = NBA \omega$$

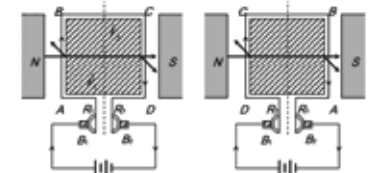
when $\omega t = 0$

$$\mathcal{E}_{Ind} = \mathcal{E}_0 \sin(\omega t)$$

AC GENERATOR



DC MOTOR



electrical machine which converts electrical energy into mechanical energy

EQUATION FOR BACK EMF

$$I = \frac{E - NBA \omega \sin(\omega t)}{R}$$

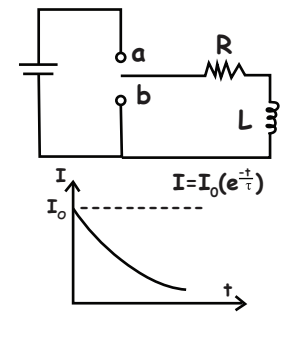
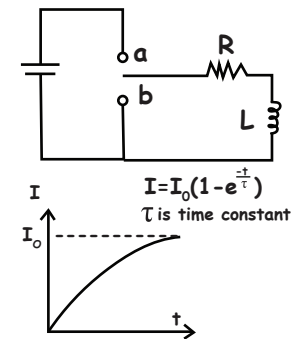
MECHANICAL POWER & EFFICIENCY OF DC MOTOR

$$\eta = \frac{\text{back emf}}{\text{supply voltage}} = \frac{P_{out}}{P_{in}} = \frac{e}{E}$$

L-R CIRCUIT

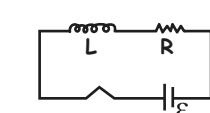
Steady state $\rightarrow$ inductor $\rightarrow$ zero resistance
For current growth in a circuit
- at $t=0$ Inductor offers infinite resistance
- at $t=\infty$ Inductor offers zero resistance

GROWTH & DECAY IN L-R CIRCUIT



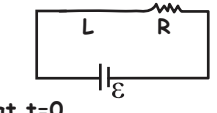
INITIAL & FINAL STATE OF L-R CIRCUIT

Initial state



at $t=0$
Initial state - inductor open
 $\mathcal{E} = V_L$

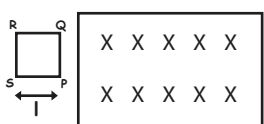
final state



at $t=\infty$
final state - replace inductor with a wire
 $I_0 = \frac{E}{R}$

MOTIONAL EMF : FARADAY'S LAW

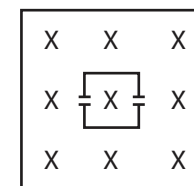
i) At $t=0 \rightarrow$ loop about to centre



iii) Loop has fully entered

$$\mathcal{E}_{Ind} = 0$$

$$I_{ind} = 0$$



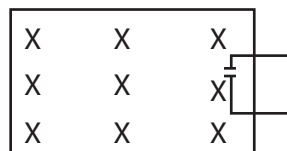
iv) field is decreasing

$$B_{ind} \otimes$$

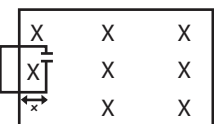
$$I_{ind} \text{ clockwise}$$

$$\mathcal{E}_{Ind} = Blv$$

$$I_{ind} = \frac{Blv}{R}$$



ii) At time $t=t$, 'x' length inside loop about to enter



$$A = lx$$

$$B_{ind} \otimes$$

$$I_{ind} \rightarrow \text{anticlockwise}$$

$$\Phi = BA = Blx$$

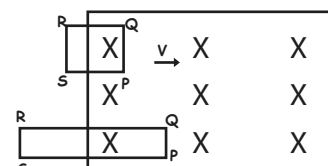
$$x = vt$$

$$\Phi = Blvt$$

$$\mathcal{E}_{Ind} = Blv$$

$$I_{ind} = \frac{\mathcal{E}_{Ind}}{R} = \frac{Blv}{R}$$

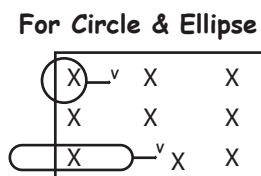
MOTION OF A SQUARE, RECTANGLE, CIRCLE & ELLIPSE IN UNIFORM MAGNETIC FIELD



For square,
 $PQ \rightarrow \mathcal{E} = Blv = \text{Constant}$
 $RQ \text{ \& } SP \rightarrow l_{eff} \text{ Parallel to velocity}$

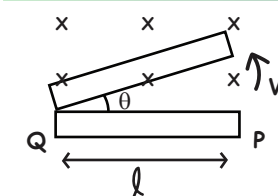
For Rectangle
 $PQ \rightarrow \mathcal{E} = Blv = \text{Constant}$
 $RQ \text{ \& } SP \rightarrow l_{eff} \text{ Parallel to velocity}$

$RS \rightarrow \mathcal{E} = 0$ (Outside B)
Q - Higher potential
P - Lower potential
Current - Anticlockwise direction

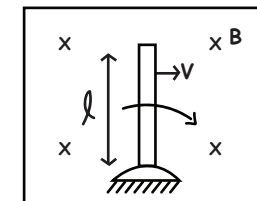


Effective length is constantly varying
Induced EMF is varying (when loop is entering, first increases reaches a maximum and then decreases)
induced current in anticlockwise direction
Induced EMF = $Bl_{eff} v$

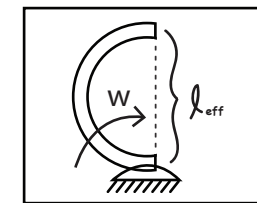
MOTIONAL EMF DUE TO ROTATIONAL MOTION



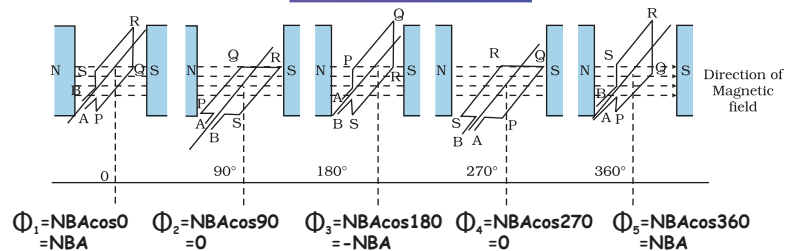
i) $\mathcal{E} = \frac{1}{2} B l^2 \omega$
Upper end is positive terminal



$$ii) \mathcal{E} = \frac{1}{2} 4BR^2 \omega$$



PERIODIC EMI



Average induced emf

$$\text{when coil is rotated from } \theta = 0^\circ \text{ to } 90^\circ \quad \mathcal{E}_{ind} = \frac{2NBA \omega}{\pi}$$

$$\text{when coil is rotated from } \theta = 90^\circ \text{ to } 180^\circ \quad \mathcal{E}_{ind} = \frac{2NBA \omega}{\pi}$$