⚡ ELECTROSTATICS

Physics

Introduction to Charge

🔬 What is Electric Charge?

- **Charge** is the property associated with matter due to which it produces and experiences electrical & magnetic effects.
- Created by **disturbing the neutrality of an atom** (remove/add electrons).
- This leads to gain/loss of charge, and also a very small change in mass (mass of electrons involved).
- SI unit: Coulomb (C) = Ampere × second | **Dimensional formula: [AT]**

> **Key Values:**
> Charge of electron: $e = 1.6 \times 10^{-19}$ C
> Mass of electron: $m_e = 9.1 \times 10^{-31}$ kg
> 1 esu = 3.3×10^{-10} C

📋 Properties of Charge

① Transferability
- A charged body in contact with uncharged body → uncharged body becomes charged.

② Charge Cannot Exist Without Mass

> **Mass can exist without charge, but charge CANNOT exist without mass.**

③ Conservation of Charge
- In a closed system, charge can neither be created nor destroyed. ΣQ = constant
- Charged particles created/destroyed only in pairs with equal & opposite charges.

④ Invariance of Charge (Independence of velocity)

> Charge is **independent of velocity.** Proved by atoms (protons have more mass & velocity but do not have more charge than electrons).

⑤ Charged Particles — Radiation
- Stationary charge → **only Electric field**
- Uniform motion → Electric field + **Magnetic field (unaccelerated)**
- Accelerated → Electric field + Magnetic field + **radiates EM waves**

⑥ Quantisation of Charge

$$Q = \pm ne \quad (n = 1, 2, 3, \ldots)$$

- Charge can only be an **integral multiple of e** (the charge of an electron).

⑦ Additivity of Charge
- Total charge = **algebraic sum** of all charges located anywhere on the body.

Charge is a SCALAR quantity.

🔌 Conductors & Insulators

Conductors	Insulators
• Valence e^- are **loosely bound** • Not permanently attached to atoms • Free electrons → charge moves easily	• Electrons are **tightly bound** • Charge **cannot be readily removed** • e.g. glass, rubber, wood

🔭 Electroscope
- Measures **degree & amount of charge**
- **Divergence** of gold leaves indicates presence & magnitude of charge

Coulomb's Law & Superposition

⚡ Coulomb's Law

$$F = \frac{Kq_1q_2}{r^2}$$

Constants:
$K = 9 \times 10^9$ N·m²·C⁻²
$K = 1 / (4\pi\varepsilon_0)$
$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$
$\varepsilon_0 = 8.85 \times 10^{-12}$ N⁻¹·m⁻²·C²
$\varepsilon_r = 1$ (air/vacuum) | $\varepsilon_r = \infty$ (conductor)

Key Properties
- ✅ **Valid only for point charges**
- Valid only when $r > 10^{-15}$ m
- **Like charges → repel | Unlike charges → attract**

- Action–reaction pair — equal force on both charges
- It is a **conservative force**
- Independent of presence/absence of other charges
- ⚠ Surrounding medium affects force via **polarisation of molecules**

Permittivity of a Medium (ε_r)

Measure of how much a medium is affected by an external field. Higher $\varepsilon_r \rightarrow$ stronger polarising effect $\rightarrow$ reduced force.

Vector Form

$$\vec{F} = \frac{Kq_1q_2}{r^2}\,\hat{r}$$

🔍 Superposition Principle

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

- **At an isolated point: $\vec{F}$_net = 0**
- All forces added as **vectors (component-wise)**

📊 Coulomb vs Gravity

Property	Electrostatic	Gravitational
Law	Inverse square ✅	Inverse square ✅
Charge/Mass	+ve & −ve	Only one kind
Nature	Attractive OR repulsive	Always attractive
Strength	K > G (much stronger)	Weaker
Acts on	Point charges	Point masses

⚖ **K > G → Electrostatic force is MUCH stronger than Gravitational force**

Electric Field

⚡ Electric Field ($\vec{E}$)

Definition: Space around a charge where its influence can be felt by another charged particle.

Basically: Force a unit +ve test charge would experience at that point.

Useful to handle **distributed / continuous bodies.**

Dimensions: $[MLT^{-3}A^{-1}]$ | Units: N/C or V/m

$$\vec{E} = \frac{\vec{F}}{q_0} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

- A charged particle is **NOT affected by its own field.**

📡 Superposition for Field Intensity

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

📐 Continuous Charge Distributions

Concept: A system of closely spaced charges forms a continuous charge distribution.

Take an element, find its $d\vec{E}$, integrate over the body:

$$\vec{E} = \int d\vec{E}$$

Charge Density Definitions

Type	Symbol	Definition
Linear charge	λ (lambda)	Charge / Length
Surface charge	σ (sigma)	Charge / Area
Volume charge	ρ (rho)	Charge / Volume

📐 $\vec{E}$ for Specific Bodies

① Ring of Charge

$$E_x = \frac{\dfrac{1}{4\pi\varepsilon_0} xQ}{(x^2 + a^2)^{3/2}}$$

$$E_{max} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{3\sqrt{3} \cdot a^2}$$

- $E_y = 0$ | $x = a/\sqrt{2}$ gives E_{max}

② Finite Line Charge

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

③ Uniform Disk

$$E_x = \frac{\sigma}{2\varepsilon_0} \cdot [1 - \frac{x}{\sqrt{x^2 + R^2}}]$$

④ Two Infinite Oppositely Charged Sheets

$$E = \frac{\sigma}{\varepsilon_0} \ (between\ the\ sheets)$$

Lines of Force & $\vec{E}$ for Spheres

∼ Lines of Force

Definition: Lines drawn such that the tangent at any point gives the direction of electric field at that point.

Properties
- Direction = direction of force experienced by a **+ve test charge**
- **No. of lines per unit area** ⊥ to lines is proportional to **magnitude of E**
- Field lines do NOT originate or terminate in free space surrounding a charge
- ✅ **Two electric field lines NEVER cross each other**
- Field lines **don't pass through conductors** — perpendicular at surface
- Tangent gives **direction of force & acceleration** on a +ve charge
- Trajectory of +ve charge ≠ field line (unless field line is straight)

Thin Spherical Shell	Conducting Solid Sphere
(i) Outside (r > R)	**(i) Inside:**
$$E = \frac{KQ}{r^2} \ (r > R)$$	$$E = 0$$
(ii) On Surface	**(ii) Outside:**
$$E = \frac{KQ}{R^2}$$	$$E = \frac{kQ}{R^2}$$
(iii) Inside (r < R)	**(iii) Surface:**
$$E = 0$$	$$E = \frac{kQ}{R^2}$$

Solid Non-Conducting Sphere

(i) Outside: Same as point charge:

$$E = \frac{KQ}{r^2}$$

(ii) Inside:

$$E = \frac{KQ(r)}{R^3}$$

(iii) Surface:

$$E = \frac{KQ}{R^2}$$

∞ Infinite Sheet of Charge

$$E_z = \frac{\sigma}{2\varepsilon_0} \ (independent \ of \ x)$$

- Field is **uniform & independent of distance** from the sheet.
- $E_y = 0$

② Semi-infinite Rod

$$E_x = E_y = \frac{\lambda}{4\pi\varepsilon_0\, r_0}$$

Electric Dipole

🧲 Electric Dipole

Definition: 2 equal and opposite point charges separated by a very small finite distance.
Diagram: (−q) ←—— 2l ——→ (+q)
Centre = midpoint (equatorial point)

Dipole Moment ($\vec{p}$)

$$\vec{p} = q \cdot (2l)$$

- Direction: **−ve to +ve** (from −q to +q)
- Vector quantity
- If more than 2 charges: take the 'centre of charge' to define dipole moment

🔬 Electric Field Due to a Dipole

Axial Position (along axis of dipole, ∥ to $\vec{p}$)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - l^2)^2}$$

- Direction: **parallel to $\vec{p}$**
- Distance r measured from **centre of dipole**

Equatorial Position (⊥ to $\vec{p}$)

$$\vec{E} = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}}{(r^2 + l^2)^{3/2}}$$

- Direction: **antiparallel to $\vec{p}$**

General Point

Resolve $\vec{p}$ into two components:
- $P_1 = P \cdot \cos\theta \rightarrow$ axial component (along r)
- $P_2 = P \cdot \sin\theta \rightarrow$ equatorial component ($\perp$ to r)

Combine using field equations:

$$E = \frac{p}{4\pi\varepsilon_0 r^3} \cdot \sqrt{1 + 3cos^2\theta}$$

$$tan\,\varphi = \frac{1}{2}\,tan\theta$$

- φ = angle between $\vec{p}$ and the line joining the centre of dipole to the point A in the field $\vec{E}$

⚡ Dipole in Uniform Electric Field

$$\vec{\tau} = \vec{P} \times \vec{E}$$

- Net force = 0 (equal & opposite forces on +q and −q)
- Net **torque ≠ 0** — tends to align the dipole along $\vec{E}$

🌀 Dipole in Non-Uniform Electric Field

Net force depends on:
- ① Orientation of dipole
- ② Dipole moment ($\vec{p}$)
- ③ Variation of $\vec{E}$

$$|\vec{F}| = p \cdot \frac{dE}{dx}$$

🔖 Quick Recap

- **Charge properties:** Transferable, conserved, quantised (Q=ne), additive, scalar
- **Quantisation:** $Q = \pm ne$ | **Conservation:** ΣQ = constant
- **Coulomb:** $F = Kq_1q_2/r^2$ — conservative, action-reaction, valid $r > 10^{-15}$ m
- $\vec{E} = \vec{F}/q_o$ | Superposition applies | $[MLT^{-3}A^{-1}]$
- **Dipole moment:** $\vec{p} = q(2l)$ direction: −q to +q
- **Axial E:** $\propto 2p/r^3$ | **Equatorial E:** $\propto p/r^3$ | **Torque:** $\tau = pE\,sin\theta$
- **Field lines NEVER cross.** E = 0 inside conductors & hollow shells.
- **K > G** → Electrostatic force >> Gravitational force